A fair payoff distribution for myopic rational agents

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Abstract

We consider the case of self-interested agents that are willing to form coalitions for increasing their individual rewards. We assume that each agent gets an individual payoff which depends on the coalition structure (CS) formed. We consider a CS to be stable if no individual agent has an incentive to change coalition from this CS. Stability is a desirable property of a CS: if agents form a stable CS, they do not spend further time and effort in selecting or changing CSs. When no stable CSs exist, rational agents will be changing coalitions forever unless some agents accept suboptimal results. When stable CSs exist, they may not be unique, and choosing one over the other will give an unfair advantage to some agents. In addition, it may not be possible to reach a stable CS from any CS using a sequence of myopic rational actions. We provide a payoff distribution scheme that is based on the expected utility of a rational myopic agent (an agent that changes coalitions to maximize immediate reward) given a probability distribution over the initial CS. To compute this expected utility, we model the coalition formation problem with a Markov chain. Agents share the utility from a social welfare maximizing CS proportionally to the expected utility of the agents, which guarantees that agents receive at least as much as their expected utility from myopic behavior. This ensures sufficient incentives for the agents to use our protocol.

1 Introduction

In the literature on coalition formation, valuation functions are typically defined only over a coalition, and the agents need to decide or negotiate a payoff distribution. We are interested in cases where the payoff distribution is defined for each partition of the agents into coalition structures (CS): each agent knows its payoff for any CS. This model corresponds to the hedonic aspect of coalition formation [1, 2, 4, 7] where the payoff of an agent, not the value of a coalition, depends only on the members of its coalition. We can view this assumption from two perspectives. The first perspective is that the environment provides a payoff to each agent. This can happen when the agents' individual goals are different but correlated and the CSs have different effects on different agents' performance. This formulation can model a community of agents that help each other improve their respective private utilities: each agent obtains a private utility which can be boosted with the help of other agents in the community. Another example is that of firms forming coalitions in a supply chain domain: each firm in a coalition provides preferred rates or discounts for its services to other members of its coalition. The benefit of each member of the coalition depends on the behaviors of other firms. Each firm in the coalition is autonomous: each sells and buys goods, and makes its own profit or loss. Note that firms still benefit from being in a coalition but the benefit varies from firm to firm in any given CS. The second perspective is to consider that the payoff distribution has already been computed using a stability criteria, e.g., the Kernel [5]. Given a CS and a valuation function, it is always possible to compute a Kernel-stable payoff distribution. Let us consider two different CSs with associated Kernel-stable payoff distributions. In both cases, the payoff distribution is stable, but an agent may prefer to form the first CS when another agent would prefer the second: even if agents are using a stable payoff distribution, agents may still have incentive to change CS.

In the papers related to hedonic coalition formation [1, 2, 4, 7], one assumption is that there is no transfer of utility. Under these assumptions it is known that the core or the set of Nash-stable equilibria may be empty. In particular, the personal goals of the agents may be conflicting, there may not be any CS that satisfies all the agents at the same time: for each CS, at least one agent may have an incentive to change coalitions. Some research deals with the search of conditions for the existence of stable coalition structures [1, 2, 4], but we want to provide a solution even when no stable CSs exist. The compromise we propose is based on allowing transfer of utility to make a CS stable.

From a societal point of view, we also want the society to perform well as a whole, hence our mechanism selects a social welfare maximizing CS. The computation of the side-payments to stabilize the CS is based on the expected utility of myopic and rational agents (i.e., agents that change coalition to maximize their immediate payoff). The computation of the expected utility uses the analysis of a Markov chain where a state of the chain corresponds to a CS and a transition corresponds to the will of an agent to change coalition. The analysis of the chain differentiate the transient states from the ergodic states[1], the latest corresponds to the Sink equilibria in [9] for games in normal forms: myopic rational agents are bound to be trapped in a set of ergodic states. The expected utility of the agent is a weighted average of the utility over the ergodic states. We view the expected utility as a means to weight the importance of an agent in the coalition formation process. We share the utility of a social welfare maximizing CS proportionally to this expected utility. Under the assumption that the initial CS is chosen at random, we show that each agent is better off following our protocol.

Most current studies on coalition formation in the multiagent community assume known valuation functions that estimate the worth of a coalition and where the valuation of any coalition is independent of the other coalitions present in the population [10, 14, 15]. However, this may not always be appropriate. In situations when the population of agents is competing for a resource or a niche, e.g., in electronic supply chains, the valuation of a coalition depends on the organization of the other agents. More generally, the presence of shared resources (if a coalition uses some resource, they will not be available to other coalitions) or conflicting goals (non-members can move the world farther from a coalition's goal state) [13] makes a valuation function depend on CS. We are especially interested in studying those situations where the worth of a coalition depends on the other coalitions that are present in the population [6, 12]. Our approach can also be used in that context as shown by our empirical example.

