An Axiomatic and Computational Analysis of Altruism, Fairness, and Stability in Coalition Formation Games

Inaugural-Dissertation

zur Erlangung des Doktorgrades
der Mathematisch-Naturwissenschaftlichen Fakultät
der Heinrich-Heine-Universität Düsseldorf

vorgelegt von

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Düsseldorf, Mai 2022

aus dem Institut für Informatik
der Heinrich-Heine-Universität Düsseldorf

Gedruckt mit der Genehmigung der
Mathematisch-Naturwissenschaftlichen Fakultät der
Heinrich-Heine-Universität Düsseldorf

Berichterstatter:

1. Prof. Dr. Jörg Rothe

2. Jun.-Prof. Dr. Robert Bredereck

3. Prof. Dr. Martin Hoefer

Tag der mündlichen Prüfung: 28. September 2022

Acknowledgements

First of all, I would like to thank my supervisor Jörg Rothe who has introduced me to the world of scientific research and guided me through my whole doctorate time. Without his guidance and constant encouragement, the publication of so many papers and the writing of this thesis would not have been possible. Also, Jörg's effort and expertise have enabled me to become a better researcher. Next, I would like to express my gratitude to Dorothea Baumeister. I'm thankful for her ongoing support, her willingness to listen to all my problems, and all her constructive and detailed feedback. Moreover, I am much obliged to my co-authors who have spared no effort to improve our joint research.

Shortly after the start of my doctoral time I was given the great opportunity to join the "NRW-Forschungskolleg Online-Partizipation," a PhD-program which is supported by the North Rhine-Westphalian funding scheme "Forschungskollegs." I very much appreciate my time as a member of the program, during which I was able to profit from its interdisciplinary scope. I thank the coordinators for their guidance and all other PhD students from the program for giving me interesting insights into their research.

Many thanks go to my colleagues who enriched my time as a doctoral student. I very much enjoyed the supportive atmosphere of our group. And I feel lucky that I was able to share an office with some of my colleagues for quite some time before the COVID-19 pandemic.

My family and friends deserve much thanks for their ongoing love and support. Particularly, I would like to thank my father for sparking my interest in computer science, my mother for always being there for me and helping me to reach all my goals, and my siblings, David and Kathi, for playing dozens of card games with me. Above all, I owe my deepest gratitude to Marcel for making my whole life so enjoyable. Thank you for all your love and affection, your faith in my abilities, and for going on so many fun adventures with me. Last but not least, I would like to thank my friends for proofreading parts of my thesis, inviting me to many exciting climbing trips, providing me with amusing riddles, and spending numerous great days together.

Zusammenfassung

Diese Arbeit beschäftigt sich mit Koalitionsbildungsspielen, welche zum Forschungsbereich der kooperativen Spieltheorie gehören. Bei diesen Spielen geht es darum, wie sich Spieler auf Grundlage ihrer individuellen Präferenzen in Gruppen, auch Koalitionen genannt, aufteilen. Bei unseren Untersuchungen konzentrieren wir uns größtenteils auf hedonische Koalitionsbildungsspiele, kurz hedonische Spiele, bei welchen vorausgesetzt wird, dass die Präferenzen der Spieler nur von ihren eigenen Koalitionen abhängen. Ein zentrales Thema in Bezug auf diese Spiele ist die Suche nach sinnvollen Formaten zur Abgabe der Präferenzen. Diese Formate sollten einfach zu erheben, möglichst ausdrucksstark und zugleich kompakt darstellbar sein. In der einschlägigen Literatur wurden bereits einige solcher Formate vorgestellt, die auch wir in dieser Arbeit behandeln werden. Ein zweiter wichtiger Punkt bei der Erforschung von hedonischen Spielen ist die Untersuchung von Stabilität, Fairness und Optimalität. Klassische Stabilitätskonzepte behandeln beispielsweise die Frage, ob einzelne Spieler oder Gruppen von Spielern einen Anreiz haben, von ihren Koalitionen abzuweichen. Zu den bekanntesten solcher Konzepte gehören Nash-Stabilität und Kernstabilität.

Auf Grundlage des aktuellen Stands der Literatur führen wir in dieser Arbeit neue Modelle für (hedonische) Koalitionsbildungsspiele ein und untersuchen diese im Hinblick auf axiomatische Eigenschaften, Stabilität, Fairness und Optimalität. Dabei spielen insbesondere Untersuchungen der Berechnungskomplexität eine wichtige Rolle.

Zuerst stellen wir verschiedene Modelle für Altruismus in Koalitionsbildungsspielen vor. Wir konzentrieren uns dabei zunächst auf den Kontext von hedonischen Spielen und erweitern die Modelle anschließend auf allgemeinere Koalitionsbildungsspiele, bei denen eine weitreichendere Form des Altruismus' möglich ist. Wir untersuchen unsere Modelle axiomatisch und vergleichen diese dabei untereinander und mit anderen Modellen. Zudem analysieren wir die Entscheidungsprobleme, die sich bei der Betrachtung klassischer Stabilitätskonzepte im Kontext von altruistischen Spielen ergeben, in Hinblick auf ihre Berechnungskomplexität.

Anschließend definieren wir drei schwellwertbasierte Fairnessbegriffe für hedonische Spiele. Diese werden in den Kontext einschlägiger Stabilitäts- und Fairnesskonzepte eingeordnet und im Hinblick auf ihre Berechnungskomplexität erforscht. Außerdem untersuchen wir den Einfluss, den unsere Fairnesskonzepte auf die soziale Wohlfahrt haben.

Schließlich führen wir ein weiteres Präferenzformat ein, bei dem die Spieler zwischen Freunden, neutralen Spielern und Feinden unterscheiden. Sie geben dementsprechend eine dreigeteilte schwache Ordnung ab. Da die Präferenzen, welche sich aus diesen Ordnungen ableiten lassen, nicht vollständig sein müssen, unterscheiden wir in den entstehenden Spielen zwischen möglicher und notwendiger Stabilität. Auch hier führen wir eine Komplexitätsanalyse der Probleme durch, die sich bezüglich bekannter Stabilitätskonzepte ergeben.

Abstract

This thesis deals with coalition formation games, which belong to the research area of cooperative game theory. In these games, players divide into groups, also called coalitions, based on their individual preferences. In our research, we mainly focus on hedonic coalition formation games, hedonic games for short, in which players' preferences are assumed to depend only on the coalitions containing themselves. A central problem in hedonic games research is finding reasonable formats for the elicitation of preferences. These preference representations should be easy to elicit, reasonably expressive, and succinct. Many such formats have already been presented in related literature, some of which we will also discuss in this thesis. A second central point in research concerning hedonic games is the investigation of stability, fairness, and optimality. For instance, common stability concepts deal with the question of whether individual players or groups of players might have an incentive to deviate from their current coalitions. Among those notions are, for example, Nash and core stability.

Based on the current state of research, we introduce new models for (hedonic) coalition formation games and investigate them with respect to axiomatic properties, stability, fairness, and optimality. In particular, investigations of the computational complexity of the associated decision problems play an important role.

We start with introducing several models for altruism in coalition formation games. First, we focus on the context of hedonic games and then extend the models to more general coalition formation games, where a broader form of altruism is possible. We conduct an axiomatic analysis of our models and compare them to related models and to each other. In addition, we study the problems, that arise when considering classical stability concepts in the context of altruistic coalition formation games, with respect to their computational complexity.

Subsequently, we define three threshold-based fairness notions for hedonic games. These notions are considered local fairness notions in the sense that the agents only have to inspect their own coalitions to decide whether a coalition structure is fair to them. We study the relations of these notions to other common stability and fairness concepts and examine them with respect to their computational complexity. Furthermore, we investigate the price of local fairness, i.e., the impact that our fairness concepts have on the social welfare.

Finally, we introduce another preference format in which players distinguish between friends, neutral players, and enemies. Accordingly, they cast their preferences by submitting a weak rankings that is separated by two thresholds. Since the preferences that can be derived from these rankings are not necessarily complete, we distinguish between possible and necessary stability in the resulting games. Again, we perform a computational complexity analysis of the problems that arise with respect to common stability concepts.

Contents

Chapter 1

Introduction

Nowadays, there is an enormous range of research that is concerned with topics of artificial intelligence (AI). In fact, AI research is not only about mimicking human intelligence but also about a variety of solution concepts that apply knowledge from different fields of science including not only natural sciences such as biology and physics but also social sciences such as sociology and economics. Two fields that have gained major interest on AI conferences are *multiagent systems* and *game theory*. While the research concerning these areas is very broad, there is not always a clear distinction between them.

Research concerning multiagent systems mainly deals with distributed problem solving, i.e, the cooperation of agents whose aim is to collectively solve some problem. Those systems are often applied to problems that might be more difficult or not at all solvable for a single agent. Inspiring examples of very successful multiagent systems can be found in nature: Ant colonies use their communication abilities and division of labor to master complex problems that would never be feasible for a single ant.

Game theory deals with the interaction among individual agents which are mostly assumed to be selfishly pursuing their own goals. Research in this area roughly started with the works due to Borel [22], Neumann [103], and Neumann and Morgenstern [104] and is commonly divided into *noncooperative* and *cooperative* game theory. While noncooperative game theory focuses on the preferences and actions of individual agents, cooperative game theory also sees individual preferences but rather focuses on the formation of groups and allows them to take joint actions. Examples of noncooperative game theory include the famous *prisoners' dilemma* [118], the *Monty Hall problem* (see, e.g., Selvin [131, 132] or the German book by Randow [120]), but also classic combinatorial games such as tic-tac-toe, nim, chess, go, or sudoku. The focus of noncooperative game theory is mainly on studying equilibria, i.e., stable states where no agent has a reason to deviate from her current strategy. In cooperative games, agents may form coalitions and take joint actions. For more background on multiagent systems and game theory see, e.g., the textbooks by Shoham and Leyton-Brown [133] and by Rothe [126].

The focus of this thesis is on *coalition formation games* which are a key topic in cooperative game theory. Their applications range from technical, engineering, and economic problems to social and even political problems. Drèze and Greenberg [53] initiated the study of coalition formation games with *hedonic* preferences. These games were later formalized by Banerjee et al. [15] and Bogomolnaia and Jackson [21]. The key idea of such games is that agents have to form partitions while only caring about the coalitions that they are part of. In the general framework of hedonic games, the agents have arbitrary preferences over all coalitions containing themselves. Yet, it is not reasonable to elicit rankings over all such coalitions in practice. Rather, reasonable preference representations are needed. Ideally, such formats should be succinct, expressive, and easy to elicit. The determination of reasonable preference representations has been a fundamental part of hedonic games research. Well-established representations include cardinal formats such as the *additive encoding* due to Bogomolnaia and Jackson [21] and the *fractional encoding* by Aziz et al. [11]. Other formats are based on the partitioning of the agents into friends and enemies [50, 111, 17] or on the usage of propositional formulas [56, 9].

Another crucial branch of hedonic games research addresses problems related to notions of *stability*, *fairness*, and *optimality*. The determination of such notions constitutes a major part of past research. In particular, several common notions of stability deal with single player deviations. For instance, a partition of the agents in a hedonic game is said to be *Nash stable* (or in a Nash equilibrium) if no agent wants to deviate to another coalition of the partition [21]. Other stability notions concern the deviation of groups. *Core stability* is probably the most important notion of group stability in hedonic games (see, e.g., the early paper by Banerjee et al. [15] and the survey by Woeginger [147]). Informally, a group of players blocks a given partition of the players with respect to the notion of core stability if all players in this group prefer it to the groups assigned by the partition. A partition is core stable (or *in the core*) if there is no blocking coalition [15]. Relevant notions of optimality include *Pareto-optimality*, *popularity*, and the maximization of *utilitarian* or *egalitarian social welfare*. Interesting notions of fairness, for instance *envy-freeness*, have been adopted from the research of fair division and resource allocation (see Foley [61] and the book chapters by Bouveret et al. [24] and by Lang and Rothe [95] for background on these topics).

Given such notions of stability, optimality, or fairness, we are interested in the identification of sufficient conditions for such notions, i.e., we ask which properties guarantee the stability, fairness, or optimality of outcomes. Also, stable, fair, or optimal outcomes might not even exist for certain preference profiles. A decent amount of research has been focusing on identifying properties that guarantee the existence of such outcomes. For instance, Bogomolnaia and Jackson [21] showed that Nash stable coalition structure are guaranteed to exist in symmetric additively separable hedonic games. Yet, it was later shown that deciding whether a Nash stable coalition structure exists in an (asymmetric) additively separable hedonic game is NP-complete [136]. Determining the complexity of such existence problems has generally been an important research aspect. For core stability and strict core stability, the existence problem has been proved to be Σ_2^p-complete for additively separable hedonic games [146, 116, 111]. Yet, there again exist conditions that simplify the existence problem. Burani and Zwicker [35] have shown that there always exist core stable outcomes for symmetric additively separable hedonic games with *purely cardinal* preferences. Dimitrov et al. [50] proved that the existence problem is trivial for friend-oriented and enemy-oriented hedonic games.

In this thesis, we build on the current state of research and introduce further succinct preference representations. Tackling the problem of finding an expressive, compact, and easy to elicit preference format, we introduce *weak rankings with double thresholds*. These rankings are more expressive than purely ordinal rankings (the *individually rational encoding* [14]), the *friends-and-enemies encoding* [50], or the *singleton encoding* [40]. Yet, our format is cognitively plausible and easy to elicit from the agents — probably easier than, for example, propositional formulas (as in the case of *hedonic coalition nets* [56] or the *boolean hedonic encoding* [9]). Furthermore, we do not make strong assumptions on the nature of the preferences (as the *anonymous encoding* [15] which only takes coalition sizes into account or the boolean hedonic encoding which was designed for dichotomous preferences) and our format is succinct. In conclusion, our format provides a satisfactory balance between the three requirements.

A second important aspect that we tackle in this thesis leads to a new branch of preference modeling. Since the beginnings of game theory, agents were commonly considered as completely rational and self-interested individuals (see Neumann and Morgenstern [104]). We challenge this assumption and aim for a more realistic representation of real-world coalition formation scenarios: We introduce *altruism* into coalition formation games. In our models, agents are not narrowly selfish but take the opinions of their friends into account when comparing different coalition structures. We present a variety of altruistic models and compare them with regard to their axiomatic properties. After concentrating on hedonic models, we also introduce models of altruism that drop the hedonic restriction. The changes we make for these nonhedonic models bring some axiomatic improvements and, in our opinion, an even more realistic model of altruism.

A third part of this thesis is concerned with fairness in hedonic games. Previous literature considers *envy-freeness* as a notion of fairness [21, 10, 148, 114]. Yet, to verify this notion, agents have to inspect the coalitions of other agents. To some extend, this is in conflict with the hedonic assumption which states that agents only care about their own coalitions. Furthermore, we want to avoid the need to compare large numbers of coalitions. Hence, we introduce three notions of *local fairness* that can be decided solely based on the agents' own coalitions and their preferences.

Besides these conceptual contributions, this thesis also contains several technical contributions. We investigate the *FEN-hedonic games* that result from lifting weak rankings with double thresholds to preferences over coalitions. We characterize stability in these games and study the problems of verifying stable coalition structures and of checking their existence. Furthermore, we not only axiomatically study altruistic coalition formation games but also provide elaborate computational analyses of the associated stability verification and existence problems. Our results cover many common stability notions such as Nash stability, core stability, Pareto optimality, and popularity. Concerning our notions of local fairness, we determine the complexity of computing local fairness thresholds and deciding whether locally fair coalitions structures exist for additively separable hedonic games. Moreover, we study the price of our local fairness notions.

