Host Community Respecting Refugee Housing[1]

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Abstract

We propose a novel model for refugee housing respecting the preferences of the accepting community and refugees themselves. In particular, we are given a topology representing the local community, a set of inhabitants occupying some vertices of the topology, and a set of refugees that should be housed on the empty vertices of the graph. Both the inhabitants and the refugees have preferences over the structure of their neighbourhood.

We are specifically interested in the problem of finding housing such that the preferences of every individual are met; using game-theoretical words, we are looking for housing that is stable with respect to some well-defined notion of stability. We investigate conditions under which the existence of a stable outcome is guaranteed and study the computational complexity of finding such a stable outcome. As the problem is NP-hard even in very simple settings, we employ the parameterised complexity framework to give a finer-grained view of the problem's complexity with respect to natural parameters and structural restrictions of the given topology.

1 Introduction

According to the last report of the United Nations High Commissioner for Refugees (UNHCR), there were 89.3 million forcibly displaced persons at the end of 2021 [66]. It is the highest number since the aftermath of World War II and it is for sure that these numbers will even grow. In May 2022, UNHCR announced that a tragic milestone of 100 million displaced persons was reached. They identified the war in Ukraine as the leading cause of the dramatic growth in the last year [57]. Russian aggression not only forced many Ukrainians to leave their homes, but even caused food insecurity and related population movement in many parts of the world, since Ukraine is among the fifth largest wheat exporters in the world [12].

It should be mentioned that political and armed conflicts are not the only causes of forced displacement [66]. One of the most common reasons for fleeing is due to natural disasters. To name just a few, in August 2022, massive floods across Pakistan affected at least two-thirds of the districts and displaced at least 33 million people [54, 55]; the numbers are not yet final. At the same time, a devastating drought in Somalia caused the internal displacement of at least 755,000 people [66]. Furthermore, it is expected that, due to climate change, extremes of the climate will become even more common in the near future [38].

Arguably, the best prevention against the phenomenon of forced displacement is not allowing it to appear at all; however, the aforementioned numbers clearly show that these efforts are not very successful. Therefore, in practise, three main solutions are assumed [43]. Voluntary *repatriation* is the most desirable but not very successful option. In many situations, repatriation is not even possible due to ongoing conflicts or a completely devastated environment. *Resettlement* and *integration* in the country of origin or abroad are more common. These two solutions require considerable effort from both the newcomers and the host community sides.

[1]Part of this work has been published in the Proceedings of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS '23) [45].

The very problematic part of forced displacement is the fact that 38% of all refugees[2] are hosted in only five countries [66]. And these are only the absolute numbers. For example, in Lebanon, every one in four people is a refugee [59]. The redistribution of refugees seems to be a natural solution to this imbalance; however, not all countries are willing to accept all people. One such example can be the Czech Republic, which refused to accept any Syrian refugees during the 2015 European migrant crisis, currently hosting the largest number of Ukrainian refugees per capita [67].

Even with working and widely accepted redistributing policies, there is still a need to provide housing in specific cities and communities. From the good examples of such integration strategies [60] it follows that one of the most important characteristics is that members of the accepting community do not feel threatened by newcomers.

Inspired by this, we propose a novel computational model for refugee housing. Our ultimate goal is to find an assignment of displaced persons into empty houses of a community such that this assignment corresponds to the preferences of the inhabitants about the structure of their neighbourhoods and, at the same time, our model also takes into consideration the preferences of the refugees themselves, as refugees dissatisfied with their neighbourhood have a strong intention to leave the community. More precisely, in our model, we are given a topology of the community, which is an undirected graph, a set of inhabitants together with their assignment to the vertices of the topology and preferences over the shape of their neighbourhood, and a set of refugees with the same requirements on the neighbourhood shape. We want to find a housing of refugees in the empty vertices of the topology such that the housing satisfies a certain criterion, such as stability.

Refugee redistribution has gained the attention of mathematicians and computer scientists very recently. The formal model for capturing refugee resettlement is double-sided matching [27, 28, 8, 3]. That is, on the input we are given a set of locations with multidimensional constraints and a set of refugees with multidimensional features. An example of a constraint can be the number of refugees the locality can accept on one side and the size of family on the refugee side. The question then is whether there exists a matching between localities and refugees respecting all constraints. According to us, this formulation of the refugees resettlement problem more concerns the global perspective of refugee redistributing, not the local housing problem, as we do in our paper. Aziz et al. [8] study mostly the complexity of finding stable matching with respect to different notions of stability, and it turns out that for most of the stability notions finding a stable matching is computationally intractable (NP-hard, in fact). Kuckuck et al. [49] later refined the model of Aziz et al. [8] in terms of hedonic games.

Our Contribution. Partly continuing the line of research in refugee resettlement, we introduce a novel model focused on the local housing of new refugees. Previous models [8, 3, 28] can be seen and used as a very effective model on the (inter-) national level to distribute refugees to certain locations, such as states or cities[3]. However, our model can be assumed as the second level of refugee redistribution; once refugees are allocated to some community, we want to house them in a way that respects preferences of both inhabitants and refugees.