The paper is organized as follows. In Section 2, we present the coalition framework and the existing stability concepts for coalition formation when in the non transferable utility case. In Section 3, we show how to build a Markov chain that models the coalition formation process, how to use it to compute the expected utility. Finally in Section 4, we present and discuss our proposed solution. We conclude and discuss future work in Section 5.

2 Coalition Framework

2.1 Problem Description

We consider a set N of n agents; N is also known as the *g*rand coalition. A coalition structure (CS) $s = \{\mathcal{S}_1, \cdots, \mathcal{S}_m\}$ is a partition of N, where $\mathcal{S}_i$ is the i^{th} coalition of agents with $\cup_{i \in [1..m]} \mathcal{S}_i = N$ and $i \neq j \Rightarrow \mathcal{S}_i \cap \mathcal{S}_j = \emptyset$. $\mathscr{S}$ is the set of all CSs. The coalition of agent i in s is noted as $s(i)$. We consider that an agent i has a preference order $\succsim_i$ over $\mathscr{S}$

[1] Ergodic states are states that the chain will keep coming back to, whereas transient states are states that the chain will eventually leave to never visit again.

and for a CS s, an agent i has a valuation $v_i(s)$. These assumptions have two consequences:

- Each agent has a private utility which depends on the other agents present in the coalition, as for hedonic coalition formation [1, 4, 2, 7]. Coalitions do not always receive a reward as a whole: each agent has a private cost and benefit which depends on the organization of the agents. Members of a coalition help each other, which can globally reduce the cost or increase the private benefit of each member. For example, soccer players have a private utility, or satisfaction, which depends on the other members of the team.

- Unlike in the hedonic coalition formation case, we are working in the more general case where the valuation of a coalition depends on the other coalitions present in the population. For an agent i such that $i \in \mathcal{C}$ and two CSs s_1 and s_2 such that $\mathcal{C} \in s_1$ and $\mathcal{C} \in s_2$, it is possible that $u_i(s_1) \neq u_i(s_2)$. In our soccer example, the satisfaction of a player in a team playing a league may also depend on how the remaining players are dispatched in the other team, for example, he may prefer that the best players are put in different teams than put altogether in a "dream team". A more generic example involves agents competing for an environmental niche. The payoff of a coalition may be higher when the competitors work alone than when the competitors also decide to team together to form a more competitive group. Ray and Vohra [12] consider this problem and propose a protocol where agents propose a coalition and a distribution of the coalitions' worth. Other agents can accept or reject the proposition. One issue is that, when proposing a coalition, an agent does not know which CS will ultimately form. Hence, the payoff distribution proposed by an agent is conditioned on the CS that is finally selected. Ray and Vohra consider that the agents' offer contains a payoff distribution for each possible CS, which is not realistic for large populations. But such elaborate offers allow them to show the existence of an equilibrium.

We further assume that there is no coordinated change of coalitions; one agent at a time can change coalition. This assumption prevents uncertainties about the state of the CS. For example, let agents i, j and k form singleton coalitions. At this point, agent i would like to join agent k, and agent j would like to join agent i, but neither i or j would like to form the grand coalition. If we allowed simultaneous moves, the resulting state would be unclear. The grand coalition may be formed though it was not the intent of agent i or j. The resultant CS could also be $\{\{i, k\}, \{j\}\}$ where agent i joined agent k, and agent j tried to join the coalition of agent i, but ended up joining an empty coalition. It could also be $\{\{i, j\}, \{k\}\}$ where agent j joined agent i, and agent k refused that both agent i and j joined it at the same time. We avoid such ambiguities with this assumption.

Finally, we assume that agents are myopic and rational, and members of a coalition accepts a new member only when all members agree. After a change of CS, it is possible, if not likely, that another agent changes its coalition, leading to a different CS. As it is computationally expensive to perform multi-steps look ahead because of the large state space, we consider myopic agents that change coalition to maximize their immediate reward. We believe it is reasonable to assume that current members can control when other agents can join a coalition. Moreover, it would not be myopic rational for a member i to accept a new agent if this meant a payoff loss for i. Hence, we also assume that all members of a coalition must agree to accept a new member and, if some member i refuses, we will say that agent i vetoes the transition. We also make the implicit assumption that members of a coalition cannot prevent a member to leave, even if some of the remaining members lose utility.