1.1 Outline

In Chapter 2, we provide the required background for this thesis and explain all concepts that are needed to comprehend the following chapters. The provided background includes an introduction to computational complexity in Section 2.1, a brief overview of graph theory in Section 2.2, and a survey of the relevant aspects of coalition formation games in Section 2.3. This survey contains not only basic definitions and observations but also references to related work. The major part of our research starts in Chapter 3 where we study different aspects of altruism in coalition formation games. More precisely, Chapter 3 divides into three sections. First, we explore altruistic hedonic games in Section 3.1. After introducing such games, we conduct an axiomatic analysis of our altruistic models and investigate the problems of verifying stable outcomes and of deciding whether stable outcomes exist in such games. In Section 3.2, we further analyze altruistic hedonic games while concentrating on the notions of popularity and strict popularity. Subsequently, we study altruism in a more general scope of coalition formation games. In particular, Section 3.3 expounds an altruistic coalition formation model which is not restricted to hedonic preferences but allows for a more far-reaching altruistic behavior. We identify some advantages that this extended model offers compared to the altruistic hedonic model and study stability in these games. In Chapter 4, we continue with research concerning notions of local fairness in hedonic games. After proposing three such notions, we relate them to other popular notions of stability, determine the computational complexity of the associated decision problems, and study the price of our local fairness notions. In Chapter 5, we introduce and study FEN-hedonic games where agents divide the other agents into friends, enemies, and neutral players while additionally ranking their friends and enemies respectively. We then investigate problems concerning the verification and existence of possibly or necessarily stable coalition structures. We conclude with Chapter 6 where we recap this thesis, highlight some important contributions, and identify some possible directions for future research.

Chapter 2

Background

In this chapter, we provide background information for all subjects studied in the following chapters. We illustrate the essential models and notions that are important for understanding this thesis. We start with an introduction to computational complexity theory in Section 2.1. In Section 2.2, we provide a brief introduction to graph theory. Furthermore, we give an insight into coalition formation in Section 2.3. For literature on the more general topic of cooperative game theory, see the textbooks by Chalkiadakis et al. [41], Shoham and Leyton-Brown [133], Peleg and Sudhölter [113], or the book chapters by Elkind and Rothe [55] and Elkind et al. [57].

2.1 Computational Complexity

A main part of this thesis will be the study of different decision problems and the determination of their computational complexity. But what is a decision problem, how do we measure its complexity, and what does it mean that a problem is 'hard' or 'easy'? We will answer these and other questions in the following section and give a short introduction to computational complexity theory. For more background on this topic we refer to the textbooks by Rothe [125, 128], Papadimitriou [112], and Arora and Barak [4].

2.1.1 Computational Problems

The objective of computational complexity theory is to classify *computational problems* based on their difficulty. In general, a computational problem can be any kind of problem that could be solved by a computer. There are different types of computational problems such as *decision problems*, *optimization problems*, and *search problems*. In this section, we will concentrate on decision problems which are basically questions that can be answered either by yes or no. We will represent any decision problem by specifying its name, an input format, and a question concerning the input. One of the most important decision problems in computational complexity theory is the boolean satisfiability problem (SAT) [66] which asks whether there is a truth assignment for a given boolean formula:[1]

SATISFIABILITY (SAT)	
Given:	A boolean formula φ in conjunctive normal form.
Question:	Is there a truth assignment for the variables in φ that satisfies φ?

Now, given any decision problem, any concrete input that satisfies the specified input requirements is called an *instance* of the problem. An instance is a *yes-instance* if and only if the answer to the specified question is 'yes' for this instance. Otherwise, the instance is called *no-instance*. Decision problems can also be represented by the set of their yes-instances. For example, SAT can be written as

$$\text{SAT} = \{\varphi \mid \varphi \text{ is a satisfiable boolean formula in conjunctive normal form }\}.$$

2.1.2 Algorithms, Runtimes, and Complexity Classes

In computer science, we use algorithms to solve problems.[2] Informally, a *deterministic algorithm* for problem A is a finite sequence of explicit instructions that, when executed for a given input I, outputs the answer to problem A for input I. Formally, algorithms can be modeled via *Turing machines* which were invented by Turing [141, 142] in 1936. We will not give a formal definition of Turing machines here but give some intuitive explanations instead. We refer to the textbooks by Rothe [125, 128] and Papadimitriou [112] for more background on Turing machines.

A Turing machine M that solves a problem A can be started with any instance I of the problem. Starting with an initial configuration that is based on the input I, the Turing machine M then does some computations which lead to further configurations. After a finite number of computation steps, M might reach a final configuration where it *accepts* the input I. The set of all inputs that M accepts is called the *language of* M and is denoted by $L(M)$. We say that M *accepts* the problem A if it accepts all its yes-instances and none of its no-instances, i.e., if $L(M) = A$.

We further distinguish between deterministic and nondeterministic Turing machines. *Deterministic Turing machines* (DTMs) represent deterministic algorithms and the computation of a DTM is a deterministic sequence of configurations. That means that its computation can be represented by a single unique path of configurations and it accepts the input exactly if it accepts the input on this one path. For a DTM M that accept language $L(M)$, we also say that it *decides* the problem $L(M)$. In contrast to that, *nondeterministic Turing machines* (NTMs) represent nondeterministic algorithms and can have more than one computation path. In the

[1] For some background on boolean formulas and propositional logic see the textbooks by Rothe [125, 128].

[2] There are problems that are not solvable by algorithms, e.g., the halting problem, but we will only concentrate on solvable problems in this thesis.

computation of a NTM, there can occur configurations for which the next computation step is not unique; rather, there might be multiple possible successor configurations. In this case, the computation can be represented by a tree where every fork of the tree represents a non-deterministic situation. For a NTM M, we say that M accepts the input I if I is accepted on at least one path of the computation tree.

When developing algorithms[3] for a given problem, there will certainly be more than one possible solution. So the following questions arise: What is the best algorithm for the given problem? And how do we even compare two algorithms? In computational complexity theory, we compare algorithms based on their *computation times* (or *runtimes*) and *space requirements*. The runtime is measured by the number of elementary computational steps that are needed when executing the algorithm. The space requirements are measured by the size of the memory that is used while executing the algorithm. In this thesis, we will concentrate only on the runtime of algorithms.

Now, the goal of algorithmics is to find algorithms that have low runtimes. But of course, the runtime of an algorithm may vary based on the concrete input instance. For example, algorithm M might solve a given problem faster than algorithm N for a given instance while N solves the same problem faster than M for another instance. So, how do we compare the runtimes of these two algorithms? Common answers to this question are to compare either the *best-case*, *average-case*, or *worst-case runtimes*. In this thesis, we will concentrate on the latter. This means that we are looking for upper bounds on the runtimes of algorithms. Further, we always measure runtimes based on the size of the input (which is usually encoded in binary). We can then group problems into *complexity classes* which specify upper bounds on their *worst-case time complexity*, i.e., given a problem A, we ask for the maximal number of computation steps that the fastest algorithm solving A might need for any instance of A.

For any computable total function $f : \mathbb{N} \to \mathbb{N}$, we define

- $\mathrm{DTIME}(f)$ as the set of all problems A for which there exists a DTM M with $L(M) = A$ that decides any instance of A of size n in time at most $f(n)$; and
- $\mathrm{NTIME}(f)$ as the set of all problems A for which there exists an NTM M with $L(M) = A$ that accepts any yes-instance of A of size n in time at most $f(n)$.

As we are only interested in the asymptotic behavior of the worst-case runtime, we group complexity classes with similar functions together. In particular, we denote the set of all polynomial functions[4] by $\mathbb{P}\mathrm{ol}$ and define all problems that can be decided by a DTM (respectively NTM) in polynomial time as follows:

$$\mathrm{P} = \bigcup_{f \in \mathbb{P}\mathrm{ol}} \mathrm{DTIME}(f) \qquad \text{and} \qquad \mathrm{NP} = \bigcup_{f \in \mathbb{P}\mathrm{ol}} \mathrm{NTIME}(f).$$

[3] Keep in mind that we use Turing machines as models of algorithms.

[4] A polynomial function f has the form $f(n) = \sum_{i=0}^{m} c_i \cdot n^i = c_m \cdot n^m + \ldots + c_2 \cdot n^2 + c_1 \cdot n + c_0$ where $n, m, c_m, \ldots, c_0 \in \mathbb{N}$.

As each DTM is also a NTM (that just does not make use of the nondeterminism), it directly follows from the definitions of P and NP that $\mathrm{P} \subseteq \mathrm{NP}$. However, it is still unknown whether $\mathrm{P} \subset \mathrm{NP}$ or $\mathrm{P} = \mathrm{NP}$ holds. This open question is known as the "P versus NP problem" [45] and is one of the seven Millennium Prize Problems [36] that were declared by the Clay Mathematics Institute in 2000. In this thesis, we will follow the common belief and assume that $\mathrm{P} \subset \mathrm{NP}$. Actually, if $\mathrm{P} = \mathrm{NP}$ would hold, many results from the literature would become meaningless. For examples of such results and further discussions on the P versus NP problem see, e.g., the book chapters by Rothe [127] and Arora and Barak [5].

For all problems in P we also say that they are *solvable in (deterministic) polynomial time* and that they can be solved *efficiently*. We sometimes also say that these problems are *easy*. For all problems that are not in P, i.e., not solvable in deterministic polynomial time, we say that they are *computationally intractable*.[5]

Additionally to the two well-known complexity classes P and NP, we will also consider the complexity class coNP which is the class of the complements of all problems in NP. It is defined as

$$\mathrm{coNP} = \{\bar{A} \mid A \in \mathrm{NP}\}$$

where, for any decision problem A, the *complement* of A is defined by

$$\bar{A} = \{I \mid I \text{ is a no-instance of } A\}.$$

Note that NP can analogously be defined as the set of all decision problems for which a yes-instance can be verified in deterministic polynomial time while coNP can be defined as the set of all decision problems for which a no-instance can be verified in deterministic polynomial time.

2.1.3 Polynomial-Time Many-One Reducibility and Hardness

We have just seen that complexity classes specify upper bounds on the complexity of the contained problems. This subsection will be on how to specify lower bounds on the complexity of problems. We use *reductions* to show that one problem is at least as complex (or *hard*) as another one.

We say that problem A is *polynomial-time many-one reducible* to problem B (denoted by $A \leq_m^p B$) if and only if there exists a polynomial-time computable total function f that maps instances of A to instances of B such that, for every instance I of A,

$$I \in A \Longleftrightarrow f(I) \in B,$$

i.e., I is a yes-instance of A if and only if $f(I)$ is a yes-instance of B. Note that the relation $\leq_m^p$ is reflexive (i.e., $A \leq_m^p A$ for any problem A) and transitive (i.e., $A \leq_m^p C$ for any problems A, B,

[5] Cobham [43] and Edmonds [54] were the first to identify the set of *tractable problems* with the class P (see the *Cobham-Edmonds thesis*).

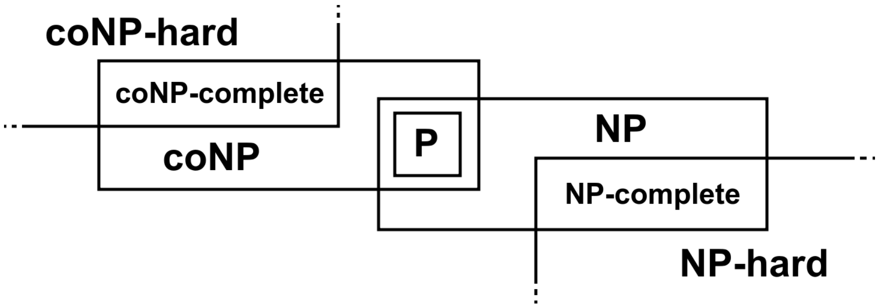

Figure 2.1: Assumed relations among the complexity classes P, NP, and coNP

and C with $A \leq_m^p B$ and $B \leq_m^p C$). Furthermore, a problem is said to be *hard* for a complexity class if it is at least as hard as every other problem in the class. Problems that are hard for a class and are also contained in it are called *complete* for the class. Formally, problem A is $\leq_m^p$-hard for a complexity class $\mathscr{C}$ if $B \leq_m^p A$ for every problem B in $\mathscr{C}$. We then also say that A is $\mathscr{C}$-hard. Moreover, problem A is $\leq_m^p$-complete for the complexity class $\mathscr{C}$ if $A \in \mathscr{C}$ and A is $\mathscr{C}$-hard. We then also say that A is $\mathscr{C}$-complete. The complexity classes P, NP, and coNP are closed under $\leq_m^p$-reducibility which means that for $\mathscr{C} \in \{\mathrm{P}, \mathrm{NP}, \mathrm{coNP}\}$ and any two problems A and B, $A \leq_m^p B$ and $B \in \mathscr{C}$ implies $A \in \mathscr{C}$.

Note that SAT was the first problem that was shown to be NP-complete. This was shown independently by Cook [44] in 1971 and by Levin [97] in 1973. While they had to use quite sophisticated constructions to show the hardness of SAT,[6] we will use the following helpful implications to proof the hardness of problems. They follow directly from the transitivity of $\leq_m^p$ and because P is closed under $\leq_m^p$:

- For any complexity class $\mathscr{C}$, if A is $\mathscr{C}$-hard and $A \leq_m^p B$ then B is $\mathscr{C}$-hard.
- If problem A is NP-complete or coNP-complete, then $A \in \mathrm{P}$ if and only if $\mathrm{P} = \mathrm{NP}$.

By the second implication and under the assumption that $\mathrm{P} \neq \mathrm{NP}$, there is no polynomial-time algorithm for any NP-complete or coNP-complete problem. Assuming that $\mathrm{P} \neq \mathrm{NP}$, $\mathrm{NP} \neq \mathrm{coNP}$, and $\mathrm{P} \neq \mathrm{NP} \cap \mathrm{coNP}$, the relations between the three complexity classes and the sets of NP-hard and coNP-hard problems can be illustrated as shown in Figure 2.1.

2.1.4 Some Hard Problems

The list of decision problems that have been shown to be NP-complete grows steadily since the Cook-Levin theorem [44, 97] was published. For instance, Karp [82] showed the NP-completeness of many problems and a collection of many hard problems can be found in the book by Garey and Johnson [66]. We will now present some decision problems that will be used in this thesis.

[6] You can find proofs of the *Cook-Levin theorem*, e.g., in Garey and Johnson [66, Section 2.6] or Rothe [125, Section 3.5.3].

One of the decision problems that Karp [82] showed to be NP-complete is EXACT COVER BY 3-SETS (X3C). It is defined as follows.

EXACT COVER BY 3-SETS (X3C)	
Given:	Integers $k \geq 2$ and $m \geq 2$, a set $B = \{b_1, \ldots, b_{3k}\}$, and a collection $\mathscr{S} = \{S_1, \ldots, S_m\}$ of 3-element subsets of B.
Question:	Is there an exact cover of B in $\mathscr{S}$, i.e., a subset $\mathscr{S}' \subseteq \mathscr{S}$ of size k such that every element of B occurs in exactly one set in $\mathscr{S}'$?