In particular, we introduce three variants of refugee housing, each targeting a certain perspective of this problem. Our simplest model, introduced in Section 3, completely eliminates the preferences of refugees and studies only the stability of the housing with respect to the preferences of the inhabitants. We call this variant *anonymous* housing. Since refugees are

[2]From the strict sociological point-of-view, not all forcibly displaced persons are classified as refugees. Slightly abusing the terminology, we will use the term refugee and displaced person interchangeably.

[3]In fact, the American resettlement agency HIAS use the matching software *Annie*^{TM} MOORE which was later improved by Ahani et al. [3].

assumed to be indistinguishable, inhabitants have preferences over the number of refugees in their neighbourhood.

As stated above, the most successful refugee integration projects have the following property in common; they try to make both inhabitants and refugees as satisfied as possible by various activities to ensure that both groups get to know each other. We believe that our *hedonic* model, where the preferences of both inhabitants and refugees are based on the identity of particular members of the second group, supports and leads to more stable and acceptable housing. This model is formally defined and studied in Section 4.

The two introduced models have some disadvantages. The first is disrespectful to the refugees' preferences, while the second is not very realistic, as it is hard to make all inhabitants familiar with all refugees and the other way around. Therefore, our last model can be seen as a compromise between these two extremes. In the *diversity* setting, introduced in Section 5, all agents (union of inhabitants and refugees) are partitioned into k types, and preferences are over the fractions of agents of each type in the neighbourhood of each agent. Another advantage of this approach is that it nicely captures also the settings where we already have number of integrated refugees and the newcomers want to have some of them in the neighbourhood, or the case of an internally displaced person.

In all the aforementioned variants of the refugee housing problem, agents have dichotomous preferences; that is, they approve some set of alternatives and do not distinguish between them. It can be seen as if the neighbourhood of some agent does not comply with his approval set, he would rather leave the local community, which is very undesired behaviour on both sides.

For all assumed variants, we show that an equilibrium is not guaranteed to exist even in very simple instances. Thus, we study the computational complexity of finding an equilibrium or deciding that no equilibrium exists. To this end, we provide polynomial-time algorithms and complementary NP-hardness results. In order to paint a more comprehensive picture of the computational tractability of the aforementioned problems, we employ a finer-grained framework of parameterised complexity to give tractable algorithms for, e. g., instances where the number of refugees or the number of inhabitants is small, or for certain structural restrictions of the topology. Additionally, we complement many of our algorithmic results with conditional lower-bounds matching the running-time of these algorithms.

Statements where proofs or details are omitted due to space constraints are marked with $\star$. A version containing all proofs and details is available in [46].

Related Work. Our model is influenced by a game-theoretic reformulation of the famous Schelling's model [61, 62] of residential segregation introduced by Agarwal et al. [2]. Here, we are given a simple undirected graph G and a set of selfish agents partitioned into k types. Every agent wants to maximise the fraction of agents of her own type in her neighbourhood. The goal is then to assign agents to the vertices of G so that no agent can improve her utility by either jumping to an unoccupied vertex or swapping positions with another agent. Follow-up works include those that study the problem from the perspective of computational complexity [48, 32] and equilibrium existence guarantees [15, 16, 44].

The second main inspiration for our model is the Hedonic Seat Arrangement problem and its variants recently introduced by Bodlaender et al. [17]. Here, the goal is to find an assignment of agents with preferences to the vertices of the underlying topology. The desired assignment should then meet specific criteria such as different forms of stability, maximising social welfare, or being envy-free. In our model, compared to Hedonic Seat Arrangement of Bodlaender et al. [17], inhabitants already occupy some vertices of the topology and we have to assign refugees to the remaining (empty) vertices in a desirable way.

Next, the problem of house allocation [1] or housing market [64] has been extensively studied in the area of mechanism design. Here, each agent owns a house and the objective is to find a socially efficient outcome using reallocations of objects. Later, You et al. [69] introduced house allocation over social networks that follows current trend in mechanism design initiated by Li et al. [51], where each individual can only communicate with his neighbours. As stated before, the house allocation is studied mainly from the viewpoint of mechanism design and as such is far from our model.

Finally, hedonic games [31, 20, 21] are a well-studied class of coalition formation games where the goal is to partition agents into coalitions and where the utility of every agent depends on the identity of other agents in his coalition. In anonymous games [11, 20], the agents have preferences over the sizes of their coalition. The most recent variants of hedonic games are the so-called hedonic diversity games [22, 19] where agents are partitioned into k types and preferences are over the ratios of each type in the coalition. The main difference between our model and (all variants of) hedonic games is that in the latter model all coalitions are pairwise disjoint; however, in our case, each agent has his own neighbourhood overlapping with neighbourhoods of other agents.

2 Preliminaries

Let $\mathbb{N}$ denote the set of positive integers. Given two positive integers $j, j' \in \mathbb{N}$, with $j \leq j'$, we call the set $[j, j'] = \{j, \ldots, j'\}$ an *interval*, and let $[j] = [1, j]$ and $[j]_0 = [j] \cup \{0\}$. Let S be a set. By 2^S we denote the set of all subsets of S and, given $k \leq |S|$, we denote by $\binom{S}{k}$ the set of all subsets of S of size k.

