# Parameterized Complexity Results for the Kemeny Rule in Judgment Aggregation

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**Abstract**

We investigate the parameterized complexity of computing an outcome of the Kemeny rule in judgment aggregation, providing the first parameterized complexity results for this problem for any judgment aggregation procedure. As parameters, we consider (i) the number of issues, (ii) the maximum size of formulas used to represent issues, (iii) the size of the integrity constraint used to restrict the set of feasible opinions, (iv) the number of individuals, and (v) the maximum Hamming distance between any two individual opinions, as well as all possible combinations of these parameters. We provide parameterized complexity results for two judgment aggregation frameworks: formula-based judgment aggregation and constraint-based judgment aggregation. Whereas the classical complexity of computing an outcome of the Kemeny rule in these two frameworks coincides, the parameterized complexity results differ.

## 1 Introduction

The area of computational social choice places computational aspects of social choice procedures at a first-class level among selection criteria for the choice between such procedures. For example, since the seminal work of Bartholdi, Tovey and Trick [3], it has been well known that for many voting rules the problem of computing who won a particular election is computationally intractable. As a result, the computational complexity of this winner determination problem plays an important role in the selection of what voting scheme to use. Traditionally, classical computational complexity theory has been used to provide qualitative information about the difficulty of relevant computational problems for social choice procedures. For instance, completeness results for the classical complexity classes NP and $\Theta_2^p$ abound in the computational social choice literature (see, e.g., [3, 8, 9, 21, 29, 40, 41]).

However, classical complexity theory—being a worst-case framework that measures the running time of algorithms in terms of only the bit-size of the input—is mostly blind to what aspects of the input underlie negative complexity results. Consequently, negative complexity results tend to be interpreted overly pessimistically. An illustrative example of this concerns the winner determination problem for the Kemeny voting rule. This problem is $\Theta_2^p$-complete in general (which rules out algorithms that work efficiently across the board), but efficient algorithms do exist for cases where the number of candidates is small [5, 6, 30, 33, 39].

The multidimensional framework of *parameterized complexity* [13, 14, 23, 38] offers a mathematically rigorous theory to analyze the computational complexity of problems on the basis of more than just the input size in bits. Therefore, using this framework, one can give much more informative complexity results that are sensitive to various aspects of the problem input—in principle, any aspect of the input can be taken into account. In the analysis of voting procedures, parameterized complexity has been used widely, to give a more accurate picture of the complexity of many related computational problems, including the problem of winner determination (see, e.g., [5, 6, 33]). In the area of judgment aggregation, however, parameterized complexity has been used to analyze the computational complexity of problems only in very few cases [4, 19]. The fundamental problem of computing the outcome of judgment aggregation procedures, as of yet, remains uninvestigated from a parameterized complexity point of view.

We hope to initiate a structured parameterized complexity investigation of the problem of computing the outcome of judgment aggregation procedures. Judgment aggregation studies the process of combining individual judgments on a set of related propositions of the members of a group into a collective judgment reflecting the views of the group as a whole [15, 26, 34, 35]. Seen from a classical complexity point of view, computing the outcome of many judgment aggregation procedures is $\Theta_2^p$-complete. However, as these negative results pertain only to the case where every possible input needs to be considered, there is a lot of room for relativizing these negative results by taking a parameterized complexity perspective and considering various (combinations of) reasonable restrictions on the inputs.

**Contribution** In this paper, we start the parameterized complexity investigation of judgment aggregation procedures by considering one of the most prominent procedures: the Kemeny procedure for judgment aggregation (or *Kemeny rule*, for short).[1] The unrestricted problem of computing an outcome for this rule is $\Theta_2^p$-complete [17, 32]. We consider a number of natural parameters for this problem—capturing various aspects of the problem input that can be expected to be small in some applications—and we give a complete parameterized complexity classification for the problem of computing the outcome of the Kemeny rule, for every combination of these parameters. The parameters that we consider are:

- the number $n$ of issues that the individuals (and the group) form an opinion on;
- the maximum size $m$ of formulas used to represent the issues;
- the size $c$ of the integrity constraint used to limit the set of feasible opinions;
- the number $p$ of individuals; and
- the maximum (Hamming) distance $h$ between any two individual opinions.

The results in this paper open up interesting and natural lines of future research. A similar parameterized complexity analysis can be performed for the problem of computing the outcome of other judgment aggregation procedures. Moreover, further parameters can be taken into account in future parameterized complexity analyses of the problem.

We develop parameterized complexity results for two formal frameworks for judgment aggregation: *formula-based judgment aggregation* and *constraint-based judgment aggregation* (the former is often simply called 'judgment aggregation' [12, 17, 32] and the latter is also called 'binary aggregation with integrity constraints' [24, 25])—we define these two frameworks in detail in Section 3. In general, the computational complexity of computing the outcome of a judgment aggregation procedure might differ for these two frameworks [16], but for the Kemeny rule this problem is $\Theta_2^p$-complete in both frameworks [17, 24].

**Nonstandard Parameterized Complexity Tools** Since the invention of parameterized complexity theory, it has been applied mostly to problems that are in NP. As a result, the most commonly used parameterized complexity toolbox is insufficient to perform a complete parameterized complexity analysis of problems that are beyond NP (such as the $\Theta_2^p$-complete problem of computing the outcome of the Kemeny procedure). Recently, various novel parameterized complexity tools have been developed that aid in analyzing the parameterized complexity of problems beyond NP [19, 20, 28]. The parameterized complexity results in this

[1] This procedure has also been called "MWA" [32], "Median rule" [37], "Simple scoring rule" [11], and "Prototype-Hamming rule" [36].

paper feature one of these innovative parameterized complexity tools: the class $\text{FPT}^{\text{NP}}[\text{few}]$, which consists of problems that can be solved by a fixed-parameter tractable algorithm that can query an NP oracle a small number of times (that is, the number of oracle queries depends only on the parameter value). We define this class in detail in Section 2, where we discuss relevant notions from parameterized complexity.

**Overview of Results** We provide a parameterized complexity classification for the problem of computing an outcome of the Kemeny rule, for all possible combinations of the parameters that we consider—both (1) in the framework of formula-based judgment aggregation and (2) in the framework of constraint-based judgment aggregation.

For the framework of formula-based judgment aggregation, we give a tight classification for each possible case. In particular, we show the following. When parameterized by any set of parameters that includes $c$, $n$ and $m$, the problem is fixed-parameter tractable (Proposition 3). Otherwise, when parameterized by any set of parameters that includes either $n$ or both $h$ and $p$, the problem is $\text{FPT}^{\text{NP}}[\text{few}]$-complete (Propositions 1, 2, 8, 9 and 10). For all remaining cases, the problem is para-$\Theta_2^{\text{p}}$-complete (Corollary 6 and Proposition 7).

For the framework of constraint-based judgment aggregation, we show the following results. When parameterized by any set of parameters that includes either $c$ or $n$, the problem is fixed-parameter tractable (Propositions 11 and 12). Otherwise, when parameterized by any set of parameters that includes $h$, the problem is W[SAT]-hard and is in XP (Propositions 13 and 14). For all remaining cases, the problem is para-$\Theta_2^{\text{p}}$-complete (Proposition 15). The results for the formula-based judgment aggregation framework are summarized in Table 1, and the results for the constraint-based framework are summarized in Table 2 (in both tables, a star denotes an arbitrary choice for the subset).

| $P_1 \subseteq \{c, n, m\}$ | $P_2 \subseteq \{h, p\}$ | complexity param. by $P_1 \cup P_2$ | |
|---|---|---|---|
| $P_1 = \{c, n, m\}$ | $*$ | in FPT | (Prop 3) |
| $*$ | $P_2 = \{h, p\}$ | in $\text{FPT}^{\text{NP}}[\text{few}]$ | (Prop 1) |
| $n \in P_1$ | $*$ | in $\text{FPT}^{\text{NP}}[\text{few}]$ | (Prop 2) |
| $P_1 \subsetneq \{c, n, m\}$ | $*$ | $\text{FPT}^{\text{NP}}[\text{few}]$-hard | (Props 8, 9 and 10) |
| $P_1 \subseteq \{c, m\}$ | $P_2 \subsetneq \{h, p\}$ | para-$\Theta_2^{\text{p}}$-hard | (Cor 6, Prop 7) |

Table 1: Parameterized complexity results for OUTCOME(KEMENY)$^{\text{fb}}$.

| $P_1 \subseteq \{c, n\}$ | $P_2 \subseteq \{h, p\}$ | complexity param. by $P_1 \cup P_2$ | |
|---|---|---|---|
| $P_1 \neq \emptyset$ | $*$ | in FPT | (Props 11 and 12) |
| $*$ | $h \in P_2$ | in XP | (Prop 13) |
| $P_1 = \emptyset$ | $*$ | W[SAT]-hard | (Prop 14) |
| $P_1 = \emptyset$ | $P_2 \subseteq \{p\}$ | para-$\Theta_2^{\text{p}}$-hard | (Prop 15) |

Table 2: Parameterized complexity results for OUTCOME(KEMENY)$^{\text{cb}}$.

