Information Fusion and Social Choice

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COST-ADT Doctoral School on computational Social Choice

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Merging

- Applications :
  - Distributed information systems
    - Databases
    - Multi-agent systems
- Propositional bases can encode different types of information :
  - knowledge
  - beliefs
  - goals
  - rules / laws
  - ...

Plan

- Propositional Base Merging
  - Logical Properties
- Merging Operators
  - Model based operators
  - Formula based operators
  - DA^2 operators
  - Vectors of conflicts
  - Defaults based operators
  - Similarity based operators
- Merging and . . .
  - . . . Belief Revision
  - . . . Social Choice
  - . . . Judgment Aggregation
- Other logical merging frameworks
- Negotiation/Conciliation

Definitions

- A set of formulae $\mathcal{L}$ build from :
  - A set of propositional symbols : $\mathcal{P} = a, b, c, \ldots$
  - Connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.
- An interpretation (world) is a function $\mathcal{P} \longrightarrow \{0, 1\}$.
- A model of a formula is an interpretation that makes it true.
- The set of models of a formula α is denoted by $mod(\alpha)$.
- A formula α is consistent if $mod(\alpha) \neq \emptyset$

- A base φ is a finite set of propositional formulae.
- A profile E is a multi-set of bases : $E = \{\varphi_1, \ldots, \varphi_n\}$.
- $\bigwedge E$ denotes the conjunction of the bases of E.
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Equivalence between profiles :

- Let E_1, E_2 be two profiles. E_1 and E_2 are equivalent, noted $E_1 \leftrightarrow E_2$, iff there exists a bijection f from $E_1 = \{\varphi_1^1, \ldots, \varphi_n^1\}$ to $E_2 = \{\varphi_1^2, \ldots, \varphi_n^2\}$ such that $\vdash f(\varphi) \leftrightarrow \varphi$.

Merging

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Merging

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Integrity Constraints

Merging

Profile

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(IC7) $\triangle_{\mu_1}(E) \wedge \mu_2 \vdash \triangle_{\mu_1 \wedge \mu_2}(E)$

(IC8) If $\triangle_{\mu_1}(E) \wedge \mu_2$ is consistent, then $\triangle_{\mu_1 \wedge \mu_2}(E) \vdash \triangle_{\mu_1}(E)$

Majority vs Arbitration

Ally, Brian and Charles have to decide what they will do this night. Brian and Ally want to go to the restaurant and to the cinema. Charles does not want to go out this night and so he does not want to go nor to the restaurant nor to the cinema.

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Majority   restaurant and cinema

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Brian	+ +
Charles	– –

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Majority	restaurant and cinema	Arbitration	restaurant xor cinema
Ally	+ +	Ally	+
Brian	+ +	Brian	+
Charles	– –	Charles	+

Majority - Arbitration

(Maj) $\exists n \ \triangle_\mu (E_1 \sqcup E_2{}^n) \vdash \triangle_\mu(E_2)$

▹ An IC merging operator is a majority operator if it satisfies (*Maj*).

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(Arb)
$$\left.\begin{array}{l} \triangle_{\mu_1}(\varphi_1) \leftrightarrow \triangle_{\mu_2}(\varphi_2) \\ \triangle_{\mu_1 \leftrightarrow \neg \mu_2}(\varphi_1 \sqcup \varphi_2) \leftrightarrow (\mu_1 \leftrightarrow \neg \mu_2) \\ \mu_1 \nvdash \mu_2 \\ \mu_2 \nvdash \mu_1 \end{array}\right\} \Rightarrow \triangle_{\mu_1 \vee \mu_2}(\varphi_1 \sqcup \varphi_2) \leftrightarrow \triangle_{\mu_1}(\varphi_1)$$

▷ An IC merging operator is an arbitration operator if it satifies (*Arb*).