Let $R = \{r_1, \ldots, r_m\}$ be a non-empty set of *refugees* and $I = \{i_1, \ldots, i_\ell\}$ be a set of *inhabitants*. The set of all *agents* is defined as $N = R \cup I$. A *topology* is a simple undirected graph $G = (V, E)$, where $|V| \geq |N|$. For a vertex v, we denote by $N(v)$ the set of its *neighbours*, formally $N(v) = \{u \mid \{u, v\} \in E\}$. The size of the neighbourhood of a vertex v is called its *degree* and is defined as $\deg(v) = |N(v)|$. The *closed neighbourhood* of vertex v is defined as $N[v] = N(v) \cup \{v\}$. In this work, we follow the basic graph-theoretical terminology [29].

An *inhabitants assignment* is an injective function $\iota\colon I \to V$. The set of vertices occupied by the inhabitants is denoted V_I and, given an inhabitant $i \in I$, we denote the set of unoccupied vertices in his neighbourhood $U_i = N(\iota(i)) \setminus V_I$. The set of all vertices that are not occupied by inhabitants is denoted $V_U = V \setminus V_I$.

The goal of every variant of our problem is to find a mapping of refugees to vertices that are not occupied by inhabitants. Formally, *housing* is an injective mapping $\pi\colon R \to V_U$. A set of vertices occupied by refugees with respect to housing π is denoted $V_\pi = \{\pi(r) \mid r \in R\}$. We denote by $\Pi_{G,\iota}$ the set of all possible housings, and we drop the subscript whenever G and ι are clear from the context.

Parameterised Complexity. We study the problem in the framework of parameterised complexity [25, 30, 58]. Here, we investigate the complexity of the problem not only with respect to an input size n, but even assuming some additional *parameter* k. The goal is to find a parameter which is small and the "hardness" can be confined to this parameter. The most favourable outcome is an algorithm with running time $f(k) \cdot n^{\mathcal{O}(1)}$, where f is any computable function. We call this algorithm *fixed-parameter tractable* and the complexity class containing all problems that admit algorithms with such running time is called FPT.

Not all combinations of parameters yield to fixed-parameter tractable algorithms. A less favourable outcome is an algorithm running in $n^{f(k)}$ time, where f is any computable function. Parameterised problems admitting such algorithms belong to complexity class XP. To exclude the existence of a fixed-parameter tractable algorithm, one can show that the parameterised problem is W[t]-hard for some $t \geq 1$. This can be done via a *parameterised reduction* from any problem known to be W[t]-hard. It could also be the case that a parameterised problem is NP-hard even for a fixed value of k; we call such problems para-NP-hard and, assuming $\mathsf{P} \neq \mathsf{NP}$, such problems do not admit XP algorithms.

Our running-time lower-bounds are based on the well-known Exponential Time Hypothesis (ETH) of Impagliazzo and Paturi [41]; see also Impagliazzo et al. [42] and the survey of Lokshtanov et al. [52]. This conjecture states that, roughly speaking, there is no algorithm solving 3-SAT in time sub-exponential in the number of variables.

3 Anonymous Refugees

In our simplest model of refugee housing, we assume refugees to be non-strategic and concern only the preferences of inhabitants. In this sense, the refugees are, from the viewpoint of inhabitants, anonymous, and the preferences only take into account the number of refugees in the neighbourhood of each inhabitant.

We formally capture this setting in the computational problem called the Anonymous Refugees Housing problem (ARH for short). A preference of every inhabitant $i \in I$ is a set $A_i \subseteq [\deg(\iota(i))]_0$ of the approved numbers of refugees in the neighbourhood. Our goal is to decide whether there is a housing $\pi\colon R \to V_U$ that respects the preferences of all inhabitants.

Definition 1. *A housing $\pi\colon R \to V_U$ is called* inhabitant-respecting *if for every $i \in I$ we have $|N(\iota(i)) \cap V_\pi| \in A_i$.*

If the approval set A_i for an inhabitant $i \in I$ consists of consecutive numbers, we say that the inhabitant i approves an interval.

Example 1. *Let the topology be a cycle with four vertices. There are two inhabitants assigned to neighbouring vertices. One of these inhabitants, call her h_1, has approval set $A_{h_1} = \{0, 1\}$, and the second one, say h_2, is not approving any refugees in his neighbourhood, that is, $A_{h_2} = \{0\}$. We have $R = \{r\}$. The only valid housing is next to the inhabitant h_1 as housing r in the neighbourhood of h_2 clearly does not respect his preference. Also note that in this particular example, all the inhabitants approve intervals.*

As our first result, we observe that even in a very simple settings, it is not guaranteed that any inhabitant-respecting refugees housing exists.

Proposition 1. *There is an instance of the ARH problem with no inhabitant-respecting refugees housing even if all inhabitants approve intervals.*

To prove Proposition 1, assume an instance with one inhabitant i and two refugees r_1 and r_2. Let the topology be K^3, the inhabitant i be assigned to an arbitrary vertex, and let $A_i = \{0\}$. There are exactly two possible refugees housings and in any of them the inhabitant i has two neighbouring refugees; therefore, there is no inhabitant-respecting housing.