**Roadmap** In the remainder of Section 1, we discuss relevant related work. Then, in Section 2, we give a brief overview of the concepts and tools from the theory of (parameterized) complexity that we use in this paper. In Section 3, we introduce the two formal judgment aggregation frameworks, and we formally define the computational problem of computing an outcome for the Kemeny rule, as well as all the parameterized variants of this problem that we consider. We provide the parameterized complexity results for all parameterized variants of the problem (for both judgment aggregation frameworks) in Section 4. Due to space restrictions, proofs of results marked with an asterisk can be found in the appendix. Finally, we conclude in Section 5.

**Related Work** Parameterized complexity theory has been used to investigate the complexity of the winner determination problem in voting (which is analogous to the problem of computing the outcome of a judgment aggregation procedure) for several voting rules [5, 6, 33]. Parameterized complexity has also been used to study various other problems in the area of judgment aggregation, such as problems related to bribery [4, 19]. The complexity of computing outcomes for the Kemeny procedure in judgment aggregation (and other procedures) has been studied from a classical complexity point of view [17, 18, 24, 32]. It has also been studied what influence the choice of formal framework to model the setting of judgment aggregation has on the (classical) complexity of computing outcomes for various judgment aggregation procedures [16].

## 2 Parameterized Complexity

We begin by briefly introducing the relevant concepts and notation from propositional logic and (parameterized) complexity theory. We use the notation $[n]$, for any $n \in \mathbb{N}$, to denote the set $\{1, \dots, n\}$.

**Propositional Logic** Propositional formulas are constructed from propositional variables using the Boolean operators $\wedge, \vee, \rightarrow$, and $\neg$. A propositional formula is *doubly-negated* if it is of the form $\neg\neg\psi$. For every propositional formula $\varphi$, we let $\sim\varphi$ denote the *complement* of $\varphi$, i.e., $\sim\varphi = \neg\varphi$ if $\varphi$ is not of the form $\neg\psi$, and $\sim\varphi = \psi$ if $\varphi$ is of the form $\neg\psi$. For a propositional formula $\varphi$, the set $\text{Var}(\varphi)$ denotes the set of all variables occurring in $\varphi$. We use the standard notion of *(truth) assignments* $\alpha : \text{Var}(\varphi) \rightarrow \{0, 1\}$ for Boolean formulas and the standard notion of *truth* of a formula under such an assignment. An assignment that makes the formula true is called a *model* of the formula.

**Classical Complexity** We assume the reader to be familiar with the most common concepts from complexity theory, such as the complexity classes P and NP. These basic notions are explained in textbooks on the topic; see, e.g., [2]. In this paper, we will also refer to the complexity class $\Theta_2^p$, that consists of all decision problems that can be solved by a polynomial-time algorithm that queries an NP oracle $O(\log n)$ times. The following problem is complete for the class $\Theta_2^p$ under polynomial-time reductions [7, 31, 44].

Max-Model
*Instance:* A satisfiable propositional formula $\varphi$, and a variable $w \in \text{Var}(\varphi)$.
*Question:* Is there a model of $\varphi$ that sets a maximal number of variables in $\text{Var}(\varphi)$ to true (among all models of $\varphi$) and that sets $w$ to true?

**Parameterized Complexity** Next, we introduce the relevant concepts of parameterized complexity theory. For more details, we refer to textbooks on the topic [10, 13, 14, 23, 38]. An instance of a parameterized problem is a pair $(x, k)$ where $x$ is the main part of the instance, and $k$ is the parameter. A parameterized problem is *fixed-parameter tractable* if instances $(x, k)$ of the problem can be solved by a deterministic algorithm that runs in time $f(k)|x|^c$, where $f$ is a computable function of $k$, and $c$ is a constant (algorithms running within such time bounds are called *fpt-algorithms*). FPT denotes the class of all fixed-parameter tractable problems. When considering multiple parameters, we take their sum as a single parameter.

Parameterized complexity also offers a *completeness theory*, similar to the theory of NP-completeness, that provides a way to obtain evidence that a parameterized problem is not fixed-parameter tractable. Hardness for parameterized complexity classes is based on fpt-reductions, which are many-one reductions where the parameter of one problem maps into the parameter for the other. More specifically, a parameterized problem $Q$ is fpt-reducible to another parameterized problem $Q'$ if there is a mapping $R$ that maps instances of $Q$ to instances of $Q'$ such that (i) $(I, k) \in Q$ if and only if $R(I, k) = (I', k') \in Q'$, (ii) $k' \leq g(k)$ for a computable function $g$, and (iii) $R$ can be computed in time $f(k)|I|^c$ for a computable function $f$ and a constant $c$. Central to the completeness theory are the classes of the Weft hierarchy, including the class W[SAT]. The parameterized complexity class W[SAT] can be characterized as the set of those parameterized problems that can be fpt-reduced to the problem MONOTONE-WSAT [1]. In this problem, the input consists of a monotone propositional formula $\varphi$—i.e., $\varphi$ contains no negations—and a positive integer $k$. The parameter is $k$, and the question is whether there exists a truth assignment that sets at most $k$ variables in $\text{Var}(\varphi)$ true and that satisfies $\varphi$. Moreover, the parameterized complexity class XP consists of all problems that can be solved in time $O(n^{f(k)})$, for some computable function $f$, where $n$ is the input size and $k$ is the parameter value.

The following parameterized complexity classes are analogues to classical complexity classes. Let K be a classical complexity class, e.g., $\Theta_2^{\text{p}}$. The parameterized complexity class para-K is then defined as the class of all parameterized problems $Q$ for which there exist a computable function $f$ and a problem $Q' \in K$ such that for all instances $(x, k)$ we have that $(x, k) \in Q$ if and only if $(x, f(k)) \in Q'$. Intuitively, the class para-K consists of all problems that are in K after a precomputation that only involves the parameter. A parameterized problem is para-K-hard if it is K-hard already for a constant value of the parameter [22].

The final parameterized complexity class that we consider is $\text{FPT}^{\text{NP}}[\text{few}]$, consisting of all parameterized problems that can be solved by an fpt-algorithm that queries an NP oracle at most $f(k)$ many times, where $f$ is some computable function and where $k$ denotes the parameter value [19, 20, 27]—an NP oracle can decide in a single time step whether an instance of an NP problems is a yes-instance or a no-instance. Intuitively, this class consists of those problems that can be reduced to SAT by a Turing reduction that runs in fixed-parameter tractable time, and queries the oracle at most $f(k)$ times. The following parameterized variant of MAX-MODEL is $\text{FPT}^{\text{NP}}[\text{few}]$-complete under fpt-reductions ($\star$).

LOCAL-MAX-MODEL
*Instance:* A satisfiable propositional formula $\varphi$, a subset $X \subseteq \text{Var}(\varphi)$ of variables, and a variable $w \in X$.
*Parameter:* $|X|$.
*Question:* Is there a model of $\varphi$ that sets a maximal number of variables in $X$ to true (among all models of $\varphi$) and that sets $w$ to true?

# 3 Judgment Aggregation

Next, we introduce the two formal judgment aggregation frameworks that we use in this paper: *formula-based judgment aggregation* (as used by, e.g., [12, 17, 32]) and *constraint-based judgment aggregation* (as used by, e.g., [24]). For both frameworks, we will also define the computational problem OUTCOME(KEMENY) of computing an outcome of the Kemeny procedure, and we will formally define the parameters that we consider.

**Formula-Based Judgment Aggregation** An *agenda* is a finite, nonempty set $\Phi$ of formulas that does not contain any doubly-negated formulas and that is closed under complementation. Moreover, if $\Phi = \{\varphi_1, \dots, \varphi_n, \neg\varphi_1, \dots, \neg\varphi_n\}$ is an agenda, then we let $[\Phi] = \{\varphi_1, \dots, \varphi_n\}$ denote the *pre-agenda* associated to the agenda $\Phi$. We denote the bitsize of the agenda $\Phi$ by $\text{size}(\Phi) = \sum_{\varphi \in \Phi} |\varphi|$. A *judgment set* $J$ for an agenda $\Phi$ is a subset $J \subseteq \Phi$. We call a judgment set $J$ *complete* if $\varphi \in J$ or $\sim\varphi \in J$ for all $\varphi \in \Phi$; and we call it *consistent* if there exists an assignment that makes all formulas in $J$ true. Intuitively, the consistent and complete judgment sets are the opinions that individuals and the group can have.