We will often make use of a restricted version of X3C. Gonzalez [68] showed that the problem remains NP-complete even when every element of the set occurs exactly three times in the 3-element subset collection.

RESTRICTED EXACT COVER BY 3-SETS (RX3C)	
Given:	An integer $k \geq 2$, a set $B = \{b_1, \ldots, b_{3k}\}$, and a collection $\mathscr{S} = \{S_1, \ldots, S_{3k}\}$ of 3-element subsets of B, where each element of B occurs in exactly three sets in $\mathscr{S}$.
Question:	Does there exist an exact cover of B in $\mathscr{S}$, i.e., a subset $\mathscr{S}' \subseteq \mathscr{S}$ of size k such that every element of B occurs in exactly one set in $\mathscr{S}'$?

We illustrate RX3C with the following example.

Example 2.1. Let $k = 3$, $B = \{1, \ldots, 9\}$, and $\mathscr{S} = \{S_1, \ldots, S_9\}$ with

$$\begin{aligned} S_1 &= \{1,2,3\}, & S_2 &= \{1,5,6\}, & S_3 &= \{1,5,9\}, \\ S_4 &= \{2,4,6\}, & S_5 &= \{2,7,8\}, & S_6 &= \{3,4,5\}, \\ S_7 &= \{3,7,8\}, & S_8 &= \{4,6,9\}, & S_9 &= \{7,8,9\}. \end{aligned}$$

Then, the question is whether there is a subset $\mathscr{S}'$ of $\mathscr{S}$ of size 3 such that each element of B occurs exactly one time in $\mathscr{S}'$. In fact, there exists such a subset, namely $\mathscr{S}' = \{S_3, S_4, S_7\} = \{\{1,5,9\}, \{2,4,6\}, \{3,7,8\}\}$. Hence, the given instance is a yes-instance of RX3C.

We now turn to the following graph problem that was shown to be NP-complete by Karp [82]. Also note that we will give some more background on graph theory in Section 2.2.

CLIQUE	
Given:	An integer $k \geq 1$ and an undirected graph $G = (V, E)$.
Question:	Is there a clique of size k in G, i.e., a subset $V' \subseteq V$ of the vertices such that there is an edge between every two vertices in V'?

2.1.5 Beyond P and NP

P, NP, and coNP are not the only complexity classes out there. In fact, there are various hierarchies of complexity classes beyond NP. For instance, Meyer and Stockmeyer [99] and Stockmeyer [134] introduced the *polynomial hierarchy* which makes use of *oracle Turing machines*. For two complexity classes $\mathscr{C}$ and $\mathscr{D}$, the class $\mathscr{C}^{\mathscr{D}}$ contains all problems that can be solved by an algorithm according to class $\mathscr{C}$ that additionally has access to an *oracle* which verifies instances of a set $D \in \mathscr{D}$ in a single computation step. The polynomial hierarchy is defined inductively by $\Delta_0^p = \Sigma_0^p = \Pi_0^p = \mathrm{P}$ and, for $i \geq 0$,

$$\Delta_{i+1}^p = \mathrm{P}^{\Sigma_i^p},$$
$$\Sigma_{i+1}^p = \mathrm{NP}^{\Sigma_i^p}, \text{ and}$$
$$\Pi_{i+1}^p = \mathrm{co}\Sigma_{\mathrm{i+1}}^{\mathrm{p}}.$$

Additionally, $\mathrm{PH} = \bigcup_{i \geq 0} \Sigma_i^p$. For the first layer of the polynomial hierarchy, the definitions imply that $\Delta_1^p = \mathrm{P}$, $\Sigma_1^p = \mathrm{NP}$, and $\Pi_1^p = \mathrm{coNP}$. The second layer is given by $\Delta_2^p = \mathrm{P}^{\mathrm{NP}}$, $\Sigma_2^p = \mathrm{NP}^{\mathrm{NP}}$, and $\Pi_2^p = \mathrm{coNP}^{\mathrm{NP}}$. For more details on the polynomial hierarchy, the reader is referred to, e.g., the textbooks by Rothe [125, 128].

There are many further interesting aspects of complexity theory such as *parameterized complexity* (see, e.g., the books by Downey and Fellows [52, 51] and Flum and Grohe [60]) and *probabilistic complexity* (see Gill [67] and, e.g., Balcázar et al. [13]).

2.2 Graph Theory

We now give some basics of graph theory. For an extensive introduction to graph theory, see, e.g., the textbooks by West [144] and Gurski et al. [70].

Formally, a *graph* is a pair $G = (V, E)$ where V is a set of vertices (or *nodes*) and E is a set of edges. In the case of an *undirected graph*, the edges are undirected and we have $E \subseteq \{\{u, v\} \mid u, v \in V, u \neq v\}$. In the case of a *directed graph*, the edges are directed and we have $E \subseteq V \times V$ where any $(u, v) \in E$ is a directed edge from u to v. By removing the directions of the edges in any directed graph (V, E), we obtain its *underlying undirected graph* $(V, \{\{u, v\} \mid (u, v) \in E\})$.

We will now illustrate some important notions of graph theory. While most of the following terms can also be defined similarly for directed graphs, we will concentrate on the case of undirected graphs. We define the following notions for any undirected graph $G = (V, E)$.

G *isomorphic* to another undirected graph $G' = (V', E')$ if there is a bijection $f : V \to V'$ with $\{u, v\} \in E \iff \{f(u), f(v)\} \in E'$. We say that two vertices u and v are *neighbors* if $\{u, v\} \in E$. The set of all neighbors of v is denoted by $N(v) = \{u \in V | \{u, v\} \in E\}$.

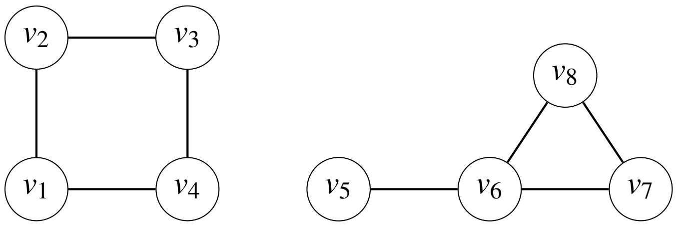

Figure 2.2: An undirected graph with two connected components from Example 2.2

A *path* from vertex v_1 to vertex v_k is a sequence $p = (v_1, \dots, v_k)$ of vertices with $k \geq 1$ and $\{v_i, v_{i+1}\} \in E$ for all $i \in \{1, \dots, k-1\}$. The *length* of a path $p = (v_1, \dots, v_k)$ is the number of contained edges, i.e., $k-1$. A *cycle* is a path $p = (v_1, \dots, v_k)$ where $\{v_k, v_1\} \in E$. The *distance* between vertices u and v is the length of a shortest path between u and v and is denoted by $d(u, v)$. If there is no path between u and v, then $d(u, v) = \infty$. The *diameter* of G is the maximal distance between two vertices in G, i.e., $\max_{u,v \in V} d(u, v)$.

A graph $G' = (V', E')$ is a *subgraph* of G if $V' \subseteq V$ and $E' \subseteq E$. Any subset $V' \subseteq V$ of the vertices *induces* a subgraph which is defined by $G|_{V'} = (V', E \cap \{\{u, v\} \mid u, v \in V'\})$. Hence, $G|_{V'}$ consists of all vertices in V' and all edges from G between the vertices in V'. G is *connected* if there exists a path from u to v for each two vertices $u, v \in V$ with $u \neq v$. Furthermore, G is a *tree* if it is connected and contains no cycles. An induced subgraph $G|_{V'}$ of G (with $V' \subseteq V$) is a *connected component* of G if $G|_{V'}$ is connected and there is no superset V'' with $V' \subset V'' \subseteq V$ for which $G|_{V''}$ is connected. Note that G can be partitioned into connected components in linear time. This can, for instance, be done via *depth-first search*. Finally, a set $V' \subseteq V$ is a *clique* in G if there is an edge $\{u, v\} \in E$ between every two vertices $u, v \in V', u \neq v$.

We complete this section with the following example which illustrates the above definitions.

Example 2.2. Let $G = (V, E)$ be an undirected graph with vertices $V = \{v_1, \dots, v_8\}$ and edges $E = \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_1\}, \{v_5, v_6\}, \{v_6, v_7\}, \{v_7, v_8\}, \{v_6, v_8\}\}$. This graph is depicted in Figure 2.2. First, it can be observed that G has two connected components: the subgraphs induced by the vertex sets $V_1 = \{v_1, v_2, v_3, v_4\}$ and $V'' = \{v_5, v_6, v_7, v_8\}$. Furthermore, $p = (v_1, v_2, v_3, v_4)$ is a path of length 3 in G. p is also a cycle because $\{v_4, v_1\} \in E$. The vertex sets $\{v_6, v_7, v_8\}$ and $\{v_5, v_6\}$ are examples of cliques in G. The distance between vertices v_4 and v_5 is ∞ since there is no path connecting these two vertices. But the distance between vertices v_5 and v_8 is 2 since this is the length of a shortest path between them. The diameter of G is ∞ since G is not connected. However, the induced subgraphs $G|_{V'}$ and $G|_{V''}$ both have a diameter of 2. The two induced subgraphs $G|_{\{v_1, v_3, v_4\}}$ and $G|_{\{v_5, v_6, v_7\}}$ are isomorphic to each other while this is not the case for the two subgraphs $G|_{V'}$ and $G|_{V''}$. Last, the induced subgraph $G|_{\{v_1, v_3, v_5, v_7, v_8\}}$ is not connected and has four connected components.

2.3 Coalition Formation Games

In this thesis, we consider *coalition formation games* as a subclass of *non-transferable utility (NTU) games*. In these games, agents form groups based on their individual preferences where, in general, any partition of the agents is a possible outcome of the game. The agents evaluate the possible outcomes based on individual preferences.

We will now give an introduction to coalition formation games. After introducing some basic concepts, we will provide some background on hedonic games. They form an important subclass of coalition formation games where agents only care about the coalitions that they belong to. Afterwards, we describe some common preference representations, including cardinal formats, representations based on the categorization into friends and enemies, and many more. We then define some stability, optimality, and fairness notions that are of interest when studying hedonic games and explain some interesting decision problems which are associated with these notions. We complete the chapter by surveying the literature on this topic and summarizing some interesting results.

For more overviews of coalition formation games, see the survey by Hajduková [72] or the textbook by Chalkiadakis et al. [41]. For more background on NTU games see, for example, Section 5.1 in the textbook by Chalkiadakis et al. [41]. For literature on hedonic games, we refer to the book chapter by Aziz and Savani [8] and the survey by Woeginger [147].

2.3.1 Basic Definitions

Let $N = \{1, \dots, n\}$ be the set of *agents* (which we also call *players*). Subsets $C \subseteq N$ of the agents are called *coalitions* and, for any player $i \in N$, we denote the set of all coalitions containing i by $\mathscr{N}^i = \{C \subseteq N \mid i \in C\}$. It holds that $|\mathscr{N}^i| = 2^{n-1}$, which means that the number of coalitions containing i is exponential in the number of agents. Coalitions that contain only one player are also called *singleton coalitions* or *singletons*, for short. The coalition N that consists of all players is also called the *grand coalition*. A *coalition structure* is a partition $\Gamma = \{C_1, \dots, C_k\}$ of the set N of agents. As for every partition, it holds that $\bigcup_{i=1}^{k} C_i = N$ and $C_i \cap C_j = \emptyset$ for all $i, j \in \{1, \dots, k\}$ with $i \neq j$. There is no general restriction on the number k of coalitions in a coalition structure which means that k can range anywhere between 1 and n. The unique coalition in Γ that contains agent i is denoted by $\Gamma(i)$. Moreover, the set of all coalition structures for a set of agents N is denoted by $\mathscr{C}_N$. Note that the size of $\mathscr{C}_N$ grows exponentially with the number n of agents and equals the nth Bell number [18, 123]. For example, the first six Bell numbers are $B_1 = 1$, $B_2 = 2$, $B_3 = 5$, $B_4 = 15$, $B_5 = 52$, and $B_6 = 203$, which means that there are 203 possible partitions of a set of six agents.

Based on these notions, a *coalition formation game* is a pair $(N, \succeq)$, where $N = \{1, \dots, n\}$ is a set of agents and $\succeq = (\succeq_1, \dots, \succeq_n)$ is the profile of preferences of the agents. For each agent $i \in N$, $\succeq_i$ denotes her preference relation which is a complete weak order over all coalition structures, i.e., $\succeq_i \subseteq \mathscr{C}_N \times \mathscr{C}_N$. For two coalition structures $\Gamma, \Delta \in \mathscr{C}_N$, we say that agent i

weakly prefers Γ *to* Δ if $\Gamma \succeq_i \Delta$, that i *prefers* Γ *to* Δ (denoted by $\Gamma \succ_i \Delta$) if $\Gamma \succeq_i \Delta$ but not $\Delta \succeq_i \Gamma$, and that i is *indifferent between* Γ *and* Δ (denoted by $\Gamma \sim_i \Delta$) if $\Gamma \succeq_i \Delta$ and $\Delta \succeq_i \Gamma$.

2.3.2 Hedonic Games

The focus of this thesis will mainly be on coalition formation games with hedonic preferences, *hedonic games* for short. They were introduced independently by Banerjee et al. [15] and Bogomolnaia and Jackson [21]. The key idea of hedonic games (going back to Drèze and Greenberg [53]) is that agents only care about the coalitions that they are part of and not about the rest of a coalition structure. More formally, let any coalition formation game $(N, \succeq)$ be given. Then, the preference $\succeq_i$ of player i is hedonic if it only depends on the coalitions that i is part of, i.e., if for any two coalition structures $\Gamma, \Delta \in \mathscr{C}_N$, it holds that $\Gamma(i) = \Delta(i)$ implies $\Gamma \sim_i \Delta$. If the preferences of all agents $i \in N$ are hedonic, $(N, \succeq)$ is also called hedonic. For such a hedonic (coalition formation) game $(N, \succeq)$, the preferences are usually represented by complete weak orders over the set of coalitions containing an agent, i.e., $\succeq_i \subseteq \mathscr{N}^i \times \mathscr{N}^i$ for all $i \in N$. For two coalitions $A, B \in \mathscr{N}^i$, we then say that i *weakly prefers* A to B if $A \succeq_i B$, that i *prefers* A to B if $A \succ_i B$, and that i is *indifferent* between A and B if $A \sim_i B$. It follows from the definition of hedonic games that $\Gamma \succeq_i \Delta$ if and only if $\Gamma(i) \succeq_i \Delta(i)$.

Note that there are subclasses of hedonic games where only coalitions of certain sizes are allowed. For example, *marriage* and *roommate games* [64, 124] are hedonic games where all coalitions must have a size of at most two. These games and many other matching models are studied in *matching theory*. For more background on this topic we refer to the book chapter by Klaus et al. [94] and the textbooks by Roth and Sotomayor [124], Manlove [98], and Gusfield and Irving [71]. For other subclasses of hedonic games, the agents are assumed to divide into two types. In *hedonic diversity games* [31], an agent's preference depends on the fractions of agents of each type in a coalition. In this thesis however, we will only concentrate on general hedonic games where agents have no types and arbitrary coalition sizes are allowed.

We now give a simple example of a hedonic game.