In the previous example, we used the fact that the inhabitant i does not approve any refugees in his neighbourhood. We call such inhabitants *intolerant*. Despite the fact that the instance does not have an inhabitant-respecting housing even if $A_i = \{1\}$, we observe that intolerant inhabitants can be safely removed.

Proposition 2 ($\star$). *Let $\mathcal{I} = (G, I, R, \iota, (A_i)_{i \in I})$ be an instance of the ARH problem, $j \in I$ be an inhabitant with $A_j = \{0\}$, and $F_j = \{\iota(j)\} \cup U_j$. $\mathcal{I}$ admits an inhabitant-respecting housing iff the instance $\mathcal{I}' = (G \setminus F_j, I \setminus \{j\}, R, \iota, (A_i)_{i \in I \setminus \{j\}})$ admits an inhabitant-respecting housing.*

Due to the definition of approval sets, inhabitants without unoccupied neighbourhood are necessarily assumed intolerant and therefore can be safely removed by Proposition 2. Hence, we assume only instances without intolerant inhabitants where every inhabitant has at least one unoccupied vertex in her neighbourhood.

Proposition 3 ($\star$). *Let $\mathcal{I} = (G, I, R, \iota, (A_i)_{i\in I})$ be an instance of the* ARH *problem and $\{u, v\} \in E(G)$ be an edge such that either $u, v \in V_I$ or $u, v \in V_U$. Then $\mathcal{I}$ admits an inhabitant-respecting housing iff the instance $\mathcal{I}' = ((V(G), E(G) \setminus \{\{u, v\}\}), I, R, \iota, (A_i)_{i\in I})$ admits an inhabitant-respecting housing.*

Proposition 3 directly implies that all graphs assumed in this section are naturally bipartite.

Theorem 1 ($\star$). *Every instance of the* ARH *problem where the topology is a graph of maximum degree 2 can be solved in polynomial-time.*

Proof sketch. Our algorithm is based on the dynamic programming approach combined with the gradual elimination of inhabitants' approval sets and exhaustive application of Proposition 2. Observe that graph of maximum degree 2 is a collection of paths and cycles [29]. We first introduce an algorithm that solves the problem on paths and then show how to improve the algorithm to solve cycles.

Let the topology be a path $P = v_1 v_2 \dots v_k$, $k \geq 3$, and suppose that the vertex v_1 is occupied by an inhabitant $i \in I$. We distinguish two cases based on A_i and show how the algorithm proceeds. If $A_i = \{1\}$, then we have to house a refugee on v_2. However, this adds one refugee in the neighbourhood of the inhabitant j occupying the vertex v_3. To capture this, we decrease the value of all elements in A_j. If there are any negative numbers in A_j after this operation, we remove all of them from the list. Then we delete v_1 and v_2 from P, decrease $|R|$ by one, and solve the problem for $P' = v_3 \dots v_k$. If $A_i = \{0, 1\}$, we have to try both possibilities. That is, we run the algorithm once with $A_i = \{0\}$ and once with $A_i = \{1\}$. If any run of the algorithm finds a solution, we also have a solution for the original instance.

The described algorithm is exponential in the worst case. To improve the running time, we tabularise the computed partial solutions. Our dynamic programming table DP has three dimensions: the first for an inhabitant, the second for an actual value of $|R|$, and the third for a shape of approval set. The stored value is either *yes* or *no* depending on whether the combination of indices yields to a inhabitant-respecting housing. There are $\mathcal{O}(n)$ inhabitants, the value of $|R|$ is also in $\mathcal{O}(n)$, and there are at most 2 different approval sets possible for each inhabitant on the path. Therefore, the size of the table is at most $\mathcal{O}(n^2)$, which is also the running time of our algorithm. $\square$

Unfortunately, as the following theorem states, the bounded-degree condition from Theorem 1 cannot be relaxed any more.

Theorem 2 ($\star$). *The* ARH *problem is* NP*-complete even if the topology is a graph of maximum degree 3 and all inhabitants approve intervals.*

Proof sketch. For NP-hardness, we present a polynomial-time reduction from a variant of the 2-BALANCED 3-SAT problem which is known to be NP-complete [65, 37, 14]. In this variant of 3-SAT, we are given a propositional formula φ with n variables $x_1, \dots, x_n$ and m clauses $C_1, \dots, C_m$ such that each clause contains at most 3 literals and every variable appears in at most 4 clauses – at most twice as a positive literal and at most twice as a negative literal. Later in this paper, we will refer to this reduction as *basic reduction.*

We construct an equivalent instance $\mathcal{I}$ of ARH as follows. We represent every variable x_i by a single *variable gadget* X_i that is a path $t_i v_i f_i$. The vertex v_i is occupied by an inhabitant with approval set $\{1\}$. All other vertices are empty and we call the vertex t_i the *t-port* and the vertex f_i the *f-port*. Every *clause* C_j is represented by a single vertex c_j occupied by an inhabitant h_j who approves the interval $[1, |C_j|]$ and is connected to the t-port of the variable gadget X_i if the variable x_i occurs as a positive literal in C_j and to the f-port of X_i if x_i occurs as a negative literal in C_j, respectively. To complete the reduction, we set $|R| = n$. □

Since the above results clearly show that the problem is very hard even in simple settings, we turn our attention to the parameterised complexity of the ARH problem. In particular, we study the problem's complexity from the viewpoint of natural parameters, such as the number of refugees, the number of inhabitants, the number of empty vertices, and various structural parameters restricting the shape of the topology.