We associate with each agenda $\Phi$ an integrity constraint $\Gamma$, that can be used to further restrict the set of feasible opinions. Such an *integrity constraint* consists of a single propositional formula. We say that a judgment set $J$ is $\Gamma$-*consistent* if there exists a truth assignment that simultaneously makes all formulas in $J$ and $\Gamma$ true. Let $\mathcal{J}(\Phi, \Gamma)$ denote the set of all complete and $\Gamma$-consistent subsets of $\Phi$. We say that finite sequences $\boldsymbol{J} \in \mathcal{J}(\Phi, \Gamma)^+$ of complete and $\Gamma$-consistent judgment sets are *profiles*, and where convenient we equate a profile $\boldsymbol{J} = (J_1, \dots, J_p)$ with the (multi)set $\{J_1, \dots, J_p\}$.

A *judgment aggregation procedure* (or *rule*) for the agenda $\Phi$ and the integrity constraint $\Gamma$ is a function $F$ that takes as input a profile $\boldsymbol{J} \in \mathcal{J}(\Phi, \Gamma)^+$, and that produces a non-empty set of non-empty judgment sets, i.e., it produces an element in $2^{2^{\Phi} \setminus \{\emptyset\}} \setminus \{\emptyset\}$. We call a judgment aggregation procedure $F$ *resolute* if for any profile $\boldsymbol{J}$ it returns a singleton, i.e., $|F(\boldsymbol{J})| = 1$; otherwise, we call $F$ *irresolute*. An example of a resolute judgment aggregation procedure is the *strict majority rule* Majority, where $\text{Majority}(\boldsymbol{J}) = \{J^*\}$ and where $\varphi \in J^*$ if and only if $\varphi$ occurs in the strict majority of judgment sets in $\boldsymbol{J}$, for all $\varphi \in [\Phi]$. We call a judgment aggregation procedure $F$ *complete* and $\Gamma$-*consistent*, if $J$ is complete and $\Gamma$-consistent, respectively, for every $\boldsymbol{J} \in \mathcal{J}(\Phi, \Gamma)^+$ and every $J \in F(\boldsymbol{J})$. The procedure Majority is not consistent. Consider the agenda $\Phi$ with $[\Phi] = \{p, q, p \rightarrow q\}$, and the profile $\boldsymbol{J} = (J_1, J_2, J_3)$, where $J_1 = \{p, q, (p \rightarrow q)\}$, $J_2 = \{p, \neg q, \neg(p \rightarrow q)\}$, and $J_3 = \{\neg p, \neg q, (p \rightarrow q)\}$. The unique outcome $\{p, \neg q, (p \rightarrow q)\}$ in $\text{Majority}(\boldsymbol{J})$ is inconsistent.

The *Kemeny aggregation procedure* is based on a notion of distance. This distance is based on the Hamming distance $d(J, J') = |\{ \varphi \in [\Phi] : \varphi \in (J \setminus J') \cup (J' \setminus J) \}|$ between two complete judgment sets $J, J'$. Intuitively, the Hamming distance $d(J, J')$ counts the number of issues on which two judgment sets disagree. Let $J$ be a single $\Gamma$-consistent and complete judgment set, and let $(J_1, \dots, J_p) = \boldsymbol{J} \in \mathcal{J}(\Phi, \Gamma)^+$ be a profile. We define the distance between $J$ and $\boldsymbol{J}$ to be $\text{Dist}(J, \boldsymbol{J}) = \sum_{i \in [p]} d(J, J_i)$. Then, we let the outcome $\text{Kemeny}_{\Phi,\Gamma}(\boldsymbol{J})$ of the Kemeny rule be the set of those $J^* \in \mathcal{J}(\Phi, \Gamma)$ for which there is no $J \in \mathcal{J}(\Phi, \Gamma)$ such that $\text{Dist}(J, \boldsymbol{J}) < \text{Dist}(J^*, \boldsymbol{J})$. (If $\Phi$ and $\Gamma$ are clear from the context, we often write $\text{Kemeny}(\boldsymbol{J})$ to denote $\text{Kemeny}_{\Phi,\Gamma}(\boldsymbol{J})$.) Intuitively, the Kemeny rule selects those complete and $\Gamma$-consistent judgment sets that minimize the cumulative Hamming distance to the judgment sets in the profile. The Kemeny rule is irresolute, complete and $\Gamma$-consistent.

We formalize the problem of computing an outcome of the Kemeny rule—in the formula-based judgment aggregation framework—with the following decision problem OUTCOME(KEMENY)$^{\text{fb}}$. Any algorithm that solves OUTCOME(KEMENY)$^{\text{fb}}$ can be used to construct some $J^* \in \text{Kemeny}(\boldsymbol{J})$, with polynomial overhead, by iteratively calling the algorithm and adding formulas to the set $L$. Moreover, multiple outcomes $J_1^*, J_2^*, \dots$ can be constructed by adding previously found outcomes as the sets $L_i$.

OUTCOME(KEMENY)$^{\text{fb}}$
*Instance:* An agenda $\Phi$ with an integrity constraint $\Gamma$, a profile $\boldsymbol{J} \in \mathcal{J}(\Phi, \Gamma)^+$ and subsets $L, L_1, \dots, L_u \subseteq \Phi$ of the agenda, with $u \geq 0$.
*Question:* Is there a judgment set $J^* \in \text{Kemeny}(\boldsymbol{J})$ such that $L \subseteq J^*$ and $L_i \not\subseteq J^*$ for each $i \in [u]$?

The parameters that we consider for the problem OUTCOME(KEMENY)$^{\text{fb}}$ are defined straightforwardly. For an instance $(\Phi, \Gamma, \boldsymbol{J}, L, L_1, \dots, L_u)$ of OUTCOME(KEMENY)$^{\text{fb}}$ with $\boldsymbol{J} = (J_1, \dots, J_p)$, we let $n = |[\Phi]|$, $m = \max\{ |\varphi| : \varphi \in [\Phi] \}$, $c = |\Gamma|$, $p = |\boldsymbol{J}|$, and $h = \max\{ d(J_i, J_{i'}) : 1 \leq i < i' \leq p \}$.

**Constraint-Based Judgment Aggregation** Let $\mathcal{I} = \{x_1, \dots, x_n\}$ be a finite set of *issues*. Intuitively, these issues are the topics about which the individuals want to combine their judgments. A truth assignment $\alpha : \mathcal{I} \to \{0, 1\}$ is called a *ballot*, and represents an opinion that individuals and the group can have. We will also denote ballots $\alpha$ by a binary vector $(b_1, \dots, b_n) \in \{0, 1\}^n$, where $b_i = \alpha(x_i)$ for each $i \in [n]$. Moreover, we say that $(p_1, \dots, p_n) \in \{0, 1, \star\}^n$ is a *partial ballot*, and that $(p_1, \dots, p_n)$ *agrees with* a ballot $(b_1, \dots, b_n)$ if $p_i = b_i$ whenever $p_i \neq \star$, for all $i \in [n]$. As in the case for formula-based judgment aggregation, we introduce an integrity constraint $\Gamma$, that can be used to restrict the set of feasible opinions (for both the individuals and the group). The integrity constraint $\Gamma$ is a propositional formula on the variables $x_1, \dots, x_n$. We define the set $\mathcal{R}(\mathcal{I}, \Gamma)$ of *rational ballots* to be the ballots (for $\mathcal{I}$) that satisfy the integrity constraint $\Gamma$. Rational ballots in the constraint-based judgment aggregation framework correspond to complete and $\Gamma$-consistent judgment sets in the formula-based judgment aggregation framework. We say that finite sequences $\boldsymbol{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ of rational ballots are *profiles*, and where convenient we equate a profile $\boldsymbol{r} = (r_1, \dots, r_p)$ with the (multi)set $\{r_1, \dots, r_p\}$.

A *judgment aggregation procedure* (or *rule*), for the set $\mathcal{I}$ of issues and the integrity constraint $\Gamma$, is a function $F$ that takes as input a profile $\boldsymbol{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$, and that produces a non-empty set of ballots. We call a judgment aggregation procedure $F$ *rational* (or *consistent*), if $r$ is rational for every $\boldsymbol{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ and every $r \in F(\boldsymbol{r})$.

As an example of a judgment aggregation procedure we consider the *strict majority rule* Majority, where $\text{Majority}(\boldsymbol{r}) = \{(b_1, \dots, b_n)\}$ and where each $b_i$ agrees with the majority of the $i$-th bits in the ballots in $\boldsymbol{r}$ (in case of a tie, we arbitrarily let $b_i = 1$). To see that Majority is not rational, consider the set $\mathcal{I} = \{x_1, x_2, x_3\}$ of issues, the integrity constraint $\Gamma = x_3 \leftrightarrow (x_1 \to x_2)$, and the profile $\boldsymbol{r} = (r_1, r_2, r_3)$, where $r_1 = (1, 1, 1)$, $r_2 = (1, 0, 0)$, and $r_3 = (0, 0, 1)$. The unique outcome $(1, 0, 1)$ in $\text{Majority}(\boldsymbol{r})$ is not rational.