Example 2.3. Let the set of players be given by $N = \{1,2,3\}$. Then, there are four different coalitions containing agent 1, namely $C_1 = \{1\}$, $C_2 = \{1,2\}$, $C_3 = \{1,3\}$, and $C_4 = \{1,2,3\}$. Here, C_1 is a singleton coalition and C_4 is the grand coalition. The set of all possible coalition structures $\mathscr{C}_N$ contains exactly five coalition structures:

$$\Gamma_1 = \{\{1\},\{2\},\{3\}\}, \ \Gamma_2 = \{\{1\},\{2,3\}\}, \ \Gamma_3 = \{\{1,2\},\{3\}\},$$
$$\Gamma_4 = \{\{1,3\},\{2\}\}, \text{ and } \Gamma_5 = \{\{1,2,3\}\}.$$

In particular, we have $\mathscr{C}_N = \{\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5\}$.

Further consider the following preference profile $\succeq = (\succeq_1, \succeq_2, \succeq_3)$ that, together with the set of agents N, defines a hedonic game $\mathscr{G} = (N, \succeq)$:

$$\{1,2,3\} \succ_1 \{1,2\} \succ_1 \{1,3\} \succ_1 \{1\},$$
$$\{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2\} \succ_2 \{2,3\},$$
$$\{3\} \succ_3 \{1,3\} \sim_3 \{2,3\} \succ_3 \{1,2,3\}.$$

For this hedonic game, agent 1 prefers coalition $\Gamma_4(1) = \{1,3\}$ to coalition $\Gamma_2(1) = \{1\}$. Therefore, 1 also prefers Γ_4 to Γ_2. In contrast, agent 3 is indifferent between Γ_4 and Γ_2 because she is indifferent between $\Gamma_4(3) = \{1,3\}$ and $\Gamma_2(3) = \{2,3\}$.

2.3.3 Preference Representations

Even when considering the restricted case of hedonic coalition formation games, it is not reasonable to elicit full preferences in practice. Collecting a full preference over $\mathscr{N}^i$ for every agent $i \in N$ would not only lead to a preference profile of exponential size (in the number of agents) but would also present an extreme cognitive burden for the agents. Hence, we are looking for succinct representations of the preferences that are still reasonably expressive and easy to elicit.

Cardinal Preference Representations

There is a broad literature that concerns the problem of finding compact representations for hedonic preferences. Commonly used representations include the *additive encoding* due to Bogomolnaia and Jackson [21], the *fractional encoding* due to Aziz et al. [11], the *modified fractional encoding* due to Olsen [110], and the *friends-and-enemies encoding* due to Dimitrov et al. [50]. All these four representations have in common that they can be specified via cardinal valuation functions, i.e., they belong to the class of *cardinal hedonic games*. In these games, each agent i assigns a cardinal value to every other agent j that indicates how much i likes j. The agents' preferences can then be inferred from their valuation functions. The four representations differ in the range of valuations and in how the preferences are inferred.

Additively Separable Hedonic Games A hedonic game $(N, \succeq)$ is *additively separable* if, for every player $i \in N$, there exists a valuation function $v_i : N \to \mathbb{Q}$ such that for any two coalitions $A, B \in \mathscr{N}^i$ it holds that

$$A \succeq_i B \iff \sum_{j \in A} v_i(j) \geq \sum_{j \in B} v_i(j).$$

Hence, an additively separable hedonic game can also be represented by a tuple (N, v) consisting of a set of agents and a collection of valuation functions. It is commonly assumed that $v_i(i) = 0$ for every $i \in N$.[7] In additively separable hedonic games, agent i's valuation of a coalition $A \in \mathcal{N}^i$ is defined as $v_i^{\text{add}}(A) = \sum_{j\in A} v_i(j)$. Additively separable hedonic games [21] were studied, e.g., by Sung and Dimitrov [137, 136], Aziz et al. [10], and Woeginger [146].

Example 2.4. Again, consider the hedonic game $\mathcal{G} = (N, \succeq)$ from Example 2.3. $\mathcal{G}$ is additively separable as it can be represented via the following valuation functions:

i	$v_i(1)$	$v_i(2)$	$v_i(3)$
1	0	2	1
2	2	0	-1
3	-1	-1	0

We validate that these valuation functions indeed lead to the preferences from Example 2.3 using agent 2 as an example. We compute agent 2's valuations for the four coalitions:

$$v_2^{\text{add}}(\{1,2\}) = 2+0 = 2, \qquad v_2^{\text{add}}(\{1,2,3\}) = 2+0-1 = 1,$$
$$v_2^{\text{add}}(\{2\}) = 0, \text{ and} \qquad v_2^{\text{add}}(\{2,3\}) = 0-1 = -1.$$

Since $v_2^{\text{add}}(\{1,2\}) > v_2^{\text{add}}(\{1,2,3\}) > v_2^{\text{add}}(\{2\}) > v_2^{\text{add}}(\{2,3\})$, agent 2's valuation function v_2 indeed corresponds to the preference $\{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2\} \succ_2 \{2,3\}$.

Fractional Hedonic Games In fractional hedonic games, the value of a coalition is the average value of the members of the coalition. Hence, given a valuation function v_i of agent i, i's fractional value for a coalition $A \in \mathcal{N}^i$ is $v_i^{\text{frac}}(A) = \frac{1}{|A|}\sum_{j\in A} v_i(j)$ and a hedonic game $(N, \succeq)$ is *fractional* if for every player $i \in N$ there exists a valuation function $v_i : N \to \mathbb{Q}$ such that for any two coalitions $A, B \in \mathcal{N}^i$ it holds that

$$A \succeq_i B \iff v_i^{\text{frac}}(A) \geq v_i^{\text{frac}}(B).$$

Again, giving a fractional hedonic game by a tuple (N, v) of agents and valuation functions, it is commonly assumed that $v_i(i) = 0$ for all agents $i \in N$. Fractional hedonic games [11] have been studied, e.g., by Bilò et al. [19], Brandl et al. [25], Kaklamanis et al. [80], and Carosi et al. [37].

Modified Fractional Hedonic Games Modified fractional hedonic games are defined analogously to fractional hedonic games besides that the valuation of a player $i \in N$ for coalition $A \in \mathcal{N}^i$ is defined by

$$v_i^{\text{mfrac}}(A) = \begin{cases} \frac{1}{(|A|-1)}\sum_{j\in A} v_i(j) & \text{if } A \neq \{i\}, \\ 0 & \text{if } A = \{i\}. \end{cases}$$

[7] This is a normalization assumption. For each additively separable preference $\succeq_i$, there exists a valuation function v_i with $v_i(i) = 0$.

Modified fractional hedonic games [110] were studied, e.g., by Elkind et al. [58], Kaklamanis et al. [80], Monaco et al. [101, 102], Bullinger [33], and Bullinger and Kober [34].

The Friends-and-Enemies-Encoding In the friends-and-enemies encoding due to Dimitrov et al. [50], each player $i \in N$ partitions the other players into a set of friends $F_i \subseteq N \setminus \{i\}$ and a set of enemies $E_i = N \setminus (F_i \cup \{i\})$. Based on this representation, Dimitrov et al. [50] distinguish between the *friend-oriented* and the *enemy-oriented* preference extension. Under the friend-oriented model, agents prefer coalitions with more friends to coalitions with fewer friends, and in the case that two coalitions contain the same number of friends, they prefer the coalition with fewer enemies. Formally, a hedonic game $(N, \succeq)$ is friend-oriented if, for any agent $i \in N$, there exist a set of friends $F_i \subseteq N \setminus \{i\}$ and a set of enemies $E_i = N \setminus (F_i \cup \{i\})$ such that for any two coalitions $A, B \in \mathcal{N}^i$ it holds that

$$A \succeq_i B \iff |A \cap F_i| > |B \cap F_i| \text{ or } \big(|A \cap F_i| = |B \cap F_i| \text{ and } |A \cap E_i| \leq |B \cap E_i|\big). \tag{2.1}$$

Analogously, a hedonic game $(N, \succeq)$ is enemy-oriented if, for any agent $i \in N$, there exist a set of friends $F_i \subseteq N \setminus \{i\}$ and a set of enemies $E_i = N \setminus (F_i \cup \{i\})$ such that for any two coalitions $A, B \in \mathcal{N}^i$ it holds that

$$A \succeq_i B \iff |A \cap E_i| < |B \cap E_i| \text{ or } \big(|A \cap E_i| = |B \cap E_i| \text{ and } |A \cap F_i| \geq |B \cap F_i|\big). \tag{2.2}$$

Friend-oriented and enemy-oriented hedonic games can be seen as the subclasses of additively separable hedonic games where the valuation functions of the agents map only to $\{-1, n\}$ and $\{-n, 1\}$, respectively. In particular, in friend-oriented hedonic games, agents assign value n to their friends and value -1 to their enemies. In enemy-oriented hedonic games, agents assign value 1 to their friends and value $-n$ to their enemies. These cardinal values assure that the resulting additively separable hedonic preferences in fact satisfy the conditions from Equations 2.1 and 2.2. Agent i's friend-oriented respectively enemy-oriented value for coalition $A \in \mathcal{N}^i$ is then given by

$$v_i^{\text{fo}}(A) = \sum_{j \in A} v_i(j) = n|A \cap F_i| - |A \cap E_i| \text{ and}$$
$$v_i^{\text{eo}}(A) = \sum_{j \in A} v_i(j) = |A \cap F_i| - n|A \cap E_i|.$$

Note that friend- and enemy-oriented hedonic games are also referred to as hedonic games with *appreciation of friends* and *aversion to enemies*. Friend- and enemy-oriented hedonic games [50] were studied, e.g, by Sung and Dimitrov [137, 136], Aziz and Brandl [7], Rey et al. [122], and Igarashi et al. [79].

Visual Presentation All these classes of *cardinal hedonic games* can be represented by complete weighted directed graphs with the agents as vertices where the weight of an edge (i, j) from agent i to agent j is i's value for j. Sometimes some edges with equal weights, e.g., all edges with weight zero, are omitted in the graph representation. In the case

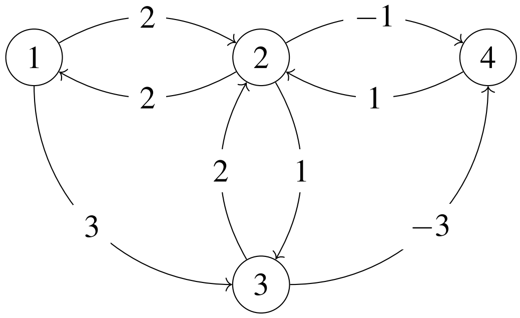

Figure 2.3: Graph representing the modified fractional hedonic game in Example 2.5. All omitted edges have weight zero.

of the friends-and-enemies encoding, all weights can be omitted. Instead, the game can be visualized by a directed graph where an edge from agent i to agent j indicates that j is i's friend. This graph is also called *network of friends*.

We call a cardinal hedonic game (N,v) *symmetric* if $v_i(j) = v_j(i)$ for all $i, j \in N$ and *simple* if $v_i(j) \in \{0,1\}$ for all $i, j \in N$. For symmetric friend-oriented and symmetric enemy-oriented hedonic games, we also say that the friendship relations are *mutual*. In this case, the network of friends is an undirected graph where an edge $\{i, j\}$ represents the mutual friendship between agents i and j.

We now give examples of a modified fractional hedonic game and a friend-oriented hedonic game, respectively.

Example 2.5. We consider a modified fractional hedonic game (N,v) with four agents $N = \{1,2,3,4\}$. The valuation functions of the agents are given by the graph in Figure 2.3 where all omitted edges represent valuations of zero. According to this graph, the valuation functions of the agents are:

i	$v_i(1)$	$v_i(2)$	$v_i(3)$	$v_i(4)$
1	0	2	3	0
2	2	0	1	−1
3	0	2	0	−3
4	0	1	0	0

We now compute the modified fractional preference of agent 2. First note that the set $\mathcal{N}^2$ of coalitions containing agent 2 has size $2^{n-1} = 2^3 = 8$. Agent 2's modified fractional valuations for these eight coalitions are given in the following table:

C	$\{2\}$	$\{1,2\}$	$\{2,3\}$	$\{2,4\}$	$\{1,2,3\}$	$\{1,2,4\}$	$\{2,3,4\}$	$\{1,2,3,4\}$
$v_2^{\text{mfrac}}(C)$	0	$2/1$	$1/1$	$-1/1$	$3/2$	$1/2$	$0/2$	$2/3$

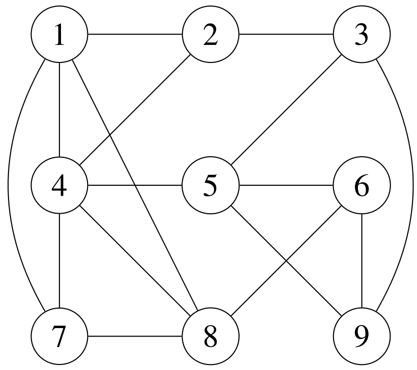

Figure 2.4: Graph representing the friend-oriented hedonic game with mutual friendship relations in Example 2.6

Sorting these valuations leads to the following modified fractional preferences of agent 2:

$$\{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2,3\} \succ_2 \{1,2,3,4\} \succ_2 \{1,2,4\} \succ_2 \{2\} \sim_2 \{2,3,4\} \succ_2 \{2,4\}.$$

Note that agent 2's additively separable preferences for the graph in Figure 2.3 differ from the above modified fractional preferences. For example, agent 2 prefers $\{1,2,3\}$ to $\{1,2\}$ under additively separable preferences.

Next, we give a short example of a friend-oriented hedonic game.

Example 2.6. We consider a friend-oriented hedonic game $(N,\succeq)$ with nine agents, i.e., $N = \{1,\dots,9\}$. The mutual friendship relations among the agents are given by the network of friends in Figure 2.4. Furthermore, we consider the two coalitions $A = \{1,2,3,4,5,6,9\}$ and $B = \{1,2,4,7,8\}$.

For agent 1, it holds that she has two friends and four enemies in A while she has four friends and no enemies in B. Therefore, 1 prefers B to A under friend-oriented preferences. Actually, B is agent 1's most preferred coalition as it contains all of her friends and none of her enemies.

Considering agent 2, we can observe that 2 has three friends and three enemies in A while she has two friends and two enemies in B. Although the proportions of friends and enemies are the same for both coalitions, agent 2 prefers coalition A to B under friend-oriented preferences. This is because she compares the absolute numbers of friends in the two coalitions, which is greater for A than for B. Using the cardinal representation of the preferences with value $n = 9$ for friends and value -1 for enemies, agent 2's friend-oriented valuations for A and B are

$$v_2^{\text{fo}}(A) = n|A\cap F_2| - |A\cap E_2| = 9\cdot|\{1,3,4\}| - |\{5,6,9\}| = 9\cdot 3 - 3 = 24 \text{ and}$$
$$v_2^{\text{fo}}(B) = n|B\cap F_2| - |B\cap E_2| = 9\cdot|\{1,4\}| - |\{7,8\}| = 9\cdot 2 - 2 = 16.$$

Preference Representations Based on Friends and Enemies

Apart from the friends-and-enemies encoding due to Dimitrov et al. [50], there has been quite some research concerning preference representations that are based on the partitioning of agents into different groups.

For instance, Ota et al. [111] study hedonic games where agents specify their preferences by partitioning the other agents into friends, enemies, and *neutral agents*. In their model, an agent's preference is independent of all agents that she is neutral to. They then distinguish between the *friend appreciation* and *enemy aversion* due to Dimitrov et al. [50] and consider the problems of verifying (strict) core stability and checking the existence of (strictly) core stable coalition structures. They show that the neutral agents have an impact of on the computational complexity of these problems.