Syncretic Assignment

A syncretic assignment is a function mapping each profile E to a total pre-order $\leq_E$ over interpretations such that :

1) If $\omega \models E$ and $\omega' \models E$, then $\omega \simeq_E \omega'$

2) If $\omega \models E$ and $\omega' \not\models E$, then $\omega <_E \omega'$

3) If $E_1 \equiv E_2$, then $\leq_{E_1} = \leq_{E_2}$

4) $\forall \omega \models \varphi_1 \; \exists \omega' \models \varphi_2 \; \omega' \leq_{\varphi_1 \sqcup \varphi_2} \omega$

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A majority syncretic assignment is a syncretic assignment which satisfies :

7) If $\omega <_{E_2} \omega'$, then $\exists n \; \omega <_{E_1 \sqcup {E_2}^n} \omega'$

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A fair syncretic assignment is a syncretic assignment which satisfies :

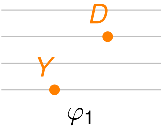
8) $$\left.\begin{array}{l} \omega <_{\varphi_1} \omega' \\ \omega <_{\varphi_2} \omega'' \\ \omega' \simeq_{\varphi_1 \sqcup \varphi_2} \omega'' \end{array}\right\} \Rightarrow \omega <_{\varphi_1 \sqcup \varphi_2} \omega'$$

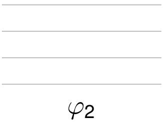
Arbitration

φ_1 φ_2

Arbitration

Arbitration





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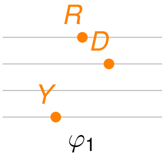

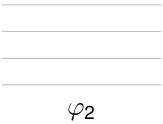

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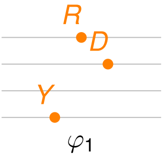

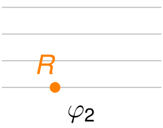

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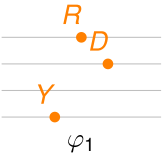

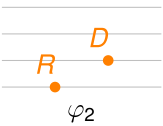

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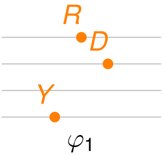

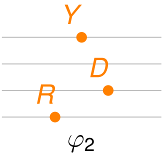

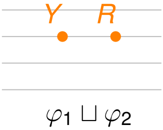

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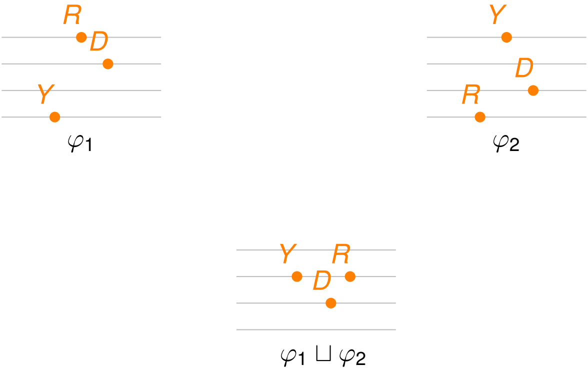

Representation Theorem

Theorem An operator is an IC merging operator if and only if there exists a syncretic assignment that maps each profile E to a total pre-order $\leq_E$ such that

$$mod(\triangle_\mu(E))) = \min(mod(\mu), \leq_E).$$

Representation Theorem

Theorem An operator is an IC merging operator (respectively IC majority merging operator or IC arbitration operator) if and only if there exists a syncretic assignment (respectively majority syncretic assignment or fair syncretic assignment) that maps each profile E to a total pre-order $\leq_E$ such that

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Idea : Select the interpretations that are the most plausible for a given profile.

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- Distance between an interpretation and a profile
  - $d_{d,f}(\omega, E) = f(d(\omega, \varphi_1), \ldots d(\omega, \varphi_n))$

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- Examples of aggregation function :
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- Examples of aggregation function :
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- Let d be any distance between interpretations.
  - $\triangle^{d,\max}$ operators satisfy (IC0-IC5), (IC7), (IC8) and (Arb).
  - $\triangle^{d,\textsc{Gmin}}$ operators are IC merging operators.
  - $\triangle^{d,\textsc{Gmax}}$ operators are arbitration operators.
  - $\triangle^{d,\Sigma}$ and $\triangle^{d,\Sigma^n}$ operators are majority operators.

Model-Based Merging

An aggregation function f is a function that associates a positive number to any tuple of positive numbers such that :

- If $x \leq y$, then $f(x_1, \ldots, x, \ldots, x_n) \leq f(x_1, \ldots, y, \ldots, x_n)$ *(monotony)*
- $f(x_1, \ldots, x_n) = 0$ if and only if $x_1 = \ldots = x_n = 0$ *(minimality)*
- $f(x) = x$ *(identity)*

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Theorem The operateur $\triangle^{d,f}$ satisfies properties (IC0-IC8) if and only if f satisfies :