The *Kemeny aggregation procedure* is defined for the constraint-based judgment aggregation framework as follows. Similarly to the case for formula-based judgment aggregation, the Kemeny rule is based on the Hamming distance $d(r, r') = |\{\, i \in [n] : b_i \neq b'_i \,\}|$, between two rational ballots $r = (b_1, \dots, b_n)$ and $r' = (b'_1, \dots, b'_n)$ for the set $\mathcal{I}$ of issues and the integrity constraint $\Gamma$. Let $r$ be a single ballot, and let $(r_1, \dots, r_p) = \boldsymbol{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ be a profile. We define the distance between $r$ and $\boldsymbol{r}$ to be $\text{Dist}(r, \boldsymbol{r}) = \sum_{i \in [p]} d(r, r_i)$. Then, we let the outcome $\text{Kemeny}_{\mathcal{I},\Gamma}(\boldsymbol{r})$ of the Kemeny rule be the set of those ballots $r^* \in \mathcal{R}(\mathcal{I}, \Gamma)$ for which there is no $r \in \mathcal{R}(\mathcal{I}, \Gamma)$ such that $\text{Dist}(r, \boldsymbol{r}) < \text{Dist}(r^*, \boldsymbol{r})$. (If $\mathcal{I}$ and $\Gamma$ are clear from the context, we often write $\text{Kemeny}(\boldsymbol{r})$ to denote $\text{Kemeny}_{\mathcal{I},\Gamma}(\boldsymbol{r})$.) The Kemeny rule is irresolute and rational.

We formalize the problem of computing an outcome of the Kemeny rule—in the constraint-based judgment aggregation framework—with the following decision problem $\text{Outcome(Kemeny)}^{\text{cb}}$. Similarly to algorithms for $\text{Outcome(Kemeny)}^{\text{fb}}$, algorithms that solve $\text{Outcome(Kemeny)}^{\text{cb}}$ can be used to construct multiple outcomes.

$\text{Outcome(Kemeny)}^{\text{cb}}$
*Instance:* A set $\mathcal{I}$ of issues with an integrity constraint $\Gamma$, a profile $\boldsymbol{r} \in \mathcal{R}(\mathcal{I}, \Gamma)^+$ and partial ballots $l, l_1, \dots, l_u$ (for $\mathcal{I}$), with $u \geq 0$.
*Question:* Is there a ballot $r^* \in \text{Kemeny}(\boldsymbol{r})$ such that $l$ agrees with $r^*$ and each $l_i$ does not agree with $r^*$?

We define the parameters that we consider for $\text{Outcome(Kemeny)}^{\text{cb}}$ as follows. For an instance $(\mathcal{I}, \Gamma, \boldsymbol{r}, l, l_1, \dots, l_u)$ of $\text{Outcome(Kemeny)}^{\text{cb}}$ with $\boldsymbol{r} = (r_1, \dots, r_p)$, we let $n = |\mathcal{I}|$, $c = |\Gamma|$, $p = |\boldsymbol{r}|$, and $h = \max\{\, d(r_i, r_{i'}) : 1 \leq i < i' \leq p \,\}$. We remark that the parameter $m$ does not make sense in the constraint-based framework, as issues are not represented by a logic formula. When needed, the parameter $m$ for $\text{Outcome(Kemeny)}^{\text{cb}}$ is defined by letting $m = 1$.

# 4 Complexity Results

In this section, we develop the parameterized complexity results for the different parameterized variants of $\text{Outcome(Kemeny)}^{\text{fb}}$ and $\text{Outcome(Kemeny)}^{\text{cb}}$ that we consider.

## 4.1 Upper Bounds for the Formula-Based Framework

We begin with showing upper bounds for $\text{Outcome(Kemeny)}^{\text{fb}}$. When parameterized either (i) by both $h$ and $p$ or (ii) by $n$, the problem is in $\text{FPT}^{\text{NP}}[\text{few}]$.

**Proposition 1.** $\text{Outcome(Kemeny)}^{\text{fb}}$ *parameterized by* $h$ *and* $p$ *is in* $\text{FPT}^{\text{NP}}[\text{few}]$.

*Proof.* The main idea behind this proof is that with these parameters, we can derive a suitable upper bound on the minimum distance of any complete and $\Gamma$-consistent judgment set to the profile $\boldsymbol{J}$, such that the usual binary search algorithm with access to an NP oracle only needs to make $O(\log h + \log p)$ many oracle queries.

We describe an algorithm $A$ that solves $\text{Outcome(Kemeny)}^{\text{fb}}$ with the required number of oracle queries. Let $(\Phi, \Gamma, \boldsymbol{J}, L, L_1, \dots, L_u)$ be an instance. The algorithm needs to determine the minimum distance $d(J, \boldsymbol{J})$ for any complete and $\Gamma$-consistent judgment set $J$ to the profile $\boldsymbol{J}$. Let $d^*$ denote this minimum distance. An upper bound on $d^*$ is given by $h(p-1)$. This upper bound can be derived as follows. Take an arbitrary $J \in \boldsymbol{J}$. Clearly $d(J, J) = 0$, and for every $J' \in \boldsymbol{J}$ with $J \neq J'$ we know that $d(J, J') \leq h$. Therefore, $d(J, \boldsymbol{J}) \leq h(p-1)$. Since $J \in \boldsymbol{J}$, we know that $J$ is complete and $\Gamma$-consistent. Therefore, the minimum distance of any complete and $\Gamma$-consistent judgment set to the profile $\boldsymbol{J}$ is at most $h(p-1)$.

The algorithm $A$ firstly computes $d^*$. Since $d^* \leq h(p-1)$, with binary search this can be done using at most $\lceil \log h(p-1) \rceil = O(\log h + \log p)$ many queries to an oracle. Then, with a single additional oracle query, the algorithm $A$ determines whether there exists a complete and $\Gamma$-consistent judgment set $J^*$ with $d(J^*, \boldsymbol{J}) = d^*$, $L \subseteq J^*$, and $L_j \not\subseteq J^*$ for each $j \in [u]$. ☐

When parameterized by the number $n$ of formulas in the pre-agenda, the number of possible judgment sets is bounded by a function of the parameter. This allows the problem to be solved in fixed-parameter tractable time, using a single query to an NP oracle for each judgment set to determine whether it it $\Gamma$-consistent.

**Proposition⋆ 2.** $\text{Outcome(Kemeny)}^{\text{fb}}$ *parameterized by* $n$ *is in* $\text{FPT}^{\text{NP}}[\text{few}]$.

When additionally parameterizing by $c$ and $m$, $\Gamma$-consistency of the judgment sets can be decided in fixed-parameter tractable time, and thus the whole problem becomes fixed-parameter tractable.

**Proposition⋆ 3.** $\text{Outcome(Kemeny)}^{\text{fb}}$ *parameterized by* $c$*,* $n$ *and* $m$ *is fixed-parameter tractable.*

## 4.2 Lower Bounds for the Formula-Based Framework

Next, we turn to parameterized hardness results for the problem $\text{Outcome(Kemeny)}^{\text{fb}}$. We begin with showing that the problem is para-$\Theta_2^{\text{p}}$-hard even when parameterized by $c$, $h$ and $m$. We will use the following lemma.

**Lemma⋆ 4.** *Let* $\varphi$ *be a propositional formula on the variables* $x_1, \dots, x_n$*. In polynomial time we can construct a propositional formula* $\varphi'$ *with* $\text{Var}(\varphi') \supseteq \text{Var}(\varphi) \cup \{z_1, \dots, z_n\}$ *such that for every truth assignment* $\alpha : \text{Var}(\varphi) \to \{0, 1\}$ *it holds that (1)* $\varphi[\alpha]$ *is true if and only if* $\varphi'[\alpha]$ *is satisfiable, and (2) if* $\alpha$ *sets exactly* $i$ *variables to true, then* $\varphi'[\alpha] \models z_i$*.*

**Proposition 5.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by $c$ and $h$ is* para-$\Theta_2^{\text{p}}$*-hard.*

*Proof.* We show that Outcome(Kemeny)$^{\text{fb}}$ is $\Theta_2^{\text{p}}$-hard already for a constant value of the parameters, by giving a reduction from Max-Model. Let $(\varphi, w)$ be an instance of Max-Model with $\text{Var}(\varphi) = \{x_1, \dots, x_n\}$ and $w = x_1$. Without loss of generality, we may assume that there is a model $\alpha$ of $\varphi$ that sets at least two variables $x_i$ to true. By Lemma 4, we can construct a suitable formula $\varphi' = c_1 \wedge \dots \wedge c_b$ with additional variables $z_1, \dots, z_n$ that represent a lower bound on the number of variables among $x_1, \dots, x_n$ that are true in models of $\varphi$.