2.2 Stability Concepts

We first start by giving the definition of stability concepts in the non-transferable utility case when the value function depends only on the members of the coalition [4]. In the following, $\succsim_i$ denotes a preference order over coalitions.

Definition 2.1. *A coalition structure s is* ***core stable*** *iff* $\nexists C \subset N \mid \forall i \in C, C \succ_i s(i)$.

Definition 2.2. *A coalition structure s is* ***Nash stable*** $(\forall i \in N)$ $(\forall C \in s \cup \{\emptyset\})$ $s(i) \succsim_i C \cup \{i\}$

Definition 2.3. *A coalition structure s is* ***individually stable*** *iff* $(\nexists i \in N)$ $(\nexists C \in s \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))$ *and* $(\forall j \in C, C \cup \{i\} \succsim_j C)$

Definition 2.4. *A coalition structure s is* ***contractually individually stable*** *iff* $(\nexists i \in N)$ $(\nexists C \in s \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))$ *and* $(\forall j \in C, C \cup \{i\} \succsim_j C)$ *and* $(\forall j \in s(i) \setminus \{i\}, s(i) \setminus \{i\} \succsim_j s(i))$

If a CS is core stable, no subset of agents has incentive to leave their respective coalition to form a new one. In a Nash stable CS s, no single agent i has an incentive to leave its coalition $s(i)$ to join an existing coalition in s or create the singleton coalition $\{i\}$. The two other criteria add a constraint on the members of the coalition joined or left by the agent. For an individually stable CS, there is no agent that can change coalition from s(i) to $C \in (s \setminus s(i)) \cup \{\emptyset\}$ yielding better payoff for itself, and the members of C should not lose utility. The contractually individual stability in addition requires that the members of s(i), the coalition left by i, should not lose utility.

The definition of Nash, individually and contractually individually stability can easily be extended to the case where the value of a coalition depends on the CS. Another criterion for a rational agent to be a member of a coalition is individual rationality [6]: an agent i would consider joining a coalition only when it is beneficial for itself. The agent compares the situation when it is on its own and when it is a member of a coalition. However, the payoff the agent gets when it is by itself depends on the CS. The minimum payoff that agent i can guarantee on its own is $r_i = \min_{s \in \mathscr{S}, \{i\} \in s} v_i(s)$ [6] (the minimum is over all the CSs where agent i forms a coalition on its own). An agent is individual rational when its payoff in a coalition with other agents is greater than the minimum payoff it can get on its own.

For some coalition formation problem, it is possible that no CS satisfies any of these stability criteria. Satisfying the individually or contractually individually stability criteria may depend on the protocol used by the agents to form coalition. For example, an academic can freely leaves its department to join a new one, provided that no member of the new department will suffer from its presence. In some cases, the coalition left is allowed to demand compensation. For example, as pointed out in [7], a player of a soccer team can join another club, but its former club can receive a compensation for the transfer. In the following, we will only assume that members of a coalition can veto the entrance of new agent in their coalition. Hence, we consider as our main stability criterion the individually stability.

2.3 Graphical representation

We can represent the relation $\succsim$ by a **preference graph of the coalition formation process**: each node is a CS, and there exists an edge from node S to node T when $\exists i \in N \mid T \succ_i S$. The **transition graph of the coalition formation process** is a directed graph where the nodes represent the CSs, and edges are valid transitions between two CSs. A transition from node s to node t is valid when

- $\exists i \in N$, $\exists C \in (t \setminus s(i)) \cup \{\emptyset\} \mid t = (s \setminus s(i) \setminus C) \cup (C \cup \{i\})$ and $t \succ s$. In other words, there is an agent i that is better off leaving its coalition $s(i)$ to either join an existing coalition in s or to form a singleton coalition.

- and $\forall j \in C$, $t \succsim_j s$, i.e., this transition is not vetoed by the members of the coalition C joined by i (of course, i is always allowed to form a singleton coalition).

Incidentally, another agent j may also prefer t over s (for example, when i moves to an existing coalition C, all agents in C may benefit). Hence, a transition may be beneficial for more than one agent. However, only the agent that changes the coalition can induce the transition. Even if it is beneficial for members of C, C's members cannot force i to leave its current coalition to join them (this action would be considered to be a group action whereas in our model, we consider only individual actions). In the case where two agents i and j that were previously forming singleton coalitions now form a coalition of two agents in the new CS, it may be difficult to interpret which agent induced the transition: as it is beneficial for both agents, an interpretation of the transition can be that agent i joins the coalition $\{j\}$ or vice versa. Our interpretation is that both agents are responsible for this transition. Hence we make an exception for this case.