- For any permutation σ, $f(x_1, \ldots, x_n) = f(\sigma(x_1, \ldots, x_n))$ *(symmetry)*
- If $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$, then $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$ *(composition)*
- If $f(x_1, \ldots, x_n, z) \leq f(y_1, \ldots, y_n, z)$, then $f(x_1, \ldots, x_n) \leq f(y_1, \ldots, y_n)$ *(decomposition)*

Example

$\mu = ((S \wedge T) \vee (S \wedge P) \vee (T \wedge P)) \rightarrow I$

$\varphi_1 = \varphi_2 = S \wedge T \wedge P$
$\varphi_3 = \neg S \wedge \neg T \wedge \neg P \wedge \neg I$
$\varphi_4 = T \wedge P \wedge \neg I$

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$\varphi_1 = \varphi_2 = S \wedge T \wedge P$ $\quad mod(\varphi_1) = \{(1,1,1,1),(1,1,1,0)\}$

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	φ_1	φ_2	φ_3	φ_4	$d_{d_H,Max}$	$d_{d_H,\Sigma}$	d_{d_H,Σ^2}	$d_{d_H,GMax}$
$(0,0,0,0)$	3	3	0	2				

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Example

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$(0,0,0,1)$	3	3	1	3	3	10	28	(3,3,3,1)
$(0,0,1,0)$	2	2	1	1	2	6	10	(2,2,1,1)
$(0,0,1,1)$	2	2	2	2	2	8	16	(2,2,2,2)
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Idea : Select some formulae from the union of the bases of the profile

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	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8	MI	Maj
$\triangle^{C1}$	√	√	√		√	√		√		√	
$\triangle^{C3}$					√	√		√	√	√	
$\triangle^{C4}$	√	√	√					√	√	√	
$\triangle^{C5}$	√	√	√		√	√		√	√	√	

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	IC0	IC1	IC2	IC3	IC4	IC5	IC6	IC7	IC8	MI	Maj
$\triangle^{C1}$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$		$\surd$	
$\triangle^{d}$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$			$\surd$
$\triangle^{S,\Sigma}$	$\surd$	$\surd$	$\surd$		$\surd$			$\surd$	$\surd$		$\surd$
$\triangle^{\cap,\Sigma}$	$\surd$	$\surd$	$\surd$			$\surd$	$\surd$	$\surd$	$\surd$		$\surd$

Example

φ_1	φ_2	φ_3
$a, b \rightarrow c$	a, b	$\neg a$

Example

$$\begin{array}{ccc} \varphi_1 & \varphi_2 & \varphi_3 \\ a, b \to c & a, b & \neg a \end{array}$$

$$\triangle_{\top}^{C1}(E) = \text{MAXCONS}(E, \top) = \{\{a, b \to c, b\},$$

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Let d be a distance between interpretations and f and g be two aggregation functions. The DA^2 merging operator $\triangle_{\mu}^{d,f,g}(E)$ is defined by :
For each $\varphi_i = \{\alpha_{i,1}, \ldots, \alpha_{i,n_i}\}$

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$$mod(\triangle_{\mu}^{d,f,g}(E))) = \{\omega \in mod(\mu) \mid d(\omega, E) \text{ is minimal}\}$$

Example

φ_1	φ_2	φ_3	φ_4
$a,\ b,\ c,\ a \wedge \neg b$	$a,\ b$	$\neg a, \neg b$	$a, a \rightarrow b$

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Vectors of conflicts

$\boxed{a,}\ \boxed{\neg a,\ b \wedge c,}\ \boxed{b \wedge d,\ e,}\ \boxed{\neg b}$

① ② ③ ④

	①	②	③	④	d_H
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- Vectors of conflicts capture all the information about the conflicts

Default based merging [Delgrande, Schaub 2007]

- Based on (supernormal) default logic
  - Let $B = \{\alpha_1, \ldots, \alpha_n\}$ be the background base
  - Let $D = \{\delta_1, \ldots, \delta_m\}$ be the set of (supernormal) defaults.
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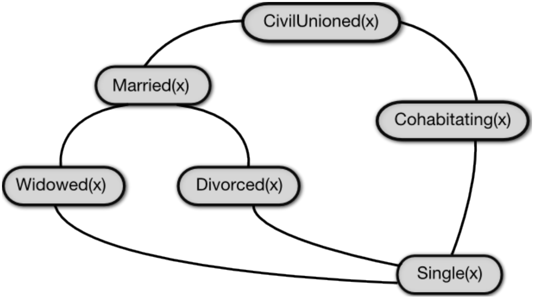
Similarity based merging [Shockaert, Prade 2009]