We construct the agenda $\Phi$ by letting $[\Phi] = \{z_w, z_{\neg w}, z_1, \dots, z_n\} \cup \{\, y_{w,i}, y_{\neg w,i} : i \in [n+1] \,\} \cup \{\, y_{i,j} : i \in [n], j \in [i] \,\} \cup \{\chi, \chi'\}$, where $z_w, z_{\neg w}$ and all $y_{w,i}, y_{\neg w,i}, y_{i,j}$ are fresh variables. We let $Y = \{\, y_{w,i}, y_{\neg w,i} : i \in [n+1] \,\} \cup \{\, y_{i,j} : i \in [n], j \in [i] \,\}$. Moreover, we let $\chi$ be such that $\chi \equiv \neg((\bigvee Y \wedge \bigvee([\Phi] \backslash Y)) \vee ((z_w \leftrightarrow x_1 \leftrightarrow \neg z_{\neg w}) \wedge \varphi'))$, and we define $\chi'$ such that $\chi' \equiv \chi$ (that is, we let $\chi'$ be a syntactic variant of $\chi$).

Then, we construct the profile $\boldsymbol{J}$ as follows. We let $\boldsymbol{J} = \{\, J_{w,i}, J_{\neg w,i} : i \in [n+1] \,\} \cup \{\, J_{i,j} : i \in [n], j \in [i] \,\}$. Each of the judgment sets in the profile includes exactly two formulas in $[\Phi]$. Consequently, the maximum Hamming distance between any two judgment sets in the profile is 4. For each $i \in [n+1]$, we let $\{y_{w,i}, z_w\} \subseteq J_{w,i}$ and $\{y_{\neg w,i}, z_{\neg w}\} \subseteq J_{\neg w,i}$. Moreover, for each $i \in [n]$ and each $j \in [i]$, we let $\{y_{i,j}, z_i\} \subseteq J_{i,j}$. It is straightforward to verify that each $J \in \boldsymbol{J}$ is consistent. Finally, we let $L = \{z_w\}$, $\Gamma = \top$, and $u = 0$.

In other words, all formulas in $[\Phi]$ are excluded in a majority of the judgment sets in the profile $\boldsymbol{J}$. However, some formulas in $[\Phi]$ are included in more judgment sets in the profile than others. The formulas $z_w$ and $z_{\neg w}$ are both included in $n+1$ sets. Each formula $z_i$ (for $i \in [n]$) is included in exactly $i$ sets. All formulas in $Y$ are included in exactly one set. Finally, the formulas $\chi$ and $\chi'$ are included in none of the sets. Intuitively, the formulas that are included in more judgment sets in the profile are cheaper to include in any candidate outcome $J^*$.

The complete judgment set that minimizes the cumulative Hamming distance to the profile $\boldsymbol{J}$ is the set $J_0 = \{\, \neg\varphi : \varphi \in [\Phi] \,\}$ that includes no formulas in $[\Phi]$. However, this set is inconsistent, which is straightforward to verify using the definition of $\chi$. It can be made consistent by adding two formulas $\varphi_1, \varphi_2$ from $[\Phi]$ (and removing their complements). The choice of $\varphi_1, \varphi_2$ that leads to a consistent judgment set with minimum distance to the profile is by letting $\varphi_1 \in \{z_w, z_{\neg w}\}$ and letting $\varphi_2 = z_\ell$, where $\ell$ is the maximum number of variables among $x_1, \dots, x_n$ set to true in any model of $\varphi$. Moreover, whenever $\varphi_1 = z_w$, the resulting judgment set is consistent if and only if there is a model of $\varphi$ that sets $\ell$ variables among $x_1, \dots, x_n$ to true, including the variable $w$. From this, we directly know that $(\varphi, w) \in$ Max-Model if and only if $(\Phi, \Gamma, \boldsymbol{J}, L) \in$ Outcome(Kemeny)$^{\text{fb}}$. This concludes our para-$\Theta_2^{\text{p}}$-hardness proof. ☐

This hardness result can straightforwardly be extended to the case where all formulas in the agenda are of constant size, by using the well-known Tseitin transformation [43] to transform a formula into CNF, leading to the following corollary.

**Corollary$^\star$ 6.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by $c$, $h$ and $m$ is* para-$\Theta_2^{\text{p}}$*-hard.*

The problem is also para-$\Theta_2^{\text{p}}$-hard when parameterized by $c$, $m$ and $p$.

**Proposition 7.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by $c$, $m$ and $p$ is* para-$\Theta_2^{\text{p}}$*-hard.*

*Proof.* We firstly show para-$\Theta_2^{\text{p}}$-hardness for the problem parameterized by $c$ and $p$, by giving a reduction from Max-Model that uses constant values of $c$ and $p$. This reduction can be seen as a modification of the $\Theta_2^{\text{p}}$-hardness proof for Outcome(Kemeny)$^{\text{fb}}$ given by Endriss and De Haan [18, Proposition 7 and Corollary 8].

Let $(\varphi, w)$ be an instance of Max-Model. We may assume without loss of generality that $\varphi$ is satisfiable by some truth assignment that sets at least one variable in $\text{Var}(\varphi)$ to true. We construct an instance $(\Phi, \Gamma, \boldsymbol{J}, L)$ of Outcome(Kemeny)$^{\text{fb}}$ as follows. Take an integer $b$ such that $b > \frac{3}{2}|\text{Var}(\varphi)|$, e.g., $b = 3|\text{Var}(\varphi)| + 1$. Let $[\Phi] = \text{Var}(\varphi) \cup \{ z_{i,j} : i \in [b], j \in [3] \} \cup \{ \varphi'_i : i \in [b] \}$, where each of the formulas $\varphi'_i$ is a syntactic variant of the following formula $\varphi'$. We define $\varphi' = (\bigvee_{j \in [3]} \bigwedge_{i \in [b]} z_{i,j}) \vee \varphi$. Intuitively, the formula $\varphi'$ is true either if (i) all variables $z_{i,j}$ are set to true for some $j \in [3]$, or if (ii) $\varphi$ is satisfied. Then we let $\boldsymbol{J} = \{J_1, J_2, J_3\}$, where for each $j \in [3]$, we let $J_j$ contain the formulas $z_{i,j}$ for all $i \in [b]$, all formulas in $\text{Var}(\varphi)$, all the formulas $\varphi'_i$, and no other formulas from $[\Phi]$. (For each $\varphi \in [\Phi]$, if $\varphi \notin J_j$, we let $\neg\varphi \in J_j$.) Clearly, the judgment sets $J_1$, $J_2$ and $J_3$ are all complete and consistent. Moreover, we let $\Gamma = \top$, and $L = \{w\}$. It is straightforward to verify that the parameters $c$ and $p$ have constant values.

We now argue that there is some $J^* \in \text{Kemeny}(\boldsymbol{J})$ with $L \subseteq J^*$ if and only if $(\varphi, w) \in$ Max-Model. To see this, we first observe that the only complete and consistent judgment sets $J$ for which it holds that $d(J, \boldsymbol{J}) < d(J_j, \boldsymbol{J})$ (for any $j \in [3]$) must satisfy that $J \models \varphi$. Moreover, among those judgment sets $J$ for which $J \models \varphi$, the judgment sets that minimize the distance to the profile $\boldsymbol{J}$ satisfy that $z_{i,j} \notin J$ for all $i \in [b]$ and all $j \in [3]$, and $\varphi'_i \in J$ for all $i \in [b]$. Using these observations, we directly get that there is some $J^* \in \text{Kemeny}(\boldsymbol{J})$ with $L \subseteq J^*$ if and only if there is a model of $\varphi$ that sets a maximal number of variables in $\text{Var}(\varphi)$ to true and that sets the variable $w$ to true.

Then, to show that the problem is also para-$\Theta_2^{\text{P}}$-hard when parameterized by $c$, $m$ and $p$, we can modify the above reduction in a way that is entirely similar to the proof of Corollary 6, replacing the formulas $\varphi'_i$ by the clauses of 3CNF formulas that have the same effect on the consistency of judgment sets as the formulas $\varphi'_i$. ☐

For all parameterizations that do not include all of the parameters $c$, $n$ and $m$, the problem Outcome(Kemeny)$^{\text{fb}}$ is $\text{FPT}^{\text{NP}}$[few]-hard. We begin with the case where $c$ can be unbounded; this proof can be extended straightforwardly to the other two cases.