Similarly, Barrot et al. [17] study hedonic games where the agents partition each other into friends, enemies, and *unknown agents*. In contrast to Ota et al. [111], Barrot et al. [17] do not assume that agents are neutral to agents that they do not know. Instead, they distinguish between *extraverted* and *introverted agents* who either appreciate the presence of unknown agents or prefer coalitions with fewer unknown agents. They then investigate the impact of unknown agents on core stability and individual stability.

Another preference representation that is based on the partitioning of agents into friends, enemies, and neutral players is described in Chapter 5. In particular, we introduce *FEN-hedonic games* where agents represent their preferences via *weak rankings with double threshold*. That means that each agent partitions the other agents into friends, enemies, and neutral players and additionally specifies weak rankings on her friends and on her enemies, respectively. For more details on FEN-hedonic games, see Chapter 5. Weak rankings with double threshold are also studied by Rey and Rey [121] who obtain preferences over coalitions by measuring the distance between any given coalition and the specified ranking.

Further Preference Representations

There are several other preference representations for hedonic games. We will now give a brief overview of some prominent of these representations.

Under the *singleton encoding* by Cechlárová and Romero-Medina [40], the agents specify rankings over single agents. Cechlárová and Romero-Medina [40] define two preference extensions that lead to so-called $\mathscr{B}$-preferences and $\mathscr{W}$-preferences, respectively. Agents with $\mathscr{B}$-preferences rank coalitions only based on the most preferred player in each coalition. Agents with $\mathscr{W}$-preferences only care about the least preferred member of their coalitions. These two preference extensions are also studied by Cechlárová and Hajduková [38, 39].

In the *individually rational encoding* due to Ballester [14], agents only rank the coalitions that they prefer to being alone. This leads to a succinct representation whenever the number of those coalitions is small.

Under the *anonymous encoding* defined by Banerjee et al. [15], the agents' preferences only depend on their coalition sizes. This means that, under anonymous preferences, the agents are indifferent among any two coalitions of the same size and do not care about the identity of the agents. A generalization of anonymous hedonic games has been studied by Darmann et al. [47]. They consider *group activity selection problems* where the agents' preferences depend on the sizes of their coalitions and on the activities allocated to their coalitions.

Elkind and Wooldridge [56] proposed a very expressive representation: *hedonic coalition nets* where the agents specify their preferences by giving a set of propositional formulas. With these formulas, the agents can specify which combination of agents they would like to have in their coalitions. For instance, agent i might specify the formula $j \wedge k \mapsto_i 8$ which means that i obtains utility 8 if she is in a coalition with agents j and k. These propositional formulas can also be more complex and contain the Boolean operators $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\neg$. An agent's total utility for a given coalition is the sum of all formulas that are satisfied by the coalition. Elkind and Wooldridge [56] show that hedonic coalition nets generalize several other preference representations such as hedonic games with $\mathscr{B}$- or $\mathscr{W}$-preferences [40], the individually rational encoding [14], additively separable hedonic games [21], and anonymous hedonic games [15].

Aziz et al. [9] consider hedonic games with *dichotomous preferences*. Formally, player i's preference is dichotomous if she can partition the set $\mathscr{N}^i$ of coalitions containing herself into two groups, satisfactory coalitions and unsatisfactory coalitions, such that she strictly prefers any satisfactory coalition to any unsatisfactory coalition and is indifferent between any two coalitions of the same group. Aziz et al. [9] introduce the *boolean hedonic encoding*, a succinct representation for hedonic games with dichotomous preferences. In this encoding, each agent's preference is given by a single propositional formula that characterizes this agent's satisfactory set of coalitions. Hedonic games with dichotomous preferences are also studied by Peters [114]. He studies the computational complexity of finding stable and optimal coalition structures in such hedonic games. While doing so, he distinguishes between several representations of such games, including the boolean encoding.

2.3.4 Stability and Optimality in Hedonic Games

Central questions in coalition formation are which coalition structures are likely to form and which coalition structures are desirable outcomes. There is a broad literature that studies such desirable properties in coalition formation. The solution concepts are concerned with *optimality*, *stability*, and *fairness*. In this section, we will consider several notions of stability and optimality.

There are various stability notions that have been proposed in the literature. Those notions mainly concern the question whether there are agents that would like to deviate from a given coalition structure. We distinguish different categories of stability notions. First, there are concepts based on *single player deviations* such as *Nash stability*, *individual stability*, or *individual rationality* that capture whether there are agents that would like to perform a deviation to another coalition on their own. Second, there exist notions of *group stability* such as *core stability* that capture whether groups of agents would want to deviate together. And third, there are notions that are based on the comparison of coalition structures such as *Pareto optimality* or *popularity*. These notions can also be seen as *optimality concepts*. Further optimality criteria are concerned with the maximization of *social welfare* or other measurements of the agents' satisfaction.

We now define some common stability notions and start with some classic notions. For any given hedonic game $(N, \succeq)$, coalition structure $\Gamma \in \mathscr{C}_N$ is said to be

- *perfect* (PF)[8] if every agent is in her most preferred coalition, i.e., every agent $i \in N$ weakly prefers $\Gamma(i)$ to every other coalition $C \in \mathscr{N}^i$.
- *individually rational* (IR) if every agent weakly prefers her current coalition to being alone, i.e., every agent $i \in N$ weakly prefers $\Gamma(i)$ to $\{i\}$.

Note that perfectness (formulated by Aziz et al. [12]) and individually rationality are two of the most extreme stability notions that we consider here. While perfectness is stronger than almost all other stability notions (except for strict popularity), individual rationality imposes only a minimal requirement and is implied by many other notions.

We continue with some further classic notions that are concerned with single player deviations and were formulated by Bogomolnaia and Jackson [21]. Coalition structure $\Gamma \in \mathscr{C}_N$ is

- *Nash stable* (NS) if no agent wants to deviate to another coalition in $\Gamma \cup \{\emptyset\}$, i.e., every agent $i \in N$ weakly prefers $\Gamma(i)$ to every coalition $C \cup \{i\}$ with $C \in \Gamma \cup \{\emptyset\}$.
- *individually stable* (IS) if no agent wants to deviate to another coalition C in $\Gamma \cup \{\emptyset\}$ and can do so without making any agent in C worse off. Formally, Γ is IS if for all agents $i \in N$ and all coalitions $C \in \Gamma \cup \{\emptyset\}$, it holds that i weakly prefers $\Gamma(i)$ to $C \cup \{i\}$ or there is a player $j \in C$ who prefers C to $C \cup \{i\}$.
- *contractually individually stable* (CIS) if no agent i wants to deviate to another coalition C in $\Gamma \cup \{\emptyset\}$ and can do so without making any agent in C or $\Gamma(i)$ worse off. Formally, Γ is CIS if for all agents $i \in N$ and all coalitions $C \in \Gamma \cup \{\emptyset\}$, it holds that i weakly prefers $\Gamma(i)$ to $C \cup \{i\}$ or there is a player $j \in C$ who prefers C to $C \cup \{i\}$ or there is a player $k \in \Gamma(i) \setminus \{i\}$ who prefers $\Gamma(i)$ to $\Gamma(i) \setminus \{i\}$.

Additionally, Sung and Dimitrov [138] introduced contractual Nash stability and some other related notions. We say that coalition structure $\Gamma \in \mathscr{C}_N$ is

- *contractually Nash stable* (CNS) if no agent i wants to deviate to another coalition in $\Gamma \cup \{\emptyset\}$ and can do so without making any agent in $\Gamma(i)$ worse off. Formally, Γ is

[8] In the context of the friends-and-enemies encoding [50], perfectness is sometimes also called "wonderful stability", e.g., by Woeginger [147], Elkind and Rothe [55], and Rey et al. [122].

CNS if for all agents $i \in N$ and all coalitions $C \in \Gamma \cup \{\emptyset\}$, it holds that i weakly prefers $\Gamma(i)$ to $C \cup \{i\}$ or there is a player $k \in \Gamma(i) \setminus \{i\}$ who prefers $\Gamma(i)$ to $\Gamma(i) \setminus \{i\}$.

We now turn to core stability which is a classic notion of group stability that was already studied by Banerjee et al. [15]. Later, core stability and strict core stability have also been extensively studied by Dimitrov et al. [50]. For any coalitions structure $\Gamma \in \mathscr{C}_N$ and any nonempty coalition $C \subseteq N$, C is said to block Γ if every agent $i \in C$ prefers C to $\Gamma(i)$. C is said to weakly block Γ if all agents $i \in C$ weakly prefer C to $\Gamma(i)$ and at least one agent $j \in C$ prefers C to $\Gamma(j)$. Coalition structure $\Gamma \in \mathscr{C}_N$ is

- *core stable* (CS) if no nonempty coalition blocks Γ.
- *strictly core stable* (SCS) if no nonempty coalition weakly blocks Γ.

Karakaya [81] and Aziz and Brandl [7] formulated some more related notions. For a coalition $C \subseteq N$, we say that coalition structure $\Delta \in \mathscr{C}_N$ is reachable from coalition structure $\Gamma \in \mathscr{C}_N$, $\Gamma \neq \Delta$, by coalition C if, for all $i, j \in N \setminus C$, it holds that $\Gamma(i) = \Gamma(j) \iff \Delta(i) = \Delta(j)$. In other words, if Δ is reachable from Γ by C, then all agents in C might deviate to other coalitions while all agents in $N \setminus C$ have to stay together as before. Then, a coalition $C \subseteq N$, $C \neq \emptyset$,

- *strong Nash blocks* coalition structure Γ if there exists a coalition structure Δ that is reachable from Γ by C such that all agents $i \in C$ prefer $\Delta(i)$ to $\Gamma(i)$.
- *weakly Nash blocks* Γ if there exists a coalition structure Δ that is reachable from Γ by C such that all agents $i \in C$ weakly prefer $\Delta(i)$ to $\Gamma(i)$ and there is an agent $j \in C$ who prefers $\Delta(j)$ to $\Gamma(j)$.
- *strong individually blocks* Γ if there exists a coalition structure Δ that is reachable from Γ by C such that all agents $i \in C$ prefer $\Delta(i)$ to $\Gamma(i)$ and there is an agent $j \in C$ such that all $k \in \Delta(j)$ weakly prefer $\Delta(k)$ to $\Gamma(k)$.

Based on these notions it holds that coalition structure $\Gamma \in \mathscr{C}_N$ is

- *strong Nash stable* (SNS) [81] if there is no coalition $C \subseteq N$ that strong Nash blocks Γ.
- *strictly strong Nash stable* (SSNS) [7] if there is no coalition $C \subseteq N$ that weakly Nash blocks Γ.
- *strong individually stable* (SIS) [7] if there is no coalition $C \subseteq N$ that strong individually blocks Γ.

We now turn to some concepts that are based on the comparison of coalition structures. For two coalition structures $\Gamma, \Delta \in \mathscr{C}_N$, we say that Δ *Pareto-dominates* Γ if every agent $i \in N$ weakly prefers $\Delta(i)$ to $\Gamma(i)$ and there is an agent $j \in N$ who prefers $\Delta(j)$ to $\Gamma(j)$. Coalition structure $\Gamma \in \mathscr{C}_N$ is

- *Pareto optimal* (PO) if there is no coalition structure that Pareto-dominates Γ.

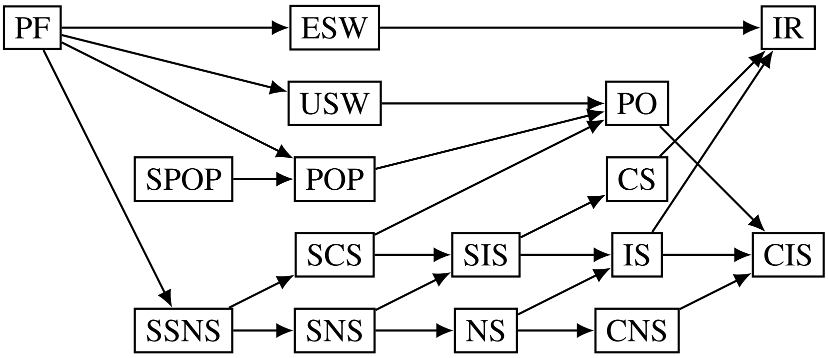

Figure 2.5: Relations among the stability and optimality notions from Section 2.3.4 where a notion A implies a notion B exactly if there is a directed path from A to B

Popularity is another notion that is based on the comparison of coalition structures. The notion was first proposed in the context of marriage games by Gärdenfors [65]. In the context of hedonic games, popularity and strict popularity were formulated by Aziz et al. [10] and Lang et al. [96]. A coalition structure $\Gamma \in \mathscr{C}_N$ is

- *popular* (POP) if for every coalition structure $\Delta \in \mathscr{C}_N$, at least as many agents prefer Γ to Δ as the other way around; formally, this means for all $\Delta \in \mathscr{C}_N$ with $\Delta \neq \Gamma$ that

$$|\{i \in N \mid \Gamma(i) \succ_i \Delta(i)\}| \geq |\{i \in N \mid \Delta(i) \succ_i \Gamma(i)\}|.$$

- *strictly popular* (SPOP) if for every coalition structure $\Delta \in \mathscr{C}_N$, more agents prefer Γ to Δ than the other way around; formally, this means for all $\Delta \in \mathscr{C}_N$ with $\Delta \neq \Gamma$ that

$$|\{i \in N \mid \Gamma(i) \succ_i \Delta(i)\}| > |\{i \in N \mid \Delta(i) \succ_i \Gamma(i)\}|.$$

We consider two further concepts that were formulated by Aziz et al. [10] and are concerned with *social welfare maximization*. For any cardinal hedonic game (N, v), we say that $\Gamma \in \mathscr{C}_N$ *maximizes*

- *utilitarian social welfare* (USW) if $\sum_{i \in N} v_i(\Gamma(i)) \geq \sum_{i \in N} v_i(\Delta(i))$ for all $\Delta \in \mathscr{C}_N$.
- *egalitarian social welfare* (ESW) if $\min_{i \in N} v_i(\Gamma(i)) \geq \min_{i \in N} v_i(\Delta(i))$ for all $\Delta \in \mathscr{C}_N$.

We also say that a coalition structure Γ is USW or ESW by which we mean that Γ maximizes USW or ESW. We further use all abbreviations from this section as nouns and adjectives; for example, we say that a coalition structure is CS (core stable) or that it satisfies CS (core stability).

There are a lot of relations among these stability and optimality notions. Some of them follow directly from the definitions, e.g., NS trivially implies IS which in turn implies CIS. The relations among all notions from this section are visualized in Figure 2.5. For more background on these relations, see, e.g., Bogomolnaia and Jackson [21] (for relations among PO, NS, IS, CIS, and CS), Sung and Dimitrov [138] (for relations among SCS, CS, NS, IS, CNS, and CIS), Aziz and Brandl [7] (for relations among SSNS, SNS, SIS, and previous notions), or Kerkmann [83] (for relations among SPOP, POP, PO, and other notions).