Theorem 3 ($\star$). *The ARH problem is W[2]-hard parameterised by the number of refugees $|R|$ even if all inhabitants approve intervals. Moreover, unless ETH fails, there is no algorithm that solves ARH in $f(|R|) \cdot n^{o(|R|)}$ time for any computable function f.*

Proof sketch. We reduce from the Dominating Set problem, which is known to be W[2]-complete [30] and, unless ETH fails, cannot be solved in $f(k) \cdot n^{o(k)}$ time for any computable function f [24]. The instance $\mathcal{I}$ of Dominating Set consists of a simple undirected graph $G = (V, E)$ and an integer $k \in \mathbb{N}$. The goal is to decide whether there is a set $D \subseteq V$ of size at most k such that each vertex $v \in V$ is either in D or at least one of its neighbours is in D.

We construct an equivalent instance $\mathcal{I}'$ of the ARH problem as follows. We start by defining the topology. For each vertex $v \in V$ we add two vertices ℓ_v and p_v. The vertex ℓ_v represents the original vertex and is intended to be free for refugees. The vertex p_v is occupied by an inhabitant h_v with $A_{h_v} = [1, |N[v]|]$. This inhabitant ensures that there is at least one refugee housed in the closed neighbourhood of p_v. The edge set of the topology G' is $\bigcup_{v \in V} \{\{p_v, \ell_w\} \mid w \in N_G[v]\}$. To complete the construction, we set $|R| = k$. □

We complement Theorem 3 with an algorithm that runs in time matching the lower-bound given in this theorem.

Proposition 4. *The ARH problem can be solved in $n^{\mathcal{O}(|R|)}$ time, where $n = |V(G)|$. That is, ARH is in XP parameterised by the number of refugees.*

Proof. Our algorithm is a simple brute-force. Let $V_U = V(G) \setminus V_I$ be the number of empty vertices and let $n = |V|$. Note that $|V_U| \leq n$. We try all subsets of V_U of size $|R|$ and for each such subset, we check in linear time whether the housing is inhabitant-respecting. This gives us the total running time $n^{\mathcal{O}(|R|)}$. □

As the number of refugees is not a parameter promising tractable algorithms even if all inhabitants approve intervals, we turn our attention to the case where the number of inhabitants is small. Our algorithm is based on integer linear programming formulation of the problem and we use the following result of Eisenbrand and Weismantel [33].

Theorem 4 ([33, Theorem 2.2]). *Integer linear programme $\mathbb{A}x \leq b$, $x \geq 0$, with n variables and m constraints can be solved in*

$$(m\Delta)^{\mathcal{O}(m)} \cdot ||b||_\infty^2$$

time, where Δ is an upper-bound on all absolute values in $\mathbb{A}$.

Theorem 5. *If all inhabitants approve intervals, then the ARH problem is fixed-parameter tractable parameterised by the number of inhabitants $|I|$.*

Proof sketch. We solve the ARH problem using an integer linear programming formulation of the problem. We introduce one binary variable x_v for every empty vertex $v \in V_U$ representing if a refugee is housed on v or not. Next, we add the following constraints.

$$\forall i \in I\colon \sum_{v \in N_G(\iota(i))} x_v \geq \mathrm{low}(i) \tag{1}$$

$$\forall i \in I\colon \sum_{v \in N_G(\iota(i))} x_v \leq \mathrm{high}(i) \tag{2}$$

$$\sum_{v \in V_U} x_v = |R|, \tag{3}$$

where for an inhabitant $i \in I$ the value $\mathrm{low}(i)$ stands for lower-end and $\mathrm{high}(i)$ stands for upper-end of the approved interval by inhabitant i, respectively. Equations (1) and (2) ensure that the number of refugees in the neighbourhood of each inhabitant is in its approved interval, while Equation (3) ensures that all refugees are housed somewhere. Using Theorem 4 we see that the given integer programme can be solved in time $|I|^{\mathcal{O}(|I|)} \cdot n^{\mathcal{O}(1)}$, as $m = 2|I| + 1$, $\Delta = 1$, and $||b||_\infty \leq n$. That is, ARH is in FPT parameterised by the number of inhabitants $|I|$. □

Note that it would be possible to provide a different ILP formulation of the problem and use the famous theorem of Lenstra Jr. [50] to show membership in FPT, however, this would yield to an algorithm with much worse (i. e., doubly-exponential) running-time.

The result from Theorem 5 cannot be easily generalised to the case with inhabitants approving general sets. However, we can show that if the number of intervals in each approval set is bounded, the problem is still fixed-parameter tractable.

Theorem 6 ($\star$)**.** *The ARH problem is fixed-parameter tractable when parameterised by the combined parameter the number of inhabitants $|I|$ and the maximum number of disjoint intervals δ in the approval sets.*

In our next result, we show that the parameter δ from Theorem 6 cannot be dropped while keeping the problem tractable.