- Associate to every propositional symbol a similarity relation (partial pre-order)
- Merging = Find the best compromise



Merging and Belief Revision

The operator $*$ is an AGM revision operator if and only if it satisfies the following properties :

- **(R1)** $\varphi * \mu$ implies μ
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- **(R3)** If μ is consistent then $\varphi * \mu$ is consistent
- **(R4)** If $\varphi_1 \equiv \varphi_2$ and $\mu_1 \equiv \mu_2$ then $\varphi_1 * \mu_1 \equiv \varphi_2 * \mu_2$
- **(R5)** $(\varphi * \mu) \wedge \psi$ implies $\varphi * (\mu \wedge \psi)$
- **(R6)** If $(\varphi * \mu) \wedge \psi$ is consistent then $\varphi * (\mu \wedge \psi)$ implies $(\varphi * \mu) \wedge \psi$

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- Links between prioritized merging and iterated revision :
  - Delgrande, Dubois, Lang. Iterated Revision as Prioritized Merging. [KR'06]

Judgment Aggregation

- A set $N = \{1, \ldots n\}$ of individuals
- A set $X = \{\alpha_1, \ldots, \alpha_m\}$ of logical formulae, called the agenda
- Each individual i gives her (consistent) judgment set about the agenda : $J_i : X \rightarrow \{0, 1\}$
- Question : how to define a consistent judgment of the group $J = f(J_1, \ldots, J_n)$ from the judgment sets of the individuals ?

Doctrinal Paradox / Discursive Paradox

	α	β	γ
1	1	0	0
2	0	1	0
3	1	1	1

- α : good researcher
- β : good teacher
- γ : hire the candidate
- $\gamma \leftrightarrow \alpha \wedge \beta$

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- Principles for judgment aggregation ?

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- Agenda
- Collective Rationality
- Systematicity

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	Ideal Process	Practical Process

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    - Non-Dictatorship
- Condorcet's Jury Theorem
  - When voters are competent and independent then majority will find the correct answer
    - 2 alternatives (yes/no questions)
    - competence
    - independence

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Clearly, there are numerous different ways to define the satisfaction of an agent given a merged base.

- Weak drastic index : the agent is considered satisfied if her beliefs/goals are consistent with the merged base.

$$i_{d_w}(\varphi, \varphi_\Delta) = \begin{cases} 1 \text{ if } \varphi \wedge \varphi_\Delta \text{ is consistent} \\ 0 \text{ otherwise.} \end{cases}$$

Strategy-Proof Merging : Satisfaction Indexes

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- Probabilistic index : the more compatible the merged base with the agent's base the more satisfied the agent.

$$i_p(\varphi, \varphi_\Delta) = \frac{\#(mod(\varphi) \cap mod(\varphi_\Delta))}{\#(mod(\varphi_\Delta))}$$

Strategy-Proof Merging : Some Results for i_{d_w}

$\#(E)$	φ	μ	$\Delta^{d_H,\Sigma}$	$\Delta^{d_H,G_{max}}$	Δ^{C1}	Δ^{C3}	Δ^{C4}	Δ^{C5}
2	φ_ω	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	**sp**
	φ	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$
> 2	φ_ω	$\top$	**sp**	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	**sp**	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	**sp**
	φ	$\top$	$\overline{sp}$	$\overline{sp}$	**sp**	**sp**	$\overline{sp}$	**sp**
		μ	$\overline{sp}$	$\overline{sp}$	**sp**	$\overline{sp}$	$\overline{sp}$	$\overline{sp}$

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Criteria for evaluating merging operators

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Merging in other frameworks

- Merging of weighted formulae
  - Benferhat-Dubois-Kaci-Prade [2000,2002,2003]
  - Meyer [2001]
- First order logic
  - Gorogiannis-Hunter [2008]
- Logic programs
  - Delgrande-Schaub-Tompits-Woltran [2009]
  - Hué-Papini-Würbel [2009]
- Constraints Networks
  - Condotta-Kaci-Marquis-Schwind [2009]
- Argumentation systems [AAAI'05, AIJ-07]
  - Dung : arguments + relation d'attaque entre arguments
    - Cadres d'argumentation partiels (PAF)
    - Distances d'édition