2.3.5 Fairness in Hedonic Games

Besides the stability concepts from the previous section, other important notions in hedonic games are concerned with *fairness*. Some of these notions are inspired from the field of *fair division* where three classic fairness criteria are *equitability*, *proportionality*, and *envy-freeness*. Further fairness notions in fair division are *jealousy-freeness* due to Gourvès et al. [69], *envy-freeness up to one good* due to Budish [32], the *max-min fair share criterion* by Budish [32], and the *min-max fair share criterion* by Bouveret and Lemaître [23]. For background on fair division theory see, e.g., the book chapters by Lang and Rothe [95] and Bouveret et al. [24].

For hedonic games, it was proposed to use envy-freeness as a notion of fairness [21, 10, 114, 115]. We say that a coalition structure $\Gamma \in \mathscr{C}_N$ is *envy-free by replacement* (EFR) if no agent envies another agent for her coalition, i.e., if for all agents $i, j \in N$ with $\Gamma(i) \neq \Gamma(j)$, agent i weakly prefers $\Gamma(i)$ to $(\Gamma(j) \setminus \{j\}) \cup \{i\}$. Perfectness is the only notion from Section 2.3.4 that implies EFR. Also, EFR does not imply any of the notion from Section 2.3.4. The following two examples illustrate EFR and some of the notions from Section 2.3.4 while showing that EFR is independent from all notions besides perfectness.

Example 2.7. Consider the hedonic game $\mathscr{G} = (N, \succeq)$ with $N = \{1,2,3\}$ and the following preference profile $\succeq = (\succeq_1, \succeq_2, \succeq_3)$:

$$\{1,2\} \succ_1 \{1,2,3\} \succ_1 \{1\} \succ_1 \{1,3\},$$
$$\{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2\} \succ_2 \{2,3\},$$
$$\{1,3\} \succ_3 \{3\} \succ_3 \{1,2,3\} \succ_3 \{2,3\}.$$

Further consider the coalition structure $\Gamma = \{\{1,2\},\{3\}\}$. We first observe that this coalition structure is not EFR because agent 3 envies agent 2 for her coalition. In particular, we have

$$(\Gamma(2) \setminus \{2\}) \cup \{3\} = \{1,3\} \succ_3 \{3\} = \Gamma(3).$$

Yet, agents 1 and 2 prefer Γ to every other coalition structure which implies that Γ is SPOP. Moreover, Γ is SSNS since there is no coalition that weakly Nash blocks Γ: First, observe that agents 1 and 2 can not be part of any weakly Nash blocking coalition because Γ is their unique most preferred coalition structure. Hence, $\{3\}$ is the only remaining coalition that could weakly Nash block Γ. However, deviating from $\{3\}$ to $\{1,2\} \cup \{3\}$ does not present an improvement to agent 3. Thus, there is no weakly Nash blocking coalition.

Note that $\mathscr{G}$ is additively separable and can be represented via the following valuations:

i	$v_i(1)$	$v_i(2)$	$v_i(3)$
1	0	2	-1
2	2	0	-1
3	1	-2	0

For these valuation functions, Γ maximizes USW and ESW. In particular, it holds that the USW for Γ is $2+2+0=4$ while the ESW for Γ is $\min\{2,2,0\}=0$.

Summing up, we have shown that Γ is SPOP, SSNS, USW, and ESW but not EFR. This shows that none of SPOP, SSNS, USW, and ESW implies EFR. Since all other notions from Section 2.3.4 except for perfectness are implied by SPOP, SSNS, USW, or ESW, none of these notions implies EFR either.

The next example shows that no notions from Section 2.3.4 is implied by EFR.

Example 2.8. Consider a very simple hedonic game $\mathscr{G}=(N,\succeq)$ with two players $N=\{1,2\}$ and the preferences $\{1\}\succ_1\{1,2\}$ and $\{2\}\succ_2\{1,2\}$. While coalition structure $\{\{1,2\}\}$ is EFR, it is neither IR nor CIS. Hence, EFR does not imply IR or CIS. Since all notions from Section 2.3.4 imply IR or CIS, none of these notions is implied by EFR.

In order to decide whether EFR is satisfied, agents have to inspect not only their own but also the coalitions of other agents. In Chapter 4, we introduce three further notions of *local fairness* that can be decided while all agents only inspect their own coalitions. The three local fairness notions, namely *min-max fairness*, *grand-coalition fairness*, and *max-min fairness*, are defined via individual threshold coalitions. In Chapter 4, we study the relations among these three local fairness notions and also relate them to other notions of stability. We further study the computational complexity of the related existence problems and of computing the threshold coalitions.

Further works studying envy-freeness in coalition formation games are due to Wright and Vorobeychik [148], Ueda [143], and Barrot and Yokoo [16]. For instance, Ueda [143] introduces and studies *justified envy-freeness*, a weakening of EFR, that is implied by CS.

2.3.6 Decision Problems for Hedonic Games

There are some natural questions that arise when studying the above stability, optimality, and fairness notions. For instance, we are interested in whether a given notion can be guaranteed for any hedonic game or whether there are hedonic games that do not allow any coalition structures that satisfy this notion. For any notion α, we are further interested in the computational complexity of the *verification problem* and the *existence problem*, which are defined as follows:

α-VERIFICATION	
Given:	A hedonic game $(N, \succeq)$ and a coalition structure $\Gamma \in \mathscr{C}_N$.
Question:	Does Γ satisfy α in $(N, \succeq)$?

α-EXISTENCE	
Given:	A hedonic game $(N, \succeq)$.
Question:	Is there a coalition structure $\Gamma \in \mathscr{C}_N$ that satisfies α in $(N, \succeq)$?

Note that there is a link between the complexities of these two problems: If α-VERIFICATION is in P for a concept α, then α-EXISTENCE is in NP as instances can be guessed nondeterministically and verified in polynomial time.

For any notion α, the following search problem is of interest as well:

α-SEARCH	
Input:	A hedonic game $(N, \succeq)$.
Output:	A coalition structure $\Gamma \in \mathscr{C}_N$ that satisfies α in $(N, \succeq)$ or "no" if there does not exist such a coalition structure.

Obviously, any upper bound on the computational complexity of α-SEARCH carries over to α-EXISTENCE, e.g., α-SEARCH $\in$ P implies α-EXISTENCE $\in$ P. Similarly, lower bounds on the computational complexity of α-EXISTENCE carry over to α-SEARCH, e.g., α-EXISTENCE being NP-hard implies α-SEARCH being NP-hard. Also, if α-VERIFICATION is in P, then α-SEARCH is in NP.

Stability Results

We will now summarize some results concerning the above problems for the stability, optimality, and fairness notions from Sections 2.3.4 and 2.3.5. Some of these results can be deduced directly from their definitions; some results are known from the literature.

Easy Verification First observe that α-VERIFICATION with $\alpha \in \{\text{IR}, \text{NS}, \text{IS}, \text{CIS}, \text{CNS}, \text{EFR}\}$ is easy for any hedonic game for which the preferences can be accessed in polynomial time. For all these notions, we can find the answer to α-VERIFICATION by iterating over all agents and checking a polynomial number (in the number of agents) of preference relations. This leads to a polynomial time algorithm if single preference relations can be checked in polynomial time. Also, whenever we can determine the agents' most preferred coalitions in polynomial time, PF-VERIFICATION is easy. For all other notions from Section 2.3.4, α-VERIFICATION is not easy in general. Indeed, it was shown that α-VERIFICATION is coNP-complete for $\alpha \in \{\text{CS}, \text{SCS}, \text{PO}, \text{POP}, \text{SPOP}, \text{USW}, \text{ESW}\}$ even if the preferences are additively separable (see Table 2.1).

Guaranteed Existence The three stability notions IR, CIS, and PO impose rather mild restrictions on coalition structures and can be fulfilled for any hedonic game. For example, for any hedonic game $(N, \succeq)$ with $N = \{1, \dots, n\}$, the coalition structure $\{\{1\}, \dots, \{n\}\}$ consisting only of singleton coalitions is IR. This follows directly from the definition of IR. Turning to PO, it can be easily seen that a PO coalition structure is guaranteed to exist by the following observations: Whenever a coalition structure Γ_2 Pareto-dominates coalition structure Γ_1, every agent weakly prefers Γ_2 to Γ_1 and at least one agent prefers Γ_2 to Γ_1. This means that the overall satisfaction grows when switching from Γ_1 to the Pareto-dominating coalition structure Γ_2. Now, assuming that there is no PO coalition structure would mean that there is an infinite sequence of coalition structures $(\Gamma_1, \Gamma_2, \dots)$. such that Γ_{i+1} Pareto-dominates Γ_i for every $i \geq 1$. Since there is only a finite number of coalition structures and since no coalition structure can occur twice in the sequence (due to the growth of satisfaction), such a sequence can not exist and there has to be a PO coalition structure. Since every PO coalition structure is CIS, this also implies the existence of a CIS coalition structure. The corresponding result for CIS was also shown by Ballester [14]. Finally, due to the guaranteed existence of these three notions, we can deduce that α-EXISTENCE is trivially in P for any hedonic game and $\alpha \in \{\text{IR}, \text{CIS}, \text{PO}\}$. In addition, Bogomolnaia and Jackson [21] show that, for any hedonic game with strict preferences,[9] there exists a coalition structure that is PO, IR, and CIS at the same time.

EFR coalition structures are guaranteed to exist for any hedonic game as well. In fact, the coalition structures $\{\{N\}\}$ consisting of the grand coalition and $\{\{1\}, \dots, \{n\}\}$ consisting only of singleton coalitions are always EFR by definition. Yet, Ueda [143] shows that there exist hedonic games where no coalition structure besides these two trivial ones is EFR.

For any cardinal hedonic game, coalition structures maximizing USW and ESW are guaranteed to exist as well. Again, this follows directly from the definitions.

Properties that Guarantee Existence For all other notions from Section 2.3.4, coalition structures that satisfy these notions are not guaranteed to exist in general hedonic games. However, some work has been done, studying properties that guarantee the existence of stable coalition structures. For example, Bogomolnaia and Jackson [21] study properties that guarantee the existence of PO, CS, NS, IS, or CIS coalition structures. They show that, for any symmetric ASHG (from here on, "additively separable hedonic game" is also abbreviated with "ASHG"), USW implies NS [21, proof of Proposition 2]. Since USW coalition structures are guaranteed to exist in ASHGs, this means that any symmetric ASHG admits a NS coalition structure. The same holds for IS and CNS coalition structures because NS implies IS and CNS. Suksompong [135] generalizes the result by Bogomolnaia and Jackson [21] and shows that NS coalition structures are even guaranteed to exist for *subset-neutral* hedonic games, a generalization of symmetric ASHGs. Moreover, Bogomolnaia and Jackson [21] show that there exists a coalition structure that simultaneously satisfies PO, IR, and IS for any ASHG with strict preferences. Banerjee et al. [15] study the existence of CS coalition

[9] That means that no player is indifferent between any two different coalitions.

structures under different restrictions of hedonic games. Motivated by the fact that there even may not be a CS coalition structure for hedonic games that satisfy rather strong restrictions, e.g., for anonymous ASHGs, they introduce the *weak top-coalition property* which guarantees the existence of a CS coalition structure. Burani and Zwicker [35] show that all symmetric ASHGs that have *purely cardinal* preference profiles admit a coalition structure that is both NS and CS. The existence of CS is also studied by Dimitrov et al. [50]. They show that CS and SCS coalition structures exist for any friend-oriented and enemy-oriented hedonic game. Furthermore, Alcalde and Revilla [1] introduce a property called *top responsiveness* that guarantees the existence of CS coalition structures. Dimitrov and Sung [48] strengthen the result of Alcalde and Revilla [1] by showing that top responsiveness even guarantees the existence of SCS coalition structures. Dimitrov and Sung [49] additionally prove that top responsiveness together with *mutuality* ensures the existence of NS coalition structures. As a counterpart to top responsiveness, Suzuki and Sung [139] introduce *bottom refuseness* (which is later called *bottom responsiveness* by Aziz and Brandl [7]). They show that, similar to top responsiveness, bottom refuseness guarantees the existence of CS coalition structures. Since friend-oriented hedonic games fulfill top responsiveness while enemy-oriented hedonic games fulfill bottom refuseness, the existence results by Alcalde and Revilla [1] for top responsiveness and by Suzuki and Sung [139] for bottom refuseness generalize the existence results by Dimitrov et al. [50] for friend-oriented and enemy-oriented hedonic games. Sung and Dimitrov [138] study the existence of CNS coalition structures and show that any hedonic game that satisfies *separability* (a generalization of additive separability) and *weak mutuality* admits a CNS coalition structure. Karakaya [81] establishes two properties that guarantee the existence of a SNS coalition structure: the *weak top-choice property* and the *descending separability* of preferences. Aziz and Brandl [7] show that the existence of a SSNS coalition structure is guaranteed in hedonic games that satisfy top responsiveness and *mutuality*. Yet, these two properties do not guarantee the existence of PF coalition structures. They also show that SIS coalition structures are guaranteed in hedonic games that satisfy *bottom responsiveness* while the existence of SNS coalition structures is guaranteed in hedonic games that satisfy *strong bottom responsiveness* and *mutuality*. Furthermore, Aziz and Brandl [7] study the existence of stable coalition structures in friend-oriented and enemy-oriented hedonic games. They show that each symmetric friend-oriented hedonic game admits a SSNS coalition structure. Moreover, each enemy-oriented hedonic game admits a SIS coalition structure and even a SNS coalition structure if the game is symmetric. They further show that SCS coalition structures are guaranteed to exist in hedonic games with strict $\mathscr{B}$-preferences [40]. Finally, Brandl et al. [25] show that CS, NS, and IS coalition structures are not guaranteed to exist in fractional hedonic games.

Complexity Results for ASHGs Without applying suiting restrictions, many classes of hedonic games do not admit stable coalition structures in general. In these cases, the related existence problems are not trivial. And even if coalition structures satisfying a given notion are guaranteed to exist, the problem of finding such coalition structures might still be intractable. Hence, there has been some research on the computational complexity of the existence and search problems for several classes of hedonic games and various stability notions. For example, Brandl et al. [25] not only show that CS, NS, and IS coalition structures are not guaranteed to exist in fractional hedonic games, but they also show that α-EXISTENCE with $\alpha \in \{\text{CS}, \text{NS}, \text{IS}\}$ is NP-hard for fractional hedonic games.