Since we assume that agents are myopically rational, for a given CS, each agent will only choose the transition that yields the maximum immediate payoff gain over all its possible legal moves. For each state, there can then be at most n outgoing edges, one for each of the n agents (this happens when every agent prefer another CS over the current one). This prunes the number of transitions from the preference graph to the transition graph.

Property 1. *A CS s is individually stable iff there is no outgoing edge from state s in the transition graph of the coalition formation.*

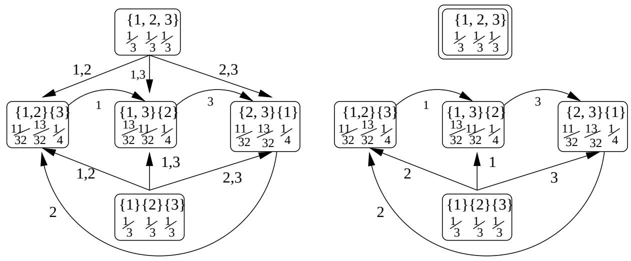
The proof is obvious given the definition of the transition graph. In Figure 1(a), we present an example with three agents where the payoff of an agent is shown below its label in a coalition. In this example, no CS is core or Nash stable. However, $\{\{1,2,3\}\}$ is individually stable. However, if the agents start from the bottom of the lattice (where each agent forms a singleton coalition) or any other CS in the mid level, the agents will be trapped in a cycle: for each CS in the mid-level, one agent benefits from leaving its coalition in that CS to join the singleton agent. We present a different scenario in Figure 1(b): the CS $\{\{0\}\{1,2\}\}$ is Nash stable, core stable (and hence individually stable), and the grand coalition is individually stable. From any CS, it is possible to reach an individually stable CS.

3 A Markov Chain model

A myopic rational agent will change coalitions if it can immediately gain utility by doing so. In this paper, we assume that the valuation is common knowledge. It is therefore possible to build and analyze the transition graph. Given the assumption that only one agent at a time can change coalition, we are now in position to estimate the probability of transition between any two CSs. For each outgoing edge e from CS s, the probability of making this transition is either

- $\frac{1}{o(s)}$, where $o(s)$ is the out degree of a node, i.e., the number of agents that want to change from s.

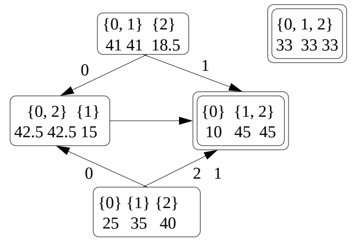
- $\frac{2}{o(s)}$ when two agents i and j that are each forming a singleton coalition merge to form the two-agent coalition $\{i,j\}$ and it is the best choice for both i and j.



Preference graph Transition model

The valuation function has one individually stable CS and contains a cycle.

(a) Coalition formation with a cycle



(b) Transition model with two individually stable CSs

Figure 1: Example of Coalition Formation problem (double boxed CS are individually stable)

	$\{1,2,3\}$	$\{1,2\}\{3\}$	$\{1,3\}\{2\}$	$\{2,3\}\{1\}$	$\{1\}\{2\}\{3\}$
$\{1,2,3\}$	1	0	0	0	0
$\{1,2\}\{3\}$	0	0	1	0	0
$\{1,3\}\{2\}$	0	0	0	1	0
$\{2,3\}\{1\}$	0	1	0	0	0
$\{1\}\{2\}\{3\}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0

Table 1: Transition Matrices for Figure 1(a)

As the probability of a transition does not depend upon the prior states of the population, the Markov assumption is verified. We have now completely defined a Markov chain. From the above specified transition model, we can construct the transition matrix P of the Markov chain. The size of the matrix is $\mathscr{B}(n) \times \mathscr{B}(n)$, where n is the number of agents and $\mathscr{B}(n)$ is the Bell function. The dimension of the matrix can be quite large, however, the matrix is sparse: for each row of the matrix, there can be only up to n positive entries[2]. In Table 1, we present the transition matrix for the example of Figure 1(a).