**Proposition 8.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by $h$, $n$, $m$ and $p$ is* $\text{FPT}^{\text{NP}}$[few]-*hard.*

*Proof.* We show $\text{FPT}^{\text{NP}}$[few]-hardness by giving an fpt-reduction from Local-Max-Model. (This reduction from Local-Max-Model is very similar to the reduction from Max-Model used in the proof of Proposition 7.) Let $(\varphi, X, w)$ be an instance of Local-Max-Model, with $X = \{x_1, \dots, x_k\}$. We construct an instance $(\Phi, \Gamma, \boldsymbol{J}, L)$ as follows. Take an integer $b$ such that $b > \frac{3}{2}|X|$, e.g., let $b = 3|X| + 1$. We let $\Phi = X \cup \{ z_{i,j} : i \in [b], j \in [3] \}$. Moreover, we let $\Gamma = \varphi' = (\bigvee_{j \in [3]} \bigwedge_{i \in [b]} z_{i,j}) \vee \varphi$. Intuitively, the formula $\Gamma$ is true either if (i) all variables $z_{i,j}$ are set to true for some $j \in [3]$, or if (ii) $\varphi$ is satisfied. Then we let $\boldsymbol{J} = \{J_1, J_2, J_3\}$, where for each $j \in [3]$, we let $J_j$ contain the formulas $z_{i,j}$ for all $i \in [b]$, and all formulas in $X$, and no other formulas in $[\Phi]$. (For each $\varphi \in [\Phi]$, if $\varphi \notin J_j$, we let $\neg\varphi \in J_j$.) Clearly, the judgment sets $J_1$, $J_2$ and $J_3$ are all complete and $\Gamma$-consistent. Finally, we let $L = \{w\}$. It is easy to verify that $h = 2b = 6k + 2$ and $n = 3b + k = 10k + 3$, where $k = |X|$, and that $m$ and $p$ are constant. Therefore, all parameter values are bounded by a function of the original parameter $k$.

We now argue that there is some $J^* \in \text{Kemeny}(\boldsymbol{J})$ with $L \subseteq J^*$ if and only if $(\varphi, X, w) \in$ Local-Max-Model. The argument for this conclusion is similar to the argument used in the proof of Proposition 7. We first observe that the only complete and consistent judgment sets $J$ for which it holds that $d(J, \boldsymbol{J}) < d(J_j, \boldsymbol{J})$ (for any $j \in [3]$) must satisfy that $J \models \varphi$. Moreover, among those judgment sets $J$ for which $J \models \varphi$, the judgment sets that minimize the distance to the profile $\boldsymbol{J}$ satisfy that $z_{i,j} \notin J$ for all $i \in [b]$ and all $j \in [3]$. Using these observations, we directly get that there is some $J^* \in \text{Kemeny}(\boldsymbol{J})$ with $L \subseteq J^*$ if and only if there is a model of $\varphi$ that sets a maximal number of variables in $X$ to true and that sets the variable $w$ to true. ☐

**Proposition⋆ 9.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by* $c$, $h$, $n$ *and* $p$ *is* $\text{FPT}^{\text{NP}}[\text{few}]$-*hard.*

**Proposition⋆ 10.** Outcome(Kemeny)$^{\text{fb}}$ *parameterized by* $c$, $h$, $m$ *and* $p$ *is* $\text{FPT}^{\text{NP}}[\text{few}]$-*hard.*

### 4.3 Upper Bounds for the Constraint-Based Framework

We now turn to showing upper bounds for Outcome(Kemeny)$^{\text{cb}}$. When parameterized by the number $n$ of issues, the number of possible ballots is bounded by a function of the parameter. This allows the problem to be solved in fixed-parameter tractable time.

**Proposition⋆ 11.** Outcome(Kemeny)$^{\text{cb}}$ *parameterized by* $n$ *is fixed-parameter tractable.*

Since the size $c$ of the integrity constraint is an upper bound on the number of issues that play a non-trivial role in the problem, this fixed-parameter tractability result easily extends to the parameter $c$.

**Proposition⋆ 12.** Outcome(Kemeny)$^{\text{cb}}$ *parameterized by* $c$ *is fixed-parameter tractable.*

Bounding the maximum Hamming distance $h$ between any two ballots in the profile gives us membership in XP.

**Proposition 13.** Outcome(Kemeny)$^{\text{cb}}$ *parameterized by* $h$ *is in* XP.

*Proof.* Let $(\mathcal{I}, \Gamma, \boldsymbol{r}, l, l_1, \dots, l_u)$ be an instance, with $\boldsymbol{r} = (r_1, \dots, r_p)$. We describe an algorithm to solve the problem in time $O(p \cdot n^h \cdot n^d)$, for some constant $d$. The main idea behind this algorithm is the fact that each ballot whose Hamming distance to every ballot in the profile is more than $h$ is irrelevant.

Take a ballot $r$ such that $d(r, r_i) > h$ for each $i \in [p]$. We show that there exists a rational ballot $r'$ with $d(r', \boldsymbol{r}) < d(r, \boldsymbol{r})$. Take any ballot in the profile, e.g., $r' = r_1$. Clearly, $r'$ is rational. Since $d(r, r_i) > h$ for each $i \in [p]$, we know that $d(r, \boldsymbol{r}) > hp$. On the other hand, for $r'$ we know that $d(r', r_i) \leq h$ for each $i \in [p]$ (and $d(r', r_1) = 0$), so $d(r', \boldsymbol{r}) \leq h(p-1)$. Therefore, $d(r', \boldsymbol{r}) < d(r, \boldsymbol{r})$.

We thus know that every rational ballot with minimum distance to the profile lies at Hamming distance at most $h$ to some ballot $r_i$ in the profile $\boldsymbol{r}$. The algorithm works as follows. It firstly enumerates all ballots with Hamming distance at most $h$ to some $r_i \in \boldsymbol{r}$. This can be done in time $O(p \cdot n^h)$. Then, similarly to the algorithm in the proof of Proposition 11, it discards those ballots that are not rational, and subsequently discards those ballots that do not have minimum distance to the profile. Finally, it iterates over all remaining rational ballots with minimum distance to determine whether there is one among them that agrees with $l$ and disagrees with each $l_j$. ☐

### 4.4 Lower Bounds for the Constraint-Based Framework

Finally, we show parameterized hardness results for Outcome(Kemeny)$^{\text{cb}}$. When parameterized by both $h$ and $p$, the problem is W[SAT]-hard.

**Proposition 14.** Outcome(Kemeny)$^{\text{cb}}$ *parameterized by* $h$ *and* $p$ *is* W[SAT]-*hard.*

*Proof.* We give an fpt-reduction from the W[SAT]-complete problem Monotone-WSat. Let $(\varphi, k)$ be an instance of Monotone-WSat. Assume without loss of generality that $k$ is divisible by 4. We construct an instance $(\mathcal{I}, \Gamma, \boldsymbol{r}, l)$ of $\text{Outcome}(\text{Kemeny})^{\text{cb}}$ as follows. We let $\mathcal{I} = \text{Var}(\varphi) \cup \{z\} \cup \{\, y_{i,j} : i \in [3], j \in [\frac{3}{4}k+1]\,\}$. Moreover, we let $\Gamma = (z \wedge \varphi') \vee (\neg z \wedge \bigvee_{i \in [3]} (\bigwedge_{j \in [\frac{3}{4}k+1]} y_{i,j}))$. We define $\boldsymbol{r} = (r_1, r_2, r_3)$ as follows. For each $r_i$, we let $r_i(w) = 0$ for all $w \in \{z\} \cup \text{Var}(\varphi')$. Moreover, for each $r_i$ and each $y_{\ell,j}$, we let $r_i(y_{\ell,j}) = 1$ if and only if $\ell = i$. It is readily verified that $r_1$, $r_2$ and $r_3$ are all rational. Finally, we let $l$ be the partial assignment for which $l(z) = 1$, and that is undefined on all remaining variables. This completes our construction. Clearly, $p = 3$. Moreover, $h = 2(\frac{3}{4}k + 1)$.

By construction of $\Gamma$, the only ballots that are rational—and that can have a smaller distance to the profile $\boldsymbol{r}$ than the ballots $r_1$, $r_2$ and $r_3$—are those ballots $r^*$ that satisfies $(z \wedge \varphi')$. The ballots $r_1$, $r_2$ and $r_3$ have distance $4(\frac{3}{4}k+1) = 3k+4$ to the profile $\boldsymbol{r}$. Any ballot $r^*$ that satisfies $(z \wedge \varphi')$ minimizes its distance to $\boldsymbol{r}$ by setting all variables $y_{i,j}$ to false. Any such ballot $r^*$ has distance $3(w + 1) = 3w + 3$ to the profile $\boldsymbol{r}$, where $w$ is the number of variables among $\text{Var}(\varphi')$ that it sets to true. Therefore, the distance of such a ballot $r^*$ to the profile $\boldsymbol{r}$ is smaller than the distance of $r_1$, $r_2$ and $r_3$ to $\boldsymbol{r}$ if and only if $w \leq k$. From this we can conclude that there is some $r^* \in \text{Kemeny}(\boldsymbol{r})$ that agrees with $l$ if and only if $(\varphi, k) \in$ Monotone-WSat. ☐

Finally, the proof of Proposition 7 can be modified to work also for the problem $\text{Outcome}(\text{Kemeny})^{\text{cb}}$ parameterized by $p$, showing para-$\Theta_2^{\text{p}}$-hardness for this case.