We will now illustrate some results from the literature. While doing so, we concentrate on the popular class of ASHGs. The results are summarized in Table 2.1. Sung and Dimitrov [136] show that, for any ASHG, α-EXISTENCE with $\alpha \in \{\text{NS}, \text{IS}\}$ is NP-complete and that α-EXISTENCE with $\alpha \in \{\text{CS}, \text{SCS}\}$ is NP-hard. Recall that all these hardness results carry over to the corresponding search problems. Gairing and Savani [62] strengthen the above result for NS by showing that NS-SEARCH is PLS-complete for symmetric ASHGs. In a follow-up work, Gairing and Savani [63] define some new stability concepts for symmetric ASHGs. In particular, they define *vote-in stability* which is equivalent to IS and *vote-out stability* which is equivalent to CNS. The combination of vote-in and vote-out stability is equivalent to CIS. They show that, for symmetric ASHGs, IS-SEARCH is PLS-complete while CNS-SEARCH is in P. Sung and Dimitrov [137] show that CS-VERIFICATION is coNP-complete for enemy-oriented hedonic games. Since enemy-oriented hedonic games are a subclass of ASHGs, the hardness extends to the general case of ASHGs. Aziz et al. [10] extend the result by showing that SCS-VERIFICATION is coNP-complete for enemy-oriented hedonic games as well. Furthermore, Aziz et al. [10] show many more results concerning the complexity of verification, existence, and search problems in ASHGs. For instance, they present an algorithm that finds a CIS coalition structure for any ASHG, i.e., CIS-SEARCH is in P. Woeginger [146] shows that C-EXISTENCE is Σ_2^p-complete for ASHGs. Afterwards, Woeginger [147] surveys the results and open problems concerning CS and SCS. Peters [116] extends the hardness result by Woeginger [146] and shows that SC-EXISTENCE is Σ_2^p-complete for ASHGs. Peters and Elkind [117] establish metatheorems that show the NP-hardness of α-EXISTENCE for several stability notions α. They apply these theorems to several classes of hedonic games such as ASHGs and fractional hedonic games. For ASHGs, their metatheorems reveal that α-EXISTENCE is NP-hard for $\alpha \in \{\text{NS}, \text{IS}, \text{CS}, \text{SCS}, \text{SSNS}, \text{SNS}, \text{SIS}\}$. Brandt and Bullinger [26] study POP and SPOP in ASHGs. They show that POP-EXISTENCE is NP-hard and coNP-hard for symmetric ASHGs. Thus, they conclude that it is likely that this problem is even Σ_2^p-complete. They also show that, for symmetric ASHGs, SPOP-EXISTENCE is coNP-hard and α-VERIFICATION with $\alpha \in \{\text{POP}, \text{SPOP}\}$ is coNP-complete. Furthermore, Aziz et al. [10] and Bullinger [33] study the combination of PO with other stability notions in ASHGs. They, e.g., show that it is hard to find coalition structures that are PO and EF or PO and IR. Last, Peters [115] and Hanaka and Lampis [73] study stability in ASHGs (and other classes of hedonic games) from the viewpoint of parameterized complexity.

Outlook Needless to say, there exists more interesting related literature. For instance, some research deals with the prices of stability, optimality, and fairness. These prices measure the losses of social welfare that come with certain stability, optimality, or fairness notions. For example, the price of NS is the worst-case ratio between the maximum social welfare and the social welfare of any NS coalition structure. Bilò et al. [19] study the price of NS in fractional hedonic games. Elkind et al. [58] consider the price of PO in additively

α	α-VERIFICATION	α-EXISTENCE	α-SEARCH
PF	in P [10]	in P [10]	in P [10]
IR	in P [10]	trivial [10]	trivial [10]
NS	in P [10]	NP-complete [136], trivial if sym [21, 135]	NP-hard [136], PLS-complete [62]
IS	in P [10]	NP-complete [136], trivial if sym [21]	NP-hard [136], PLS-complete [63]
CIS	in P [10]	trivial [14]	in P [10]
CNS	in P	in NP, trivial if sym [21, 138]	in NP, in P if sym [138, 63]
CS	coNP-complete [137]	Σ_2^p-complete [146, 116, 111], trivial if fn. 1 holds	Σ_2^p-complete [146, 116, 111]
SCS	coNP-complete [10]	Σ_2^p-complete [116, 111]	Σ_2^p-complete [116, 111]
PO	coNP-complete [10, 33]	trivial	in P if fn. 2 holds, in P if fn. 3 holds
POP	coNP-complete [10, 26]	NP-hard [10, 26], coNP-hard [26]	NP-hard [10, 26], coNP-hard [26]
SPOP	coNP-complete [26]	coNP-hard [26]	coNP-hard [26]
USW	coNP-complete [10]	trivial [10]	NP-hard [10]
ESW	coNP-complete [10]	trivial [10]	NP-hard [10]
EFR	in P [10]	trivial [10, 143]	trivial [10, 143]

1 If the game is symmetric and preferences are *purely cardinal* [35], if the game is friend-oriented [50], or if the game is enemy-oriented [50].
2 If all preferences are strict [10].
3 If the game is *mutually indifferent* [33]. Note that symmetry implies mutual indifference.

Table 2.1: Computational complexity of the problems from Section 2.3.6 for additively separable hedonic games and the stability, optimality, and fairness notions from Sections 2.3.4 and 2.3.5. Some additional results are given for subclasses of additively separable hedonic games, e.g., "if sym" indicates that a result holds for symmetric additively separable hedonic games.

separable, fractional, and modified fractional hedonic games. Brânzei and Larson [29] investigate the price of CS in *coalitional affinity games* which are equivalent to ASHGs. In Chapter 4, we study the price of *local fairness* in ASHGs.

Another recent branch of research studies the dynamics of deviations in hedonic games. Bilò et al. [19] study best-response Nash dynamics in fractional hedonic games. Hoefer et al. [77] analyze the impact of structural constraints (locality and externality) on the convergence in hedonic games. Carosi et al. [37] introduce *local core stability* and study the convergence of local core dynamics in simple symmetric fractional hedonic games. They also study the price of local core stability. Brandt et al. [27] investigate how deviations according to the notion of IS converge in various classes of hedonic games including anonymous and fractional hedonic games. Brandt et al. [28] study dynamics based on single-player deviations in ASHGs.

Further interesting research concerns the robustness of stability against the deletion of agents (*agent failure*) [79], strategyproof mechanisms that prevent strategical agent behavior [59], or hedonic games where the communication of the agents is restricted by an underlying graph such that agents can only form a coalition if they are connected in the graph [78].

Chapter 3

Altruism in Coalition Formation Games

In game theory, it is usually assumed that the agents are completely self-interested and act perfectly rational to accomplish their individual goals. Hence, the agents are assumed to always take those actions that lead them to their own optimal outcomes. This idea is related to the notion of the *homo economicus*. However, there has been some recent research from evolutionary biologists that shows that this approach is obsolete. In 2020, Hare and Woods [74] rephrased the Darwinian evolutionary thesis "survival of the fittest" with the thesis "survival of the friendliest". They studied the social behavior of several animal species, including dogs but also chimpanzees and bonobos. They observe that species with highly developed social skills and friendly behavior towards other individuals of their own and other species have an evolutionary advantage. They even argue that friendliness was essential for the success of the human species.

Along the same lines, there has been some research that attempts to integrate social aspects into models of cooperative game theory. Some authors introduce social aspects as altruism via a social network among the agents [6, 20, 76, 2]. Others directly integrate an agent's degree of selfishness or altruism into her utility function [75, 42, 3, 119]. Rothe [129] surveyed the approaches to altruism in game theory.

In the following three sections, we will study altruism in the scope of coalition formation games. In Section 3.1, we introduce *altruistic hedonic games* where agents are not narrowly selfish but also take the opinions of their friends into account when comparing two coalitions. We distinguish between several models of altruism and investigate them with respect to their axiomatic properties and the computational complexity of the associated decision problems. We continue our study in Sections 3.2, concentrating on the notions of popularity and strict popularity. In Section 3.3, we extend the models of altruism to the more general scope of coalition formation games and show that this extension brings some axiomatic advantages.

Related work Since the first introduction of altruistic hedonic games (see the preceding conference version [107] of the paper that we present in the next section [91]), there has appeared some follow-up research concerning aspects of altruism in hedonic games. For example, Schlueter and Goldsmith [130] introduce so-called *super altruistic hedonic games*. In their model, agents also behave altruistically towards agents that are further away in a social network but weight their altruistic consideration with their distances to them. This approach is related to the social distance games by Brânzei and Larson [30]. Bullinger and Kober [34] also generalize the preceding models of altruistic hedonic games. They introduce what they call *loyalty* in hedonic games. For any cardinal hedonic game, they consider agents to be loyal to any other agent that yields a positive utility when being with her in a coalition of size two. Miles [100] provides a useful online tool that can be used to simulate altruistic, friend-oriented, fractional, or additively separable hedonic games.

3.1 Altruistic Hedonic Games

This section is about the following journal article that introduces and studies *altruistic hedonic games*.

Publication (Kerkmann et al. [91])

A. Kerkmann, N. Nguyen, A. Rey, L. Rey, J. Rothe, L. Schend, and A. Wiechers. "Altruistic Hedonic Games". In: *Journal of Artificial Intelligence Research* 75 (2022), pp. 129–169

3.1.1 Summary

While previous literature on hedonic games focuses mainly on selfish players, this work introduces and studies *altruism in hedonic games*. The main idea while introducing our concepts of altruism is that players do not only care about their own valuations of coalitions but also about the valuations of others. We assume that the players have mutual friendship relations which are represented by a *network of friends*. We then assume that agents care about all their friends, i.e., their neighbors in the network of friends. When introducing the altruistic behavior, we incorporate the opinions of an agent's friends into her utility. While doing so, we make sure that the game is still hedonic: Agents only care about their own coalitions; hence, they only consider those friends that are in the same coalition. We focus on friend-oriented valuations of coalitions [50] and distinguish three degrees of altruism. First, we define a *selfish-first* degree where an agent first looks at her own friend-oriented valuation of a coalition and, only in the case that she values two coalitions the same, she looks at the her friends' valuations. Second, in the case of *equal-treatment* preferences, an agent treats herself and her friends in her coalition the same and aggregates all valuations with equal weights. Last, we introduce *altruistic-treatment* preferences where an agent first asks her friends for their valuations and, only in the case that her friends value two coalitions the same, she decides based on her own friend-oriented valuations. When aggregating the friends' valuations, we further distinguish between two aggregation methods. For *average-based* hedonic preferences, we aggregate the valuations by taking the average and, for *minimum-based* hedonic preferences, we aggregate by taking the minimum. This change of the aggregation function might seem minor but in fact makes a major difference in the altruistic behavior.

After introducing the different models of altruism in hedonic games, we differentiate them from the literature and study some axiomatic properties. In particular, our models can express preferences that can not be expressed by other models known from the related literature. Furthermore, they satisfy some desirable properties such as reflexivity, transitivity, polynomial-time computability of single preferences, and anonymity. After finishing our axiomatic study, we further consider the problems of verifying stable coalition structures in altruistic hedonic games and of deciding whether a stable outcome exists for a given altruistic hedonic game.

We study both problems for several common stability notions, such as Nash stability, core stability, and perfectness. While studying these problems, we not only concentrate on altruistic hedonic games where all agents act according to the same average-based or minimum-based degree of altruism but also consider the case of general altruistic hedonic games where each agent might individually behave according to a different degree of altruism. For selfish-first altruistic hedonic games, we provide a complete picture of the complexity of all considered problems. In particular, we show that there exist individually rational, Nash stable, individually stable, and contractually individually stable coalition structures for any altruistic hedonic game. For selfish-first altruistic hedonic games, even core stable and strictly core stable coalition structures are guaranteed to exist and the existence of perfect coalition structures can be decided in polynomial time. Concerning the verification problem, we proof that, for general altruistic hedonic games, individual rationality, Nash stability, individual stability, and contractual individual stability can be verified in polynomial time while core stability and strict core stability verification are coNP-complete. For selfish-first altruistic hedonic games, we further show that perfectness verification is in P.

3.1.2 Personal Contribution and Preceding Versions

This journal paper largely extends and improves multiple preceding conference papers that were published by Nhan-Tam Nguyen, Anja Rey, Lisa Rey, Jörg Rothe, and Lena Schend at AAMAS'16 [107], by Alessandra Wiechers and Jörg Rothe at STAIRS'20 [145], and by Jörg Rothe at AAAI'21 [129]. Parts of the AAMAS'16 paper were also presented at CoopMAS'16 [108] and COMSOC'16 [109]. Furthermore, Jörg Rothe and I presented some axiomatic properties of altruistic hedonic games at COMSOC'21 [86].

The modeling of the average-based altruistic hedonic games is due to the authors of the AAMAS'16 paper [107] and the modeling of the minimum-based variation is due to the authors of the STAIRS'20 paper [145].

My contributions are the merging and reorganization of the individual conference papers, additional related work in Section 2, the revision of various proofs from the AAMAS'16 paper [107] (the proofs of Propositions 4.2, 5.2, 6.14, and 6.17, Theorems 5.3, 5.5, 5.6, 6.5, and 6.6, and Lemma 6.1), additional visualizations (Figure 2 and Tables 2 and 4), the extension of various results to the more general case where agents might act according to different degrees of altruism, and additional results. In particular, I contributed all results concerning the properties of min-based altruistic preferences (see Section 5.3), the detailed results regarding the property of friend-dependence in Theorem 5.4, the results for type-I-monotonicity under average-based EQ and AL preferences in Theorem 5.6, Lemmas 6.3 and 6.13, Example 6.7, Theorem 6.9, Corollaries 6.10, 6.12, and 6.15, and the proofs of Propositions 6.11 and 6.16.

The writing of this journal paper was done jointly with all co-authors. The finalization and polishing were done by Jörg Rothe and me.

3.1.3 Publication

The full article [91] is appended here.

3.2 Popularity and Strict Popularity in Average-Based and Minimum-Based Altruistic Hedonic Games

The next article studies the problem of verifying popular and strictly popular coalition structures in average-based and minimum-based altruistic hedonic games.

Publication (Kerkmann and Rothe [88])

A. Kerkmann and J. Rothe. "Popularity and Strict Popularity in Average-Based and Minimum-Based Altruistic Hedonic Games". Submitted to the *47th International Symposium on Mathematical Foundations of Computer Science*. 2022

3.2.1 Summary

Considering hedonic games, the question of what accounts for a 'good' coalition structure naturally arises. There are several notions of stability in hedonic games that indicate whether an agent or a group of agents have an incentive to deviate from a given coalition structure. These concepts include, e.g., Nash stability, individual stability, and core stability. By contrast, this work studies popularity and strict popularity in hedonic games. These two concepts measure whether a given coalition structure is preferred to every other possible coalition structure by a (strict) majority of the agents.

We study popularity and strict popularity in (minimum-based) altruistic hedonic games [107, 145, 91] and determine the complexities of two decision problems. First, we consider the problem of verifying whether a given coalition structure in a given altruistic hedonic game is (strictly) popular. Second, we consider the existence problem which asks whether there exists a (strictly) popular coalition structure for a given altruistic hedonic game. While the complexity of these problems has been partly determined for strict popularity by Nguyen et al. [107] and Wiechers and Rothe [145], the problems have not been considered before for the notion of popularity. We solve all cases of strict popularity verification in (minimum-based) altruistic hedonic games that were left open by Nguyen et al. [107] and Wiechers and Rothe [145]. Furthermore, we completely determine the complexity of popularity verification in (minimum-based) altruistic hedonic games for all degrees of altruism. Our results reveal that all considered verification problems are coNP-complete. Additionally, we obtain some coNP-hardness results for strict popularity existence in equal-treatment and altruistic-treatment altruistic hedonic games. Besides, we infer that popularity verification is also coNP-complete for friend-oriented hedonic games.

3.2.2 Personal Contribution and Preceding Versions

A preliminary version of this paper has been accepted for publication at AAMAS'22 [87].

All technical results of the paper are my contribution. The writing and polishing was done jointly with Jörg Rothe.

3.2.3 Publication

The full article [88] is appended here.

3.3 Altruism in Coalition Formation Games

The following article studies altruism in the more general scope of coalition formation games.