Iterated Merging

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$$\Vert$$

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Iterated Merging

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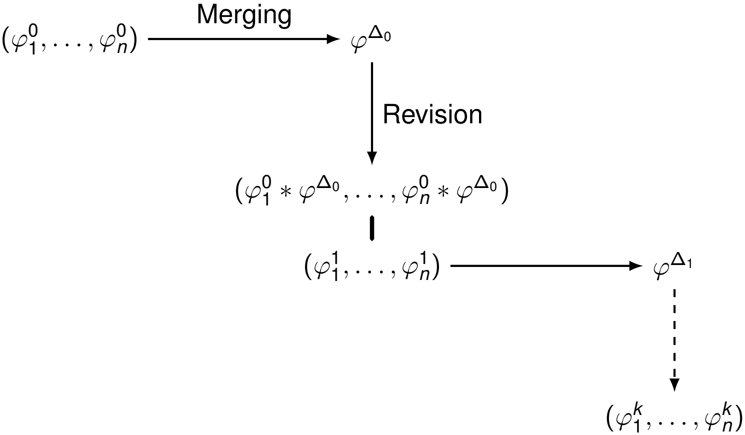

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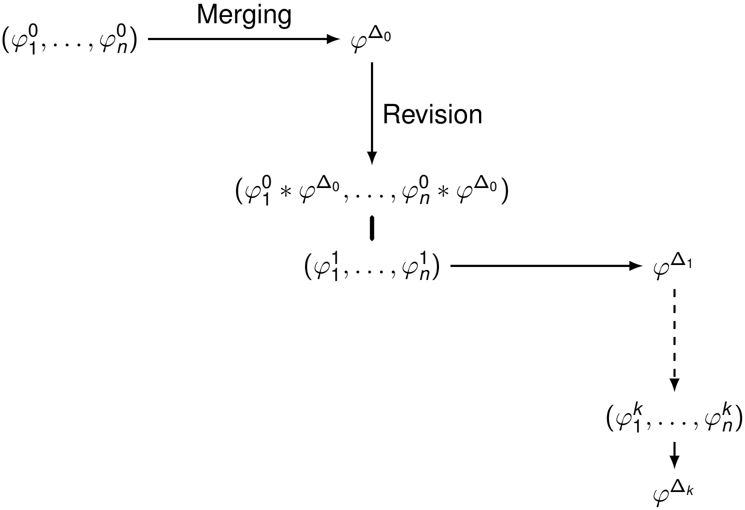

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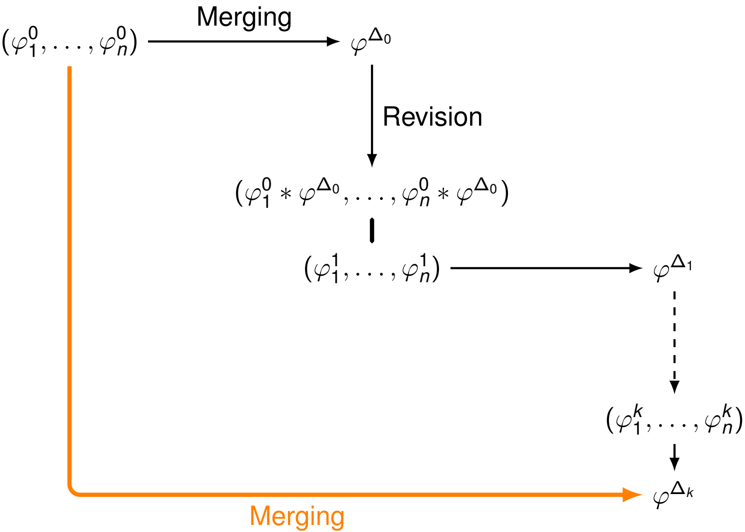

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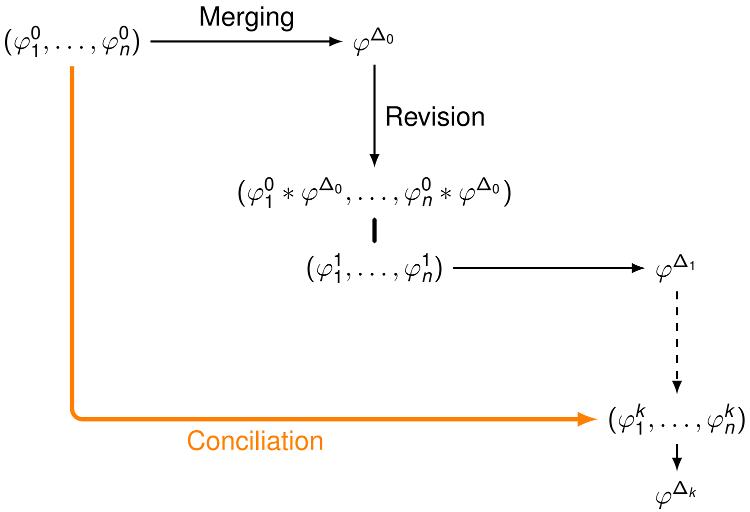