**Proposition⋆ 15.** $\text{Outcome}(\text{Kemeny})^{\text{cb}}$ *parameterized by $p$ is* para-$\Theta_2^{\text{p}}$*-hard.*

# 5 Conclusion

We gave the first parameterized complexity results for the fundamental problem of computing outcomes of judgment aggregation procedures. We studied parameterized variants of this problem for the Kemeny rule, for all combinations of the parameters $c$, $h$, $n$, $m$ and $p$. Moreover, we performed this parameterized complexity analysis for two formal frameworks for judgment aggregation: formula-based and constraint-based judgment aggregation.

Interestingly, for many combinations of parameters, the complexity of the problem differs between the two frameworks—which is in contrast with the fact that the problem has the same complexity in both frameworks when viewed from a classical complexity point of view. This reflects the ability of the framework of parameterized complexity to more accurately indicate what aspects of the problem input contribute to the complexity of the problem. The two judgment aggregation frameworks distribute the aspects of the problem differently over various parts of the problem input.

Future work includes extending the parameterized complexity investigation for computing outcomes of the Kemeny rule to different parameters. For instance, in particular for the constraint-based judgment aggregation framework, restricting the maximum degree of variables in the integrity constraint might lead to more positive parameterized complexity results. Other natural parameters that could be considered are width measures that capture the amount of structure in the logic formulas in the problem input. Moreover, it would be interesting to perform a similar parameterized complexity analysis for other judgment aggregation procedures.

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## Appendix: Additional Proofs

In this appendix, we provide proofs for those results in the main text for which a proof was omitted (these results were marked with an asterisk).

**Proposition 16.** Local-Max-Model *is* $\text{FPT}^{\text{NP}}[\text{few}]$*-complete.*

*Proof.* Membership in $\text{FPT}^{\text{NP}}[\text{few}]$ can be shown routinely. We show hardness by giving an fpt-reduction from the problem of deciding, given a family $(\varphi_i, \varphi'_i)_{1 \leq i \leq k}$ of pairs of propositional formulas, whether there exists some $1 \leq \ell \leq k$ such that $\varphi_\ell$ is satisfiable and $\varphi'_\ell$ is unsatisfiable. The parameter for this problem is $k$. This problem is known to be complete for $\text{FPT}^{\text{NP}}[\text{few}]$ [20].

Let $(\varphi_i, \varphi'_i)_{1 \leq i \leq k}$ be an instance of this problem. We assume without loss of generality that the formulas $\varphi_i$ and $\varphi'_i$ are all variable-disjoint. We construct an instance $(\psi, Z, w)$ of Local-Max-Model as follows. We consider the following disjoint sets of propositional variables:

$$\begin{aligned} V &= \bigcup_{1 \leq i \leq k} (\text{Var}(\varphi_i) \cup \text{Var}(\varphi'_i)), \\ X &= \{ x_i, x'_i : 1 \leq i \leq k \}, \\ Y &= \{ y_i, y'_i : 1 \leq i \leq k \}, \text{ and} \\ W &= \{w\}. \end{aligned}$$

We let $Z = X \cup Y \cup W$.

We then define the formula $\psi$ to be the conjunction of the following propositional formulas. Firstly, we ensure that whenever some $x_i$ is true, then $\varphi_i$ must be satisfied, and whenever some $x'_i$ is true, then $\varphi'_i$ must be satisfied. We do so by means of the following formula:

$$\bigwedge_{1 \leq i \leq k} ((x_i \rightarrow \varphi_i) \wedge (x'_i \rightarrow \varphi'_i)).$$

Then, we ensure that the variables $y_i$ and $y'_i$ get the same truth value as the variables $x_i$ and $x'_i$ (respectively):

$$\bigwedge_{1 \leq i \leq k} ((x_i \leftrightarrow y_i) \wedge (x'_i \leftrightarrow y'_i)).$$

Finally, we ensure that $w$ can only be true if there is some $1 \leq i \leq k$ such that $x_i$ is true and $x'_i$ is false:

$$w \leftrightarrow \bigvee_{1 \leq i \leq k} (x_i \wedge \neg x'_i).$$

The satisfying assignment of $\psi$ that sets as many variables in $Z$ to true as possible satisfies as many of the formulas $\varphi_i$ and $\varphi'_i$ as possible. Therefore, the model of $\psi$ that sets a maximal number of variables in $Z$ to true sets $w$ to true if and only if there is some $1 \leq \ell \leq k$ such that $\varphi_\ell$ is satisfiable and $\varphi'_\ell$ is unsatisfiable. □

*Proof of Proposition 2.* The main idea behind this proof is that the number of possible judgment sets is bounded by the parameter, that is, there are only $2^n$ many possible complete judgment sets. We describe an algorithm $A$ that solves the problem in fixed-parameter tractable time by querying an NP oracle at most $2^n$ many times. Let $(\Phi, \Gamma, \boldsymbol{J}, L, L_1, \dots, L_u)$ be an instance. Firstly, the algorithm $A$ enumerates all possible complete judgment sets $J_1, \dots, J_{2^n} \subseteq \Phi$. Then, for each such set $J_i$, the algorithm uses the NP oracle to determine whether $J_i$ is $\Gamma$-consistent. Each judgment set $J_i$ that is not $\Gamma$-consistent is discarded. This can be done straightforwardly using $2^n$ many calls to the NP oracle—one for each set $J_i$. (The number of oracle calls that are needed can be improved to $O(n)$ by using binary search on the number of $\Gamma$-consistent sets $J_i$.)

Then, for each of the remaining ($\Gamma$-consistent) judgment sets $J_i$, the algorithm $A$ computes the cumulative Hamming distance $d(J_i, \boldsymbol{J})$ to the profile $\boldsymbol{J}$. This can be done in polynomial time. Then, those $J_i$ for which this distance is not minimal—that is, those $J_i$ for which there exists some $J_{i'}$ such that $d(J_{i'}, \boldsymbol{J}) < d(J_i, \boldsymbol{J})$—are discarded as well. The remaining judgment sets $J_i$ then are exactly the complete and $\Gamma$-consistent judgment sets with a minimum distance to the profile $\boldsymbol{J}$.

Finally, the algorithm goes over each of these remaining sets $J_i$, and checks whether $L \subseteq J_i$ and $L_j \not\subseteq J_i$ for all $j \in [u]$. This can clearly be done in polynomial time. If this check succeeds for some $J_i$, the algorithm $A$ accepts the input, and otherwise, the algorithm rejects the input. □

*Proof of Proposition 3.* We describe an fpt-algorithm $A$ that solves the problem. Let $(\Phi, \Gamma, \boldsymbol{J}, L, L_1, \ldots, L_u)$ be an instance. The algorithm $A$ works exactly in the same way as the algorithm in the proof of Proposition 2. The only difference is that in order to check whether a given judgment set $J_i$ is $\Gamma$-consistent, it does not need to make an oracle query. Determining whether a given judgment set $J_i$ is $\Gamma$-consistent can be done in a brute-force fashion (e.g., using truth tables) in time $2^{c+nm} \cdot |J_i|$, since there are at most $c + nm$ propositional variables involved. Therefore, the algorithm runs in fixed-parameter tractable time. □

*Proof of Lemma 4.* Let $\varphi$ be a propositional formula on the variables $x_1, \ldots, x_n$. We construct the formula $\varphi'$ as follows. We introduce propositional variables $z_{i,j}$ and $z_i$ for each $i \in [n]$ and each $j \in [i]$. Intuitively, the variables $z_{i,j}$ encodes whether among the variables $x_1, \ldots, x_i$ at least $j$ variables are set to true, and the variables $z_i$ encode whether among the variables $x_1, \ldots, x_n$ exactly $i$ variables are set to true.