As agents change from one CS to another, the chain moves from one state to another. A state of a Markov chain is either transient or ergodic: ergodic states are states that the chain will keep coming back to, whereas transient states are states that the chain will eventually leave to never visit again. In the long term, the chain will be in one of the ergodic states. The ergodic states form multiple strongly connected components. If the size of such a strongly connected component is one, it means that the corresponding CS is individually stable (it may also be core or Nash stable, but not necessarily). The study of the Markov chain will tell us, given a probability distribution over the initial state, the probability to reach each strongly connected component, and, once reached, what is the proportion of time spent in each ergodic states. Hence, the value of the expected utility is an average over the

[2] $\mathscr{S}$ can be represented by a lattice where each CS at a given level of the lattice contains the same number of coalitions. For each level i in the lattice, an agent has at most i actions: joining one of the existing $i-1$ coalitions and forming a singleton coalition if it is not already forming one. As there are n levels, the maximum number of transitions from a CS is n.

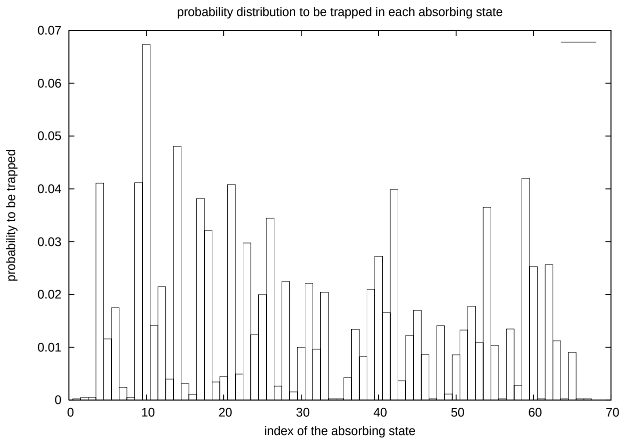


Figure 2: Example of the ART domain: probability to be in an ergodic state

possible stable CSs, and the CSs that are parts of some cycle. More formally, let $\mathscr{E}$ be the set of ergodic states of the Markov chain. For each strongly connected component $X \subset \mathscr{E}$, we compute the probability $p_X$to reach X, and then for each state s of X, we compute the fraction of time p_s spent in s in the limit (the chain may visit a state in X more often then another). If a CS s is at least individually stable, then the size of the corresponding component is one, and $p_s = 1$. For each ergodic state $s \in \mathscr{E}$, let $X(s)$ be the strongly connected component of s. The expected utility $E(v_i)$ is then $E(v_i) = \sum_{s \in \mathscr{E}} p_{X(s)} \cdot p_s \cdot v_i(s)$.

In Figure 2, we present an example issue from the Agent Reputation and Trust testbed [8]. In the testbed, agents provide appraisals about artifacts and compete for a pool of clients. To improve their appraisals, agents can ask other agents for appraisals for artifacts and reputation of other agents. We consider collusion of agents: agents can form a coalition where members provide their truthful appraisals, which benefits all members. In a domain with 8 agents, we computed the valuation function and the associated Markov chain for a particular instance, and the outcome is presented in Figure 2. In that instance, the Markov chain contains 4,140 CSs, 26,641 transitions, 62 stable CSs and 5 additional ergodic states which correspond to some strongly connected components.

4 A Fair Payoff Distribution for Myopic Rational Agent

It is possible that some coalition formation problem do not have any stable CS. To operate efficiently, we require that the agents remain in a CS. We propose that the agent forms a CS $s^\star$ that maximizes social welfare. However, $s^\star$ may not be stable, hence we want to share the utility $u^\star$ of $s^\star$ that provides the agent an incentive to stay in that CS.

The utility function, as a whole, tells how good the agent is. A first candidate is to share $u^\star$ proportionally to the average utility over all the CSs. This assumes that each CS is equally important and we believe it is not so. Another candidate is to consider an average over the stable CSs. However, such stable CSs may not always exist, and even if they do, there may not be a path allowing to reach a stable CS (as in the example of Figure 1(a)). If we assume any CS is likely to be the initial CS, we can compute an expected utility when the agents are myopic, rational, and when members of a coalition can veto the entrance of new members. The expected utility is a great metric to determine and compare the strength of each agent in the coalition formation process. We will show that the payoff obtained is at least its expected utility, which is a sufficient incentive for using our proposed payoff distribution.

4.1 Choice of Final Payoff Distribution and Corresponding CS

The expected value $E(v_i)$we computed using the Markov chain assumes that the initial CS is chosen uniformly over $\mathscr{S}$, in other word, it is no biased by the initial CS. $E(v_i)$ reflects the utility that agent i receives on average when all agents are myopically rational. We consider that this value represents the strength of an agent given the valuation function. Agents with high $E(v_i)$ should obtain a larger payoff than agents with lower $E(v_i)$.