Theorem 7 ($\star$)**.** *The ARH problem is W[1]-hard parameterised by the number of inhabitants $|I|$.*

By careful guessing, we can prove that for the combined parameters the number of refugees and the number of inhabitants, we may obtain fixed-parameter tractable algorithm. Observe that this parameterisation do not bound the instance size, as the number of vertices of the topology can be much larger than $|I| + |R|$.

Lemma 1 ($\star$)**.** *The ARH problem is fixed-parameter tractable when parameterised by the number of refugees $|R|$ and the number of inhabitants $|I|$ combined.*

The last assumed natural parameter is the number of empty vertices the refugees can be assigned to. Note that $|V_U| \geq |R|$. This parameterisation yields, in contrast to Theorem 3, to a simple algorithm running in FPT time which is, despite its simplicity, optimal assuming the Exponential Time Hypothesis.

Theorem 8 ($\star$)**.** *The ARH problem can be solved in $2^{\mathcal{O}(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time and, unless ETH fails, there is no algorithm solving ARH in $2^{o(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time even if all inhabitants approve intervals.*

In the remainder of this section, we present complexity results concerning various structural restrictions of the topology. Arguably, the most prominent structural parameter is the tree-width of a graph that, informally speaking, expresses its tree-likeness, and which is usually small in real-life networks [56]. Unfortunately, we can show the following stronger intractability result.

Many problems that are computationally hard with respect to tree-width are studied from the viewpoint of less restrictive parameters. Vertex cover number is a frequent representative of such parameters [53, 23, 47]; however, in the ARH problem, not even this restriction of the topology leads to a tractable algorithm.

Theorem 9 ($\star$)**.** *The ARH problem is W[1]-hard parameterised by the vertex cover number* $\mathrm{vc}(G)$ *of the topology.*

It is well-known, and easy to see, that the tree-width of a graph is at most its vertex-cover number. Hence, due to Theorem 9, we directly obtain the following result for tree-width.

Corollary 1. *The ARH problem is W[1]-hard parameterised by the tree-width* $\mathrm{tw}(G)$ *of the topology* G*.*

Nevertheless, if we additionally restrict the approval sets, we obtain the following algorithmic result.

Theorem 10. *The ARH problem is fixed-parameter tractable parameterised by the vertex cover number* $\mathrm{vc}(G)$ *if all inhabitants approve intervals.*

Proof sketch. Let $M \subseteq V$ be a minimum size vertex cover of G and let $k = |M|$. First, we guess the number $k' \leq k$ of refugees assigned to the modulator vertices (by guessing we mean an iteratively trying all possibilities). Next, we guess a k'-sized subset $S \subseteq M \cap V_U$ of empty vertices that are occupied by refugees in our hypothetical solution. If no such set S exists, we return *no*.

Otherwise, we remove all empty vertices from M, all vertices occupied by inhabitants that are not part of the vertex cover M, and, finally, use Theorem 5 to solve the reduced instance with $k - k'$ refugees. It is easy to see that both steps can be performed in FPT time and the theorem follows. ☐

By the same argumentation used in the proof of Theorem 10, we obtain the following last result of this section.

Corollary 2 ($\star$)**.** *The ARH problem is fixed-parameter tractable when parameterised by the vertex cover number* $\mathrm{vc}(G)$ *and the maximum number of disjoint intervals* δ *combined.*

4 Fully Hedonic Preferences

Our second model of refugee housing improves upon the previous model by introducing individual preferences of refugees. Naturally, refugees are no longer anonymous and the identity of every particular refugee matters. The preferences of the inhabitants are again dichotomous, and for every inhabitant $i \in I$ the approval set is a subset of 2^R. Similarly, for a refugee $r \in R$, the approval set A_r is a subset of 2^I. Our goal is to find housing that conforms to the preferences of both groups.

Definition 2. *A housing* $\pi \colon R \to V_U$ *is called* respecting *if for every* $i \in I$ *we have* $N(\iota(i)) \in A_i$ *and for every* $r \in R$ *we have* $N(\pi(r)) \in A_r$*.*

In other words, a housing π is respecting if every inhabitant and every refugee approves its neighbourhood. We study the problem of deciding whether there is a respecting housing in the instance with hedonic preferences under the name Hedonic Refugees Housing (HRH for short).

Example 2. *Let the topology be a cycle with four vertices. There are two inhabitants h_1 and h_2 assigned to neighbouring vertices and two refugees r_1 and r_2 to house. The approval set of inhabitant h_1 is $A_{h_1} = \{\{r_1\}, \{r_2\}\}$, that is, h_1 approves only one refugee in her neighbourhood regardless of the identity. The second inhabitant approves the set $A_{h_2} = \{\{r_2,\}, \{r_1, r_2\}\}$. In other words, the inhabitant h_2 is dissatisfied with having only the refugee r_1 in the neighbourhood; however, he is fine with neighbouring with both the refugees. For the refugees, we have $A_{r_1} = \{\{h_1\}\}$ and $A_{r_2} = \{\{h_2\}\}$. Housing r_1 in the neighbourhood of h_1 and r_2 in the neighbourhood of h_2 is clearly respecting.*

Observe that, since both inhabitants and refugees have preferences only over the other set of individuals, we can remove all edges between empty and occupied vertices, respectively. Hence, all graphs assumed in this section are again bipartite.