We let $\varphi'$ be a conjunction of several formulas. The first conjunct of $\varphi'$ is the original formula $\varphi$. Then, we add the following conjunct:

$$z_{1,1} \leftrightarrow x_1.$$

Moreover, for each $i \in [n]$ such that $i > 1$, we add:

$$\left( x_i \leftrightarrow \bigwedge_{j \in [i]} (z_{i,j} \leftrightarrow z_{i-1,j-1}) \right) \wedge \left( \neg x_i \leftrightarrow \bigwedge_{j \in [i]} (z_{i,j} \leftrightarrow z_{i-1,j}) \right),$$

where for any $i \in [n]$, $z_{i,0}$ abbreviates $\top$. Finally, for each $i \in [n]$, we add:

$$z_i \leftrightarrow (z_{n,i} \wedge \neg z_{n,i+1}),$$

where $z_{n,n+1}$ abbreviates $\bot$.

It is straightforward to verify that the formula $\varphi'$ satisfies the required properties. □

*Proof of Corollary 6.* We can modify the proof of Proposition 5 as follows. We replace the formula $\neg\chi$ (and its syntactic variant $\neg\chi'$) by a 3CNF formula that has the same effect. By using the standard Tseitin transformation [43], we can transform $\neg\chi$ into a 3CNF formula $\psi$ such that for each truth assignment $\alpha : \text{Var}(\neg\chi) \to \{0, 1\}$ it holds that $\neg\chi[\alpha]$ is true if and only if $\psi[\alpha]$ is satisfiable. Moreover, we can do this in such a way that the variables in $\text{Var}(\psi) \backslash \text{Var}(\neg\chi)$ are fresh variables. Similarly, we transform $\neg\chi'$ into a 3CNF formula $\psi'$. Let $\psi = c_1 \wedge \cdots \wedge c_b$ and $\psi' = c'_1 \wedge \cdots \wedge c'_b$.

For all judgment sets $J \in \boldsymbol{J}$, we had that $\neg\chi, \neg\chi' \in J$. Instead, we now ensure that for all $J \in \boldsymbol{J}$, we have $c_i, c'_i \in J$ for all $i \in [b]$. It is straightforward to verify that for each $J \in \boldsymbol{J}$

it holds that $J \models \neg\chi$, and that for any $J^* \in \text{Kemeny}(\boldsymbol{J})$ it holds that $J^* \models \neg\chi$. Therefore, after this transformation, we have the same set $\text{Kemeny}(\boldsymbol{J})$ of outcomes. Moreover, the maximum Hamming distance between any two judgment sets in the profile $\boldsymbol{J}$ is 4. □

*Proof of Proposition 9.* We can show $\text{FPT}^{\text{NP}}[\text{few}]$-hardness by modifying the reduction from LOCAL-MAX-MODEL used in the proof of Proposition 8. Rather than using the formula $\varphi'$ as the integrity constraint $\Gamma$, we let $\Gamma = \top$, and we add $b$ many syntactic variants $\varphi'_1, \dots, \varphi'_b$ of $\varphi'$ (and their negations) to the agenda $\Phi$—that is, the formulas $\varphi'_i$ for $i \in [b]$ are all syntactically different from each other, but for each such formula $\varphi'_i$ it holds that $\varphi' \equiv \varphi'_i$. The judgment sets $J_1$, $J_2$ and $J_3$ in the profile $\boldsymbol{J}$ all include each of these formulas $\varphi'_i$.

As a result, the parameter value $h$ remains the same. The value of the parameter $p$ remains a constant, and the value of the parameter $n$ increases only by $b$, so it remains bounded by a function of the original parameter $k$.

It is straightforward to verify that there are enough syntactic variants of the formula $\varphi'$ in all judgment sets in the profile that for any complete and consistent judgment set $J^*$ that minimizes the distance to the profile, it must hold that $J^* \models \varphi'$. Therefore, we get that the modified reduction is a correct reduction from LOCAL-MAX-MODEL, and thus that the problem is $\text{FPT}^{\text{NP}}[\text{few}]$-hard. □

*Proof of Proposition 10.* We show $\text{FPT}^{\text{NP}}[\text{few}]$-hardness by modifying the (already modified) reduction from LOCAL-MAX-MODEL given in the proof of Proposition 9. In this reduction, the agenda included a small number of formulas $\varphi'_i$, that were each of unbounded size. By using the same trick that we used in the proof of Corollary 6, we can use the standard Tseitin transformation [43] to transform each of these formulas into a 3CNF formula $\varphi''_i$ that will have the same effect. Then, rather than including $\varphi'_i$ in the agenda $\Phi$, we include all clauses of the formula $\varphi''_i$ in the agenda $\Phi$. Then, in the judgment sets $J_1$, $J_2$ and $J_3$ in the profile $\boldsymbol{J}$, we also include the clauses of $\varphi''_i$ instead of the single formula $\varphi'_i$, for all $i \in [b]$.

As a result, the number $n$ of formulas in $\Phi$ is not bounded by a function of the original parameter $k$ anymore, but the maximum size $m$ of any formula in the agenda $\Phi$ is now bounded by a constant. Using the arguments used in the proofs of Corollary 6 and Proposition 9, it is then straightforward to verify the correctness of this modified reduction. □

*Proof of Proposition 11.* The main idea behind this proof is that the number of possible ballots is bounded by the parameter, that is, there are only $2^n$ many possible (rational) ballots. We describe an algorithm $A$ that solves the problem in fixed-parameter tractable time. Let $(\mathcal{I}, \Gamma, \boldsymbol{r}, l, l_1, \dots, l_u)$ be an instance. Firstly, the algorithm $A$ enumerates all possible ballots $r_1, \dots, r_{2^n} \in \{0,1\}^n$. Then, for each such ballot $r_i$, the algorithm determines whether $r_i$ is rational, by checking whether $\Gamma[r_i]$ is true. This can be done in polynomial time. Each irrational ballot is discarded.

Then, for each of the remaining (rational) ballots $r_i$, the algorithm $A$ computes the cumulative Hamming distance $d(r_i, \boldsymbol{r})$ to the profile $\boldsymbol{r}$. This can also be done in polynomial time. Then, those $r_i$ for which this distance is not minimal—that is, those $r_i$ for which there exists some $r_{i'}$ such that $d(r_{i'}, \boldsymbol{r}) < d(r_i, \boldsymbol{r})$—are discarded as well. The remaining ballots $r_i$ then are exactly those rational ballots with a minimum distance to the profile $\boldsymbol{r}$.

Finally, the algorithm goes over each of these remaining ballots $r_i$, and checks whether $l$ agrees with $r_i$ and whether for all $j \in [u]$, $l_j$ does not agree with $r_i$. If this check succeeds for some $r_i$, the algorithm $A$ accepts the input, and otherwise, the algorithm rejects the input. □

*Proof of Proposition 12.* Since $|\Gamma| = c$, we know that the number of propositional variables in $\Gamma$ is also bounded by the parameter $c$. Take an instance $(\mathcal{I}, \Gamma, \boldsymbol{r}, l, l_1, \dots, l_u)$. Then, let $\mathcal{I}' = \text{Var}(\Gamma) \subseteq \mathcal{I}$ be the subset of issues that are mentioned in the integrity constraint $\Gamma$. We know that any outcome $r^* \in \text{Kemeny}(\boldsymbol{r})$ agrees with the majority of ballots in $\boldsymbol{r}$ on every issue in $\mathcal{I}\backslash\mathcal{I}'$ (in case of a tie, either choice works). Therefore, all that remains is to determine whether there are suitable choices for the issues in $\mathcal{I}$ (to obtain some $r^* \in \text{Kemeny}(\boldsymbol{r})$ that agrees with $l$ and does not agree with $l_j$ for all $j \in [u]$). By Proposition 11, we know that this is fixed-parameter tractable in $|\mathcal{I}'|$. Since $|\mathcal{I}'| \leq c$, we get fixed-parameter tractability also for OUTCOME(KEMENY)$^{\text{cb}}$ parameterized by $c$. □

*Proof of Proposition 15.* We modify the $\Theta_2^{\text{p}}$-hardness reduction used in the proof of Proposition 7 to work also for the case of OUTCOME(KEMENY)$^{\text{cb}}$ for a constant value of the parameter $p$. Instead of adding the formulas $\varphi_i'$ to the agenda $\Phi$, as done in the proof of Proposition 7, we let $\Gamma = \varphi'$. The remaining formulas in the agenda $\Phi$ were all propositional variables, and thus we can transform the instance $(\Phi, \Gamma, \boldsymbol{J}, L)$ that we constructed for OUTCOME(KEMENY)$^{\text{fb}}$ into an instance $(\mathcal{I}, \Gamma, \boldsymbol{r}, l)$, where $\boldsymbol{r}$ and $l$ are constructed entirely analogously to $\boldsymbol{J}$ and $L$. Clearly, $p = 3$. Moreover, by a similar argument to the one that is used in the proof of Proposition 7, we get that $(\mathcal{I}, \Gamma, \boldsymbol{r}, l) \in$ OUTCOME(KEMENY)$^{\text{cb}}$ if and only if $(\varphi, w) \in$ MAX-MODEL. □

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