To be used in a real world application, it is not desirable to have agents continuously change coalitions: agents should form a stable CS and have no incentive to further change coalition. To maximize the agents' payoff, we choose as the final CS $s^\star$ one of the CSs that maximizes social welfare. This CS may not be a stable, but it guarantees maximal total payoff to the agents. As we view the expected utility value as a measure of the strength of each agent, we propose a distribution of $v(s^\star)$ to all agents proportional to the expected payoff of the agents, i.e., we prescribe the payoff to agent i to be

$$u_i = \frac{E(v_i)}{\sum_{j\in N} E(v_j)} v(s^\star).$$

Note that this value is guaranteed to be at least as good as $E(v_i)$, as shown by Property 2. So, when agents share the payoff we propose, they are guaranteed to have at least the expected value when they were changing coalitions to maximize their immediate reward, and in general, they may get more. In addition, the payoff distribution is Pareto Optimal as we share the value of a social welfare maximizing CS (if an agent gets more utility, at least another agent must lose some). We believe that these incentives are sufficient for the agents to accept our proposed value. Not only is the payoff distribution fair, as the share of utility the agents receive is proportional to their expected utility over the chain, but the outcome is also efficient as it maximizes social welfare.

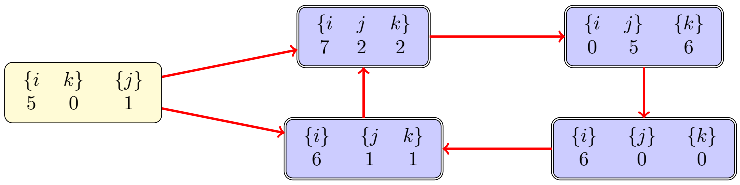
Property 2. *$u_i = \frac{E(v_i)}{\sum_{j\in N} E(v_j)} v(s^\star) \geq E(v_i)$, i.e., the payoff of an agent is at least as good as the expected utility that an agent would get on average if the agents are myopically rational.*

Proof. Let $\mathscr{E}$ denote the set of the ergodic states of a Markov chain. For player i, the expected payoff is a weighted average over the ergodic states: $E(v_i) = \sum_{s\in\mathscr{E}} \alpha_s v_i(s)$, where α_s is the weight of the ergodic state s and we have $\sum_{s\in\mathscr{E}} \alpha_s = 1$. The transient states are only used to determine the probability of leading to one of the ergodic sets: the α_s's are determined by the transient and the ergodic states (when there is a cycle or a regular sub-chain).

$$\begin{array}{rcl}
\forall s,\ v(s) & \leq & v(s^\star) \\
\forall s,\ \alpha_s \cdot v(s) & \leq & \alpha_s \cdot v(s^\star) \text{ as } \alpha_s \geq 0 \\
\sum_{s\in\mathscr{E}} \alpha_s \cdot v(s) & \leq & \sum_{s\in\mathscr{E}} \alpha_s \cdot v(s^\star) \\
\sum_{s\in\mathscr{E}} \alpha_s \cdot v(s) & \leq & v(s^\star) \cdot \sum_{s\in\mathscr{E}} \alpha_s \\
\sum_{s\in\mathscr{E}} \alpha_s \cdot v(s) & \leq & v(s^\star), \text{ as } \sum_{s\in\mathscr{E}} \alpha_s = 1 \\
\sum_{s\in\mathscr{E}} \alpha_s \sum_{j\in N} v_j(s) & \leq & v(s^\star) \\
\sum_{j\in N} \sum_{s\in\mathscr{E}} \alpha_s v_j(s) & \leq & v(s^\star) \\
\sum_{j\in N} E(v_j) & \leq & v(s^\star) \\
E(v_i) & \leq & \frac{E(v_i)}{\sum_{j\in N} E(v_j)} v(s^\star) \text{ as } E(v_i) \geq 0.
\end{array}$$

□

Another important question is to determine whether the payoff distribution v_i is individually rational: is an agent guaranteed to get as much as when an agent is forming a singleton coalition? The minimum payoff an agent can guarantee for itself is $r_i = \min_{s \in \mathscr{S},\, \{i\} \in s} v_i(s)$. For example, consider the three-agent example in Figure 3. The value obtained by i is $\frac{209}{36} = 5.806$, which is lower than 6, the minimum payoff that agent i receives when it forms a singleton coalition. This means that the payoff obtained by an agent in a coalition from our protocol is less than the worst payoff obtained by the agent when it forms a singleton coalition. Although possible in the general case, this may not be likely in practice: the worst case scenario for an agent should be when it forms a singleton coalition and when all other agents in the population try to minimize its payoff. As shown by Property 3, if the worst payoff for an agent occurs when it is forming a singleton coalition, our protocol is individual rational.