Now, we show how the results from Section 3 carry over to the hedonic setting studied in this section. Our first theorem shows that the hedonic setting is also computationally hard on graphs of constant degree. The construction is very similar to the one used to prove Theorem 2.

Theorem 11 ($\star$)**.** *The HRH problem is NP-complete even if the topology is a graph of maximum degree* 3.

From the proof of Theorem 11, we can easily distil the following general reduction from the ARH problem to the HRH problem. Let $\mathcal{I}$ be an ARH instance. For every empty vertex $v \in V_U$ we add into R' one refugee r_v with approval set $A_{r_v} = N(v)$. Next, for every inhabitant $i \in I$ we add a new inhabitant h_i approving the set $A_{h_i} = \{\binom{R_{h_i}}{a} \mid a \in A_i\}$, where $R_{h_i} = \{r_v \mid v \in N(v) \cap V_U\}$. To ensure that only $|R|$ refugees are housed, we extend the construction by a single star with $|V_U| - |R|$ leaves and occupy the centre of the star with an inhabitant g approving the set $\binom{R'}{|V_U|-|R|}$. Moreover, we have to add $\{g\}$ to the approval set of every refugee $r_v \in R'$. It is not difficult to see that the instances are indeed equivalent; however, the reduction is not polynomial-time, the approval sets can be at worst exponential in the number of empty vertices. Hence, the reduction works only in cases where the number of empty vertices is at most logarithmic in the size of the topology. Unfortunately, this is not the case for most of our polynomial-time reductions, however, we are able to show similar results using different techniques.

Theorem 12 ($\star$)**.** *The HRH problem is W[1]-hard when parameterised by the combined parameter the vertex cover number* $\mathrm{vc}(G)$ *of the topology and the number of inhabitants* $|I|$.

Proof sketch. We reduce from the W[1]-hard Multicoloured Clique problem [36]. We recall that here we are given a k-partite graph $G = (V_1 \cup \dots \cup V_k, E)$ and the goal is to find a complete sub-graph with k vertices such that it contains a vertex from every V_i, $i \in [k]$.

Let $G = (V_1 \cup \dots \cup V_k, E)$ be an instance of the Multicoloured Clique problem such that all colour classes V_i are of the same size n_G. We construct an equivalent instance $\mathcal{I}'$ of the HRH problem as follows. For every vertex set V_i, where $i \in [k]$, we introduce a *vertex-selection gadget* S_i which is a star with n_G leaves. We call an arbitrary but fixed leaf a *selection leaf.* This selection leaf serves for a vertex of colour i that should be part of the clique and is the only connection of the vertex-selection gadget with the rest of the topology. Let $\{v_i^1, \dots, v_i^{n_G}\}$ be a set of vertices in the colour class V_i. We introduce one refugee r_i^p for every v_i^p, $p \in [n_G]$ and one inhabitant s_i which is assigned to the centre c_i of S_i and approves the set $\{\{r_i^1, \dots, r_i^{n_G}\}\}$. Every refugee r_i^p approves the set $\{\{s_i\}\}$.

Then, for every pair of distinct colours $i, j \in [k]$, we introduce one additional *guard vertex* $g_{i,j}$ securing that there is an edge between vertices selected in incident vertex-selection gadgets. We connect this guard vertex $g_{i,j}$ to selection leaves of vertex-selection gadgets V_i and V_j. Moreover, we introduce an inhabitant $h_{i,j}$ assigned to $g_{i,j}$ with approval set $\{\{r_i^p, r_j^q\} \mid \{v_i^p, v_j^q\} \in E \text{ and } p, q \in [n_G]\}$. That is, $h_{i,j}$ approves exactly those pairs of vertices from V_i and V_j that are connected by an edge.

To be able to house any refugee to selection leaves, we have to extend their approval sets. Thus, for every refugee r_i^p, where $i \in [k]$ and $p \in [n_G]$, we add to the approval set the set $\{s_i\} \cup \{h_{i,j} \mid j \in [k] \setminus \{i\}\}$. This finishes the construction.

□

As a final result of this section, we prove that the HRH is NP-hard even for graphs of tree-width at most 3.

Theorem 13 ($\star$)**.** *The HRH problem is para-NP-hard parameterised by the tree-width* $\mathrm{tw}(G)$ *of the topology.*

5 Diversity Preferences

In the anonymous refugee housing, we are not assuming the preferences of individual refugees. Thanks to this property, the model is as simple as possible. The fully hedonic setting from Section 4 precisely captures preferences of both the refugees and the inhabitants. On the other hand, the fully hedonic model is not very realistic, as it is hard to acquaint all inhabitants with all refugees.

Hence, we introduce the third model of refugees housing, where both the inhabitants and the refugees are partitioned into types and agents from both groups have preferences over fractions of agents of each type in their neighbourhood.

Such diversity goals, where agents are partitioned into types and the preferences of agents are based on the fraction of each type in their neighbourhood or coalition, was successfully used in many scenarios such as school choice [7, 9, 10], public housing [13, 40], hedonic games [22, 18, 19, 26, 39], multi-attribute matching [4], or employee hiring [63].

