Sponsored Search, Market Equilibria, and the Hungarian Method

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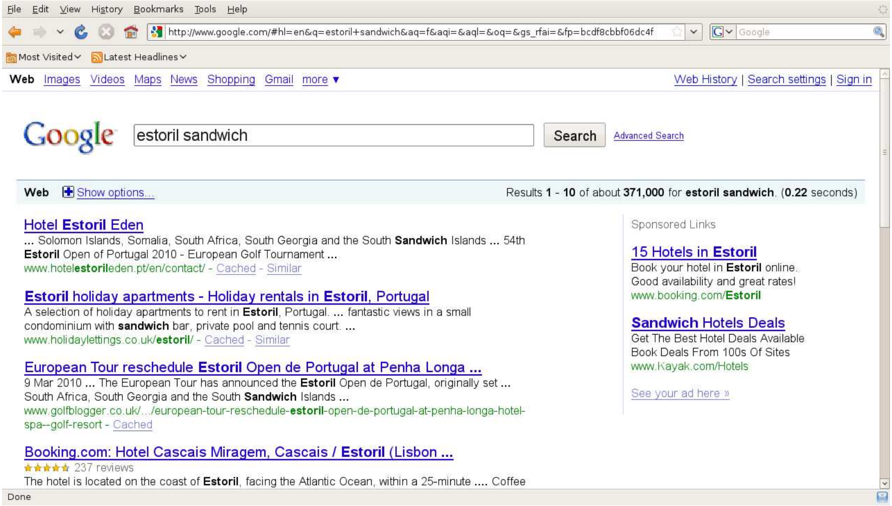
Motivation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Problem Statement

Input:

- Set of n advertisers (or bidders) I, set of k ad slots (or items) J
- Preferences, given as utility function $u_{i,j}(p_j) = v_{i,j} - p_j$
- Constraints on the prices: Reserve prices $r_{i,j}$, maximum prices $m_{i,j}$

Output:

- Matching $\mu \subseteq I \times J$ between bidders and items
- Prices p_j for each item $j \in J$

Goals: Outcome (= matching plus prices) should be:

- Feasible: Constraints on prices are satisfied
- Stable: Every bidder is "happy" with what she gets
- Bidder optimal: Every bidder is as "happy" as possible

Known Results & Our Contribution

Known results:

- Without budgets: Always exists, can be computed efficiently, is truthful [Shapley & Shubik, '72; Leonard, '83; Demange et al., '85]
- With budgets: Exists, can be computed efficiently, is truthful but only if input is in "general position" [Aggarwal et al., '09]

Our contribution:

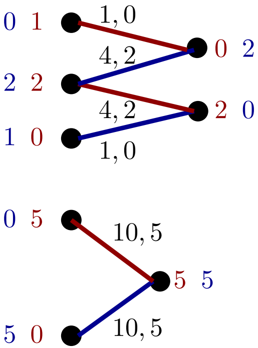
- With slightly different feasibility and stability notions a bidder optimal outcome always exists and can be computed efficiently
- Any mechanism that finds a bidder optimal outcome for these notions is truthful for per-item reserve prices and maximum prices in "general position"

Feasible, Stable, Bidder Optimal

Aggarwal et al.'s definitions:

- Feasible, if for all $(i, j) \in \mu$:
  - $r_{i,j} \leq p_j \leq m_{i,j}$
- Stable, if for all $(i, j) \in I \times J$:
  - $p_j \geq m_{i,j}$, or
  - $p_j < m_{i,j}$ and
    - $u_i \geq v_{i,j} - p_j$, or
    - $u_i \geq v_{i,j} - r_{i,j}$
- Bidder optimal, if for every feasible and stable (μ', p')
  - $u_i \geq u'_i$ for all i

Example:



Feasible, Stable, Bidder Optimal (Cont'd)

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Our definitions:

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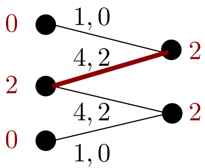
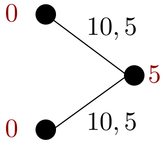
Feasible, Stable, Bidder Optimal (Cont'd)

Example (cont'd):





Our definitions:

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  - $u_i \geq u_i'$ for all i

Existence and Computation

Theorem: Modified Hungarian Method finds feasible, stable, and bidder optimal outcome in $O(nk^3 \log(k))$ steps.

Proof sketch:

- Define feasible first choice graph for a given vector of prices p such that any matching μ in this graph that matches all bidders is feasible and stable
- Start with prices all zero and repeatedly raise prices of over-demanded items by as little as possible, until all overdemand is resolved
- Use Hall's Theorem to show that price increases are required by any feasible and stable matching, conclude that prices are the smallest prices at which a feasible and stable matching exists
- Show that smallest prices correspond to bidder optimal utilities $\square$

Truthfulness

An algorithm is truthful if

- For every bidder i with utility functions $u_{i,1}(\cdot), \ldots, u_{i,k}(\cdot)$ and
- Any two inputs $(u'_{i,j}(\cdot), r_{i,j}, m'_{i,j})$ and $(u''_{i,j}(\cdot), r_{i,j}, m''_{i,j})$ with $u'_{i,j}(\cdot) = u_{i,j}(\cdot)$ & $m'_{i,j} = m_{i,j}$ for i and all j and $u'_{k,j}(\cdot) = u''_{k,j}(\cdot)$ & $m'_{k,j} = m''_{k,j}$ for $k \neq i$ and all j and matchings μ' with p' and μ'' with p''
- We have $u_{i,j'}(p'_{j'}) \geq u_{i,j''}(p''_{j''})$ where $(i,j) \in \mu$ and $(i,j'') \in \mu''$

Formalizes notion of "lying does not pay off":

Even if bidder i misreports her utility functions and maximum prices she will not achieve a higher utility with the matching and prices computed by the algorithm.

Truthfulness (cont'd)

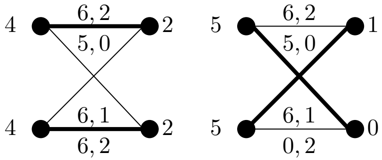
Theorem: Modified Hungarian Method is truthful if the reserve prices are per-item and during the execution of the algorithm no two maximum prices are reached at the same time.

Proof sketch:

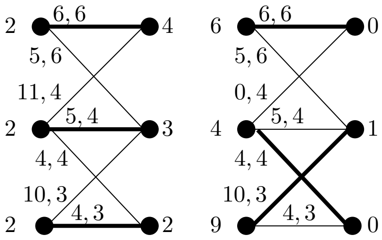
- Show that in the bidder optimal outcome at least one item is sold at the reserve price and argue that this implies that not all bidders can (strictly) benefit from misreporting
- Show that if not all, but some bidders (strictly) benefit from misreporting, then at least one of the "truthful" bidders must be "unhappy" in the bidder optimal outcome for the "falsified input", which yields a contradiction □

Truthfulness (cont'd)

Not truthful for bidder-item dependent reserve prices:



Not truthful when maximum prices are reached at the same time:



Summary and Future Work

Summary

- With slightly different feasibility and stability notions a bidder optimal outcome always exists and can be computed efficiently
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Future Work

- More general utility functions
- One-to-many and many-to-many matchings

That's it. Thanks a lot!

Slides and related working papers:

http://people.epfl.ch/paul.duetting/