There is a cycle with 4 states, hence, the proportion spent in each state is $\frac{1}{4}$. The value of the optimal CS is 11. The minimum value of agent i when it is in a singleton coalition is 6.

$E(v_i) = \frac{1}{4}(7+0+6+6) = 4.75 \qquad v_i = \frac{4.75}{4.75+2+2.25} \cdot 11 = 5.8056 < 6 = \frac{216}{36}$

$E(v_j) = \frac{1}{4}(2+5+0+1) = 2 \qquad v_j = \frac{2}{4.75+2+2.25} \cdot 11 = 2.4444 > 0$

$E(v_k) = \frac{1}{4}(2+6+0+1) = 2.25 \qquad v_k = \frac{2.25}{4.75+2+2.25} \cdot 11 = 2.75 > 0$

Figure 3: Case where the protocol is not individual rational: i's payoff is lower than r_i, i's minimum payoff when it forms a singleton.

Property 3. *If* $(\forall s \in \mathscr{S})\ v_i(s) \geq r_i = \min_{s \in \mathscr{S} | \{i\} \in s}$*, then* $u_i \geq r_i$*, i.e., the payoff distribution* u_i *is individually rational.*

Proof. The hypothesis $\forall s \in \mathscr{S}$, $v_i(s) \geq r_i$ means that for any CS, the valuation of agent i is at least equal to i's minimum valuation when it forms a singleton coalition, i.e., the payoff of an agent in a coalition with at least another agent should be at least the minimum payoff the agent receives when it is on its own in a singleton coalition. Hence, we have $\sum_{s \in \mathscr{E}} \alpha_s v_i(s) \geq \sum_{s \in \mathscr{E}} \alpha_s r_i$, and then $E(v_i) \geq r_i$ as $\sum_{s \in \mathscr{E}} \alpha_s = 1$. From Proposition 2, we have $u_i \geq E(v_i) \geq r_i$. □

4.2 Computational Complexity of the centralized algorithm

We now consider the complexity of computing the payoff distribution if a centralized entity was used. To compute the canonical form of a stochastic matrix, we first need to compute the communication classes of the matrix and this operation is polynomial in the size of the matrix ($O(\mathscr{B}(n)^2)$). Then, to determine the canonical form of the matrix, we need to find the permutation matrix, which can also be done in quadratic time, hence in $O(\mathscr{B}(n)^2)$. To compute the limit behavior of the Markov chain, either a matrix has to be inverted (which can be done in $O(\mathscr{B}(n)^3)$, or a linear system needs to be solved (iterative methods can also be used here). The complexity is then $O(\mathscr{B}(n)^3)$. The fact that the matrix is sparse should allow for faster computation. The search of the optimal CS is $O(\mathscr{B}(n))$ if the brute force method is applied. As we consider valuation function that depends on CS, we cannot use the faster algorithm in [11]. The computation of the side-payments and the execution of the payments has linear complexity. Hence, the complexity of the protocol is $O(\mathscr{B}(n)^3)$.

4.3 Experiments with Random Valuation Function

We now experiment with random valuation functions. The valuation of a coalition $\mathcal{C}$ for a particular CS is drawn from a uniform distribution in $[0, \mathcal{C}]$. Using this distribution, it is on average better to have coalitions containing many agents, but the valuation function is not superadditive. The valuation of each member of $\mathcal{C}$ is distributed randomly: each member $i \in \mathcal{C}$ receives $w_i \cdot v(\mathcal{C})$ with z_i drawn from a uniform distribution in $[0, 1]$ and $w_i = \frac{z_i}{\sum_{j \in \mathcal{C}} z_i}$. We now present the result of a particular valuation function with 6 agents where the number of CSs is 203. The associated Markov chain has 54 transient states and 149 ergodic states. The associated transition matrix of size $203 \times 203 = 41209$ has only 735 positive entries, the matrix is quite sparse. The CS with maximal social welfare is not individually stable. The value of the agents are shown in Table 4.3: the second column (avg) represents the average payoff of an agent over all CSs, the third column $\overline{v}_i$ is the expected utility of an agent computed with the Markov chain, the fourth column w_i is the share of the value of the optimal CS and the last column v_i is payoff of the agents from our protocol. Note that in the example, the value allocated to the agents from our protocol is much larger than the expected value from traversing the Markov chain.