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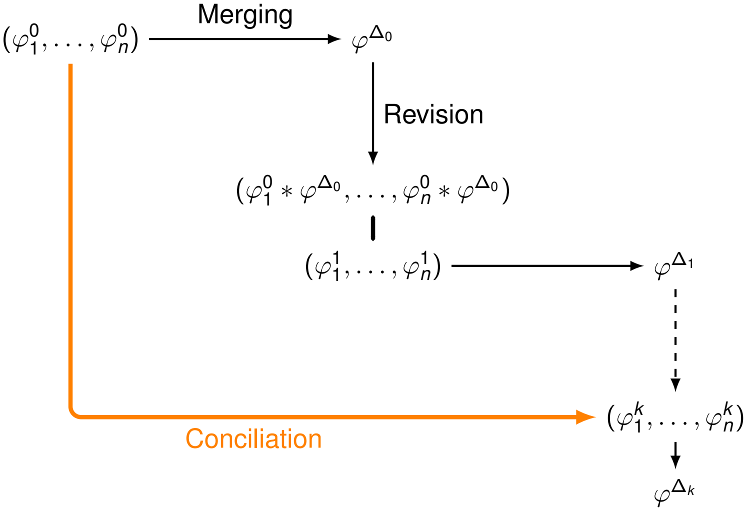

- Merging $(\varphi_1, \ldots, \varphi_n) \longrightarrow \varphi_\Delta$
- Conciliation $(\varphi_1, \ldots, \varphi_n) \longrightarrow (\varphi_1^*, \ldots, \varphi_n^*)$

Negotiation - Conciliation

Let $E = (\varphi_1, \ldots, \varphi_n)$ be a profile of belief/goal bases.

Two questions :

- What are the beliefs/goals of the group of agents ?
  - Merging (vote, social choice, MCDM, ...)
- Can the agents find a consensual position ?
  - Conciliation (negotiation, bargaining, ...)

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  - Some sources have to concede to solve the conflicts

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  - Each source gives her base
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The solution to a belief profile E for a Belief Game Model $\mathcal{N} = \langle g, \blacktriangledown \rangle$, noted $\mathcal{N}(E)$, is the belief profile $E_{\mathcal{N}}$, defined as :

- $E_0 = E$
- $E_{i+1} = \blacktriangledown_{g(E_i)}(E_i)$
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Belief Game Model

A choice function is a function $g : \mathcal{E} \to \mathcal{E}$ such that :

- $g(E) \subseteq E$
- If $\bigwedge E \not\equiv \top$, then $\exists \varphi \in g(E)$ s.t. $\varphi \not\equiv \top$
- If $E \leftrightarrow E'$, then $g(E) \leftrightarrow g(E')$

A weakening function is a function $\blacktriangledown : \mathcal{K} \to \mathcal{K}$ such that :

- $\varphi \vdash \blacktriangledown(\varphi)$
- If $\varphi = \blacktriangledown(\varphi)$, then $\varphi \leftrightarrow \top$
- If $\varphi \leftrightarrow \varphi'$, then $\blacktriangledown(\varphi) \leftrightarrow \blacktriangledown(\varphi')$

Example : Database Class [Revesz, 1994]

- $g = d_D^{\Sigma}, \blacktriangledown = \delta$

$\varphi_1 = \{100, 001, 101\}$ $\varphi_2 = \{010, 001\}$ $\varphi_3 = \{111\}$

$$mod(\varphi_1 \wedge \varphi_2 \wedge \varphi_3) = \emptyset$$

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	φ_1	φ_2	φ_3	Σ	g
φ_1		0	1	1	
φ_2	0		1	1	
φ_3	1	1		2	•

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	φ_1	φ_2	φ_3	Σ	g
φ_1		0	0	0	
φ_2	0		1	1	•
φ_3	0	1		1	•

Skipped something ?

◂ Back to Condorcet's Jury Theorem

◂ Back to Unanimity

◂ Back to Default-based merging