Publication (Kerkmann et al. [90])

A. Kerkmann, S. Cramer, and J. Rothe. "Altruism in Coalition Formation Games". Submitted to the *Annals of Mathematics and Artificial Intelligence*. 2022

3.3.1 Summary

Inspired by the altruistic hedonic games by Nguyen et al. [107], this work introduces *altruism in general coalition formation games*. While extending the framework of Nguyen et al. [107], we model agents to behave altruistically to *all their friends*, not only to the friends in their current coalitions (as it is the case for altruistic hedonic games). The model is grounded on the *friends-and-enemies encoding* by Dimitrov et al. [50] where players can be represented by the vertices of an undirected graph with the edges representing mutual friendship relations. We then consider the *friend-oriented valuations* of the agents and distinguish between the three *degrees of altruism* introduced by Nguyen et al. [107]: selfish first, equal treatment, and altruistic treatment. We further distinguish between a sum-based and minimum-based aggregation of valuations. We show that our resulting altruistic models satisfy some desirable properties and argue that it is not reasonable to exclude any of an agent's friends from her altruistic behavior. We show that all our models lead to *unanimous* preferences while the altruistic hedonic games by Nguyen et al. [107] (and the min-based altruistic hedonic games by Wiechers and Rothe [145]) can lead to equal-treatment and altruistic-treatment preferences that are not unanimous. Moreover, we show that our models also fulfill some basic properties introduced by Nguyen et al. [107] but our models fulfill more types of *monotonicity* than the altruistic hedonic models. After completing the axiomatic study of altruistic coalition formation games, we consider some common stability notions from the context of hedonic games. We extend the notions to the more general context of our work and study the computational complexity of the associated *verification* and *existence problems*. We obtain broad results for the case of selfish-first preferences and initiate the study for the other two degrees of altruism. In particular, we show that the verification and existence problems are in P for individual rationality, Nash stability, and individual stability in all our altruistic models (all three degrees of altruism and both aggregation functions). For core stability, popularity, and strict popularity verification, we obtain coNP-completeness results for the selfish-first models. Core stability and strict core stability existence are trivial for selfish-first altruistic coalition formation games as there always exist strictly core stable coalition structures in these games. Furthermore, we obtain several upper bounds on the complexity of perfectness verification and existence.

3.3.2 Personal Contribution and Preceding Versions

Me and Jörg Rothe published a work about sum-based altruism in coalition formation games at IJCAI'20 [84]. This journal article merges the IJCAI'20 paper [84] with a Bachelor's thesis about min-based altruism by Simon Cramer [46] and further results that were partly presented by me and Jörg Rothe at COMSOC'21 (with nonarchival proceedings [86]). Parts of this work were also presented at the 16th and 17th International Symposium on Artificial Intelligence and Mathematics (ISAIM'20 with nonarchival proceedings [85] and ISAIM'22 without any proceedings).

The model that we present in this work extends a model introduced by Nguyen et al. [107]. The presented extension of the model to sum-based altruistic coalition formation games and all technical results concerning sum-based altruistic coalition formation games are my contribution. Furthermore, I contributed all axiomatic results from Section 3 and extended some results for sum-based altruistic coalition formation games to the min-based case (viz., Example 3, Theorem 5, Corollary 1, and Proposition 8).

The writing and polishing of the paper was done jointly with all co-authors.

3.3.3 Publication

The full article [90] is appended here.

Chapter 4

Local Fairness in Hedonic Games via Individual Threshold Coalitions

This chapter summarizes the following journal article in which we introduce and study three local fairness notions for hedonic games:

Publication (Kerkmann et al. [93])

A. Kerkmann, N. Nguyen, and J. Rothe. “Local Fairness in Hedonic Games via Individual Threshold Coalitions”. In: *Theoretical Computer Science* 877 (2021), pp. 1–17

4.1 Summary

In this work, we introduce and study three notions of *local fairness* in hedonic games. Previous literature by Bogomolnaia and Jackson [21], Aziz et al. [10], Wright and Vorobeychik [148], and Peters [114, 115] considers *envy-freeness* as a notion of fairness in hedonic games. However, this notion requires agents to inspect other coalitions than their own. In contrast to this notion, our *local* fairness notions can be decided solely based on the agents' own coalitions and their individual preferences.

We define the three local fairness notions *min-max fairness*, *grand-coalition fairness*, and *max-min fairness* based on three different threshold coalitions. For each agent, these thresholds are solely defined on her individual preference. Moreover, a coalition structure is fair for an agent if she weakly prefers her coalition in this coalition structure to her threshold coalition.

After introducing the three local fairness notions, we show that they form a strict hierarchy: max-min fairness implies grand-coalition fairness which in turn implies min-max fairness. We also relate the three notions to other stability notions that are known from the literature, such as Nash stability, core stability, envy-freeness by replacement, and individual rationality. We then study the problem of computing the fairness thresholds and determine the complexity of this problem in the context of additively separable hedonic games. We also determine subclasses of hedonic games where fair coalition structures are guaranteed to exist. Since this does not hold for general additively separable hedonic games, we also ask for the complexity of determining whether a fair coalition structure exists in a given additively separable hedonic game.

Afterwards, we study the minimum and maximum price of local fairness which describe the best-case and worst-case loss of social welfare of a coalition structure that satisfies fairness compared to the coalition structure with maximum utilitarian social welfare. In doing so, we concentrate on min-max fairness which is the weakest of our three local fairness notions and constrains the set of possible coalition structures less than the other two notions. For symmetric additively separable hedonic games, we show that the maximum price of min-max fairness is not bounded by a constant but the minimum price of min-max fairness is always one.

Finally, we discuss an alternative fairness notion and argue that there is no local fairness notion stronger than individual rationality such that fair coalition structures exist for every hedonic game.

4.2 Personal Contribution and Preceding Versions

This journal publication extends a preliminary conference version by Nhan-Tam Nguyen and Jörg Rothe [105] that was also presented at CoopMAS'16 [106]. Contents that I contributed to our work are additional writing and improved presentation throughout the paper (e.g., the reordering of definitions in Section 2, the improvement of Figure 1, the reorganization of Sections 3.1 and 3.3, additional Footnotes 3, 4, 5, and 6, the extension of Definition 8, and the remark after Corollary 6), additional related work in Section 1.3, the examples and explanations in Propositions 1 and 2, the first example and explanation in Proposition 3, Section 4.1 (where a preliminary version of Theorem 4 was contained in [105]), the first part of Theorem 6 that shows the membership of Min-Max-Exist in NP, Observation 6, and the argumentation for Proposition 4.

4.3 Publication

The full article [93] is appended here.

CHAPTER 5

Hedonic Games with Ordinal Preferences and Thresholds

In this chapter, we summarize the following journal article in which we introduce and study a new preference representation in hedonic games where agents submit ordinal rankings that are separated by two thresholds:

Publication (Kerkmann et al. [92])

A. Kerkmann, J. Lang, A. Rey, J. Rothe, H. Schadrack, and L. Schend. "Hedonic Games with Ordinal Preferences and Thresholds". In: *Journal of Artificial Intelligence Research* 67 (2020), pp. 705–756

5.1 Summary

In this work, we introduce and study a new class of hedonic games which we call *FEN-hedonic games*. In these games, the agents partition the other agents into friends, enemies, and players that they are neutral to. Additionally, they submit a weak order on their friends and on their enemies, respectively. The resulting preference representation is referred to as *weak ranking with double threshold*. Based on this representation, we then infer preferences over coalitions using the *responsive extension principle*. Since the resulting polarized responsive extensions are not always complete, we consider agents to *possibly* or *necessarily prefer* a coalition to another one if this preference holds for *at least one* or *all* completions of their polarized responsive extensions. Afterwards, we introduce so-called *optimistic* and *pessimistic preference extensions*.

Using these extensions, we then characterize stability in FEN-hedonic games. In addition, we study the problems of verifying stable coalition structures in FEN-hedonic games and of checking whether stable coalition structures exist. While doing so, we distinguish between *possible* and *necessary stability*, depending on whether there exists at least one extended preference profile that satisfies stability or whether all extended preference profiles satisfy stability. While these verification and existence problems for possible and necessary stability are in P for the strongest and weakest notion that we consider, namely for *perfectness* and *individual rationality*, we also show some hardness results for some other stability notions. For example, we show that possible and necessary Nash stability verification are in P, while possible and necessary Nash stability existence are NP-complete. We also show that possible verification is coNP-complete for core stability, strict core stability, Pareto optimality, popularity, and strict popularity. Also, necessary verification is coNP-complete for Pareto optimality, popularity, and strict popularity. Finally, we close our work with a short discussion and some directions for future work.

5.2 Personal Contribution and Preceding Versions

This journal paper largely extends two conference papers published by Jérôme Lang, Anja Rey, Jörg Rothe, Hilmar Schadrack, and Lena Schend at AAMAS'15 [96] and by me and Jörg Rothe at AAMAS'19 [89]. The modeling is due to the authors of the AAMAS'15 paper [96]. The writing of this journal paper was done jointly with all co-authors. The ideas of all technical results that also appear in the AAMAS'19 paper [89] are my contribution. Also some technical parts from the preceeding AAMAS'15 paper [96] that were revised for this journal paper are my contribution as well. Parts of the technical results of this journal paper have already appeared, in preliminary form, in my Master's Thesis [83]. However, their presentation and many of their proofs were improved in the journal paper.

5.3 Publication

The full article [92] is appended here.

Chapter 6

Conclusion and Future Work Directions

Based on the current state of research in the field of cooperative game theory, we have proposed new preference formats for hedonic games, established several models of altruism, and studied stability, optimality, and fairness in multiple classes of hedonic games. We will now summarize the contributions of this thesis and highlight some directions for future research.

In Chapter 3, we started our study with topics of altruism. Evolutionary biology has revealed that selfishness is not always a means to success in the real world, but rather friendliness constituted an essential advantage in the evolution of certain species, including humans [74]. Motivated by this fact and with the aim to provide a more realistic model of real world scenarios, we introduced several models of altruism in coalition formation games. In Section 3.1, we presented *altruistic hedonic games* that model agents to behave altruistic towards their friends in a given network. We distinguished between three degrees of altruism and between two ways of aggregating the agents' preferences. We studied the six resulting models with respect to their axiomatic properties, showing that they fulfill some desirable properties while they can represent situations that can not be represented by other preference formats from the literature. We then conducted a computational analysis concerning stability verification and existence, focusing on Nash, individual, contractually individual, core, and strict core stability, individual rationality, and perfectness. For selfish-first altruistic hedonic games, we have settled the complexity of all considered problems. We further initiated the study for altruistic hedonic games where the agents behave according to different degrees of altruism. An important direction for future research is the completion of this study, i.e., the determination of the complexity of all considered verification and existence problems in this case.

In Section 3.2, we continued our study of altruistic hedonic games, focusing on the notions of popularity and strict popularity. We have solved all open cases of popularity and strict popularity verification, showing that verification is coNP-complete for popularity and strict popularity and all considered models of altruism. We even proved the coNP-hardness of strict popularity existence under equal- and altruistic-treatment. Yet, we suspect that these existence problems might be even harder. It is an interesting question for future research whether popularity and strict popularity existence are even Σ_2^p-complete in altruistic hedonic games. An interesting side result of our study is that popularity verification is also coNP-complete for friend-oriented hedonic games.

We have extended our models of altruism in Section 3.3. While altruistic hedonic games model agents to be altruistic to their friends in their current coalitions, we have additionally proposed *altruistic coalition formation games* where agents behave altruistic to all their friends, not only to those in the same coalition. We have seen that this removal of the hedonic restriction brings some axiomatic advantages. Particularly, altruistic coalition formation preferences are unanimous, which is not the case for all altruistic hedonic preferences. Furthermore, altruistic coalition formation preferences fulfill more cases of monotonicity than altruistic hedonic preferences. We have also initiated the study of stability in altruistic coalition formation games. Our results include characterizations of stability in altruistic coalition formation games and computational bounds on the complexity of the associated verification and existence problems.

There are several possible future work directions in the scope of Chapter 3. So far, altruism in coalition formation games was always handled as a static model where agents were only acting according to one selected degree of altruism. We are interested in models where agents individually and dynamically may choose to what degree they wish to act altruistically, which seems to be more realistic: Agents can be expected to act most altruistically when they see that others are suffering, and they are more egoistic if everyone around them is doing well. This can also be observed in reality where solidarity with others increases when social crises occur. We regard modeling such situations as a promising topic for future research. Other research in the scope of altruistic coalition formation could concern the relationship between altruism and fairness: Do altruistic preferences favor the formation of fair outcomes? Furthermore, it could be interesting to apply altruistic coalition formation games to other valuation functions. While we currently use friend-oriented valuations as a basis of our model, one might also consider general additively separable or fractional valuations.

We continued with aspects of fairness in Chapter 4 where we introduced three notions of *local fairness* for hedonic games. We showed that the three notions form a strict hierarchy and related them to other common notions of stability, fairness, and optimality. We intensively studied the three notions of altruism for additively separable hedonic games. Our studies concerning the local fairness notions provide a diverse potential for follow-up research. For instance, it would be interesting to extend the studies concerning the price of local fairness in additively separable hedonic games and find restrictions to the players' preferences such that the price of local fairness is bounded by a nontrivial constant. Another appealing future direction is the investigation of local fairness in other classes of hedonic games such as fractional or modified fractional hedonic games.

Moreover, we provided an elaborate study of *FEN-hedonic games*. In Chapter 5, we introduced the corresponding preference representation that is composed of weak ordinal rankings over the agents which are separated by two thresholds. The new format adds to existing literature that deals with the separation of the agents into friends and enemies (see, e.g., Dimitrov et al. [50], Sung and Dimitrov [137, 136], Rey et al. [122], Ota et al. [111], and Barrot et al. [17]). It is succinct, easy to elicit from the agents, and reasonably expressive. We have examined a variety of stability notions in the context of FEN-hedonic games, distinguishing between possible and necessary satisfaction of these notions. An intriguing topic for future research could be the integration of altruism in FEN-hedonic games. It might be a challenging goal to create a model that uses the rather expressive and at the same time simple representation of weak rankings with double thresholds and lifts these rankings to altruistic preferences over coalitions.

In conclusion, this thesis has made a significant contribution to the field of altruism in coalition formation games, expanded the research on the topic of fairness in hedonic games, and provided a comprehensive study of hedonic games with ordinal preferences and thresholds (FEN-hedonic games). Nevertheless, many exciting questions remain that future research might seek to answer.

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List of Figures

List of Tables

Eidesstattliche Erklärung

entsprechend §5 der Promotionsordnung vom 15.06.2018

Ich versichere an Eides Statt, dass die Dissertation von mir selbständig und ohne unzulässige fremde Hilfe unter Beachtung der „Grundsätze zur Sicherung guter wissenschaftlicher Praxis an der Heinrich-Heine-Universität Düsseldorf" erstellt worden ist.

Des Weiteren erkläre ich, dass ich eine Dissertation in der vorliegenden oder in ähnlicher Form noch bei keiner anderen Institution eingereicht habe.

Teile dieser Dissertation wurden bereits in den folgenden Schriften veröffentlicht, zur Publikation angenommen oder zur Begutachtung eingereicht:

- Kerkmann et al. [91] mit den vorläufigen Versionen [107, 108, 109, 145, 129, 86];
- Kerkmann and Rothe [88] mit der vorläufigen Version [87];
- Kerkmann et al. [90] mit den vorläufigen Versionen [84, 85, 46, 86];
- Kerkmann et al. [93] mit den vorläufigen Versionen [105, 106]
- Kerkmann et al. [92] mit den vorläufigen Versionen [96, 89, 83].

Meine Anteile an diesen Schriften werden auf den Seiten 36, 39, 41, 44 und 46 erläutert.

Ort, Datum	Anna Maria Kerkmann