agent	avg	$\overline{v_i}$	w_i	v_i
0	0.50	0.61	0.17	0.96
1	0.49	0.63	0.17	0.99
2	0.50	0.60	0.16	0.93
3	0.51	0.64	0.18	1.00
4	0.56	0.54	0.15	0.85
5	0.50	0.58	0.16	0.90

Table 2: Agents utilities for a random valuation function

4.4 Discussion on the payoff distribution

Our protocol uses global properties of the valuation function and shares the utility of the optimal CS, $s^\star$, in a fair manner. The distribution of the valuation of $s^\star$, however, is not according to the actual coalitions present in $s^\star$. In other words, given the payoff function v_i, it is possible that, for each coalition $\mathcal{C} \in s^\star$, $\sum_{i \in \mathcal{C}} u_i \neq \sum_{i \in \mathcal{C}} v_i(s^\star)$.

This is different from the traditional assumption in game theory where agents share the value of their coalition. For some agents i, $v_i(s^\star) > u_i$, which may not appear fair. What we propose to the agents is to sign a binding contract to form $s^\star$ and receive u_i as a payoff. If one agent does not want to sign the contract, the agents can form a random CS and try to find a stable CS[3]. From Proposition 2, we see that the expected utility from such a process is at most as good as the value proposed by the protocol and hence the agents have an incentive to accept the guaranteed value while saving on the "cost" of continual change. Hence, on one hand, we want the entire population of agents to cooperate and work together, which has a flavor of using the grand coalition. On the other hand, we want to use the synergy between the agents, and thus form a CS that maximizes social welfare. The reward the agent obtain is designed to be fair for all agents and reflects the performance of the agents over all CSs.

To compute the expected utility of an agent, we have assumed that the coalition formation process starts in a CS picked randomly from a uniform distribution. Of course, some

[3]Note that some agents may benefit from starting the coalition formation process in $s^\star$, hence, if some agents deviate, other agents should force the restart the coalition formation process from a random CS

probability distribution for the initial CSs will benefit some agents in detriment of others. We believe that the probability distribution of the initial CS is part of the definition of the coalition formation problem, and agents do not have any control over it. It is from the entire definition of the coalition formation problem that we compute the expected utility, which we use as a measure of the strength of an agent. If it the distribution is not uniform, the probability to reach the strongly connected components will be different (some components may not be reachable). In addition, the search of the CS that maximizes social welfare should be performed on the subset of CSs that are reachable from the set of possible initial CSs. Minor modification of our computations are needed to address these changes.

5 Conclusion, current and future work

Myopic rational agents who receive a private payoff that depends on the CS may never reach an agreement on the CS to be formed. It may be possible that for each CS, at least one agent has an incentive to change coalition. We designed a protocol that computes a payoff distribution so that agents are guaranteed to have at least the expected utility from a process where each agent would change coalition to maximize its immediate reward. The protocol assumes that 1) the valuation function provides a payoff for each individual agent given a CS and 2) the agents are myopically rational. The payoff function we propose is based on the value of a social welfare maximizing CS and on the expected utility of the agents if they try to change coalitions to maximize their immediate reward. Following our protocol, the agents form the optimal CS, which makes the multiagent system efficient from the viewpoint of a system designer. The valuation of the optimal CS is shared proportionally to the expected utility of the agents. We argue that this is a fair distribution as the payoff obtained by an agent reflects the behavior of the agents over the entire space of CSs, i.e., it is a global property of the valuation function. When the agents follow our protocol, they are guaranteed to have a payoff which is at least their expected value if all agents try to maximize their immediate reward.

The drawback of our approach is its computational cost: the agents needs to build a Markov chain where the number of states is equal to the number of the CSs, which is exponential in the number of agents. Although the corresponding transition matrix is sparse, this method may not be suitable for large number of agents (10 and more). The agents can approximate the expected value by simulating the Markov chain. In that case, they only need to be able to evaluate the best coalitional move from a given CS.

Because of the computational cost, we are studying algorithms to approximate the computation of the Markov chain. By sampling the chain, we can obtain rapidly good estimate of the expected utility of the agents. Another current line of research is the design of protocols and the issue of revealing the valuation function. In the general case, agents have to reveal their valuation, and protocol as [3] can help us ensure that no agent can take advantage of knowledge asymmetry. When agents are sharing a niche, e.g., when the valuation function represent a share attributed to each agent, the agents only need to reveal a preference order over the CSs and no agent has incentive to lie unilaterally.

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