Before we formally define the computational problem of our interest, let us introduce further notation. Let $N = I \cup R$ be a set of agents partitioned into k types $T = T_1, \ldots, T_k$. For a set $S \subseteq N$, we define a *palette* as a k-tuple $\left(\frac{|T_i \cap S|}{|S|}\right)_{i \in [k]}$ if $|S| \geq 1$ and k-tuple $(0, \ldots, 0)$ if $S = \emptyset$. Given an agent $a \in N$, her approval set is a subset of the set $\left\{\left(\frac{|T_i \cap S|}{|S|}\right)_{i \in [k]} \mid S \subseteq 2^N\right\}$.

Definition 3. *A housing* $\pi \colon R \to V_U$ *is called* diversity respecting *if for every inhabitant* $i \in I$ *the palette for the set* $\{h \in I \mid \iota(h) \in N(\iota(i))\} \cup \{r \in R \mid \pi(r) \in N(\iota(i))\}$ *is in* A_i*, and for every refugee* $r \in R$ *the palette for the set* $\{h \in I \mid \iota(h) \in N(\pi(r))\} \cup \{r' \in R \mid \pi(r') \in N(\pi(r))\}$ *is in* A_r*.*

The Diversity Refugees Housing problem (DRH for short) then asks whether there is a diversity respecting housing π. Note that this time, we are not allowed to drop edges between two inhabitants or two refugees and, thus, the graphs assumed in this section are no longer bipartite.

Example 3. *Let the topology be a cycle with four vertices. There are two agents of type* T_1*. One of these agents is an inhabitant* h_1 *approving* $\{(1, 0), (1/2, 1/2)\}$ *and the second one is a refugee* r *approving only agents of his own type, that is,* $A_r = \{(1, 0)\}$*. The type* T_2 *contains one inhabitant* h_2 *approving the set* $\{(1, 0)\}$*. Inhabitants are assigned such that* *they are neighbours. There are two possible housings for the refugee r. She can be either neighbour of h_1 or h_2. Since she accepts only agents of her own type in the neighbourhood, the only diversity respecting housing is next to inhabitant h_1.*

Note that the topology in the diversity setting is no longer bipartite graph. We can show, using similar construction as in Theorem 2, that the tractability condition based on the bounded degree cannot be surpassed even in this model.

Theorem 14 ($\star$)**.** *The DRH problem is NP-complete even if the topology is a graph of maximum degree* 3*.*

In Theorem 14 we exploit the number of types to ensure that every refugee is housed in the right house. Therefore, the number of types was as large as the number of empty vertices. In the following result, we show that the DRH problem is computationally hard even if the number of types is small.

Theorem 15 ($\star$)**.** *The DRH problem is NP-complete even if there are two types of agents.*

Proof sketch. Given an instance $\mathcal{I} = (U, \mathcal{F}, k)$ of the Set Cover problem, we construct an equivalent instance $\mathcal{I}'$ of DRH as follows. For every element $u_i \in U$ we add one vertex v_i and assign to it an inhabitant h_i. The inhabitant h_i is of type T_1 and his approval set is $\{(0, 1)\}$. Next, for every subset $F \in \mathcal{F}$, we create one vertex v_F that is adjacent to every v_i such that $u_i \in F$. To finalise the construction, we add k refugees $r_1, \ldots, r_k$ of type T_2 approving the set $\{(1, 0)\}$. $\square$

Note that the NP-hardness proved in Theorem 15 can be strengthened to a single type of agents; however, we find this situation not very natural in the context of DRH.

Additionally, it is known that the Set Cover problem is W[2]-complete and cannot be solved in $f(k) \cdot n^{o(k)}$ time, unless the ETH fails [25]. This gives us the following final result.

Corollary 3 ($\star$)**.** *The DRH problem is W[2]-hard when parameterised by the number of refugees $|R|$ even if there are only two types of agents and, unless ETH fails, there is no algorithm solving DRH in $f(R) \cdot n^{o(R)}$ time for any computable function f.*

6 Conclusions

We initiated the study of a novel model of refugee housing. The model mainly targets the situations where refugees need to be accommodated and integrated in the local community. This distinguishes us from the previous settings of refugee resettlement.

Our results identify some tractable and intractable cases of finding stable outcomes from the viewpoint of both the classical computational complexity and the finer-grained framework of parameterised complexity. To this end, we believe that other notions of stability inspired, for example, by the model of Schelling games of Agarwal et al. [2] or by exchange-stability of Alcalde [5], should be investigated.

Natural way to tackle the intractability of problems in computational social choice is to restrict the preferences of agents [35]. One such restriction that should be investigated, especially in the case of anonymous setting, are the single-peaked preferences [6] that were successfully used in similar scenarios; see, e.g., [15, 68, 22] or the survey of Elkind et al. [35]. Beside that, we are interested in the anonymous setting in which every inhabitant $i \in I$ approves an interval $[0, u_i]$, where $u_i \geq 0$ is an inhabitant-specific upper-bound on the number of refugees in her neighbourhood.

Finally, there are many notions measuring quality of an outcome studied in the literature in both the context of Schelling and hedonic games [34, 2, 8], and we believe that these notions should be investigated even in the context of refugee housing. In this line, the most prominent notion is the social-welfare of an outcome.

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