UNIVERSITY OF GUADALAJARA
CUCEA Postgraduate Department
PhD in Information Technologies

Use of meta-rankings on a Group Decision Support System

Presenter:
José G. Robledo-Hernández

Thesis adviser:
PhD Laura Plazola-Zamora

Why use DSS?

- Higher decision quality
- Improved communication
- Cost reduction
- Increased productivity

[Udo & Guimaraes, Empirically assessing factors related to DSS benefits, *European Journal of Information Systems* (1994) **3,** 218–227.]

Web-based DSS

- Modern DSS provide their users with a broad range of capabilities:
  - Information gathering & analysis
  - Model Building
  - Collaboration
  - Decision implementation
- The Internet and World Wide Web technologies has promoted a broad resurgence in the use of Decision Technologies to support decision-making tasks.

[Bhargava H.K., Power D.J., Sun D., 2005]

GDSS definition

- Group Decision Support System is a combination of computer, communication and decision technologies to support problem formulation and solution in group meetings

[DeSanctis & Gallupe, 1987].

- Interactive computer-based environments which support concerted and coordinated team effort towards completion of join tasks

[Zamfirescu C. ,2001]

- A computer-based system to support a meeting

[Aiken M., 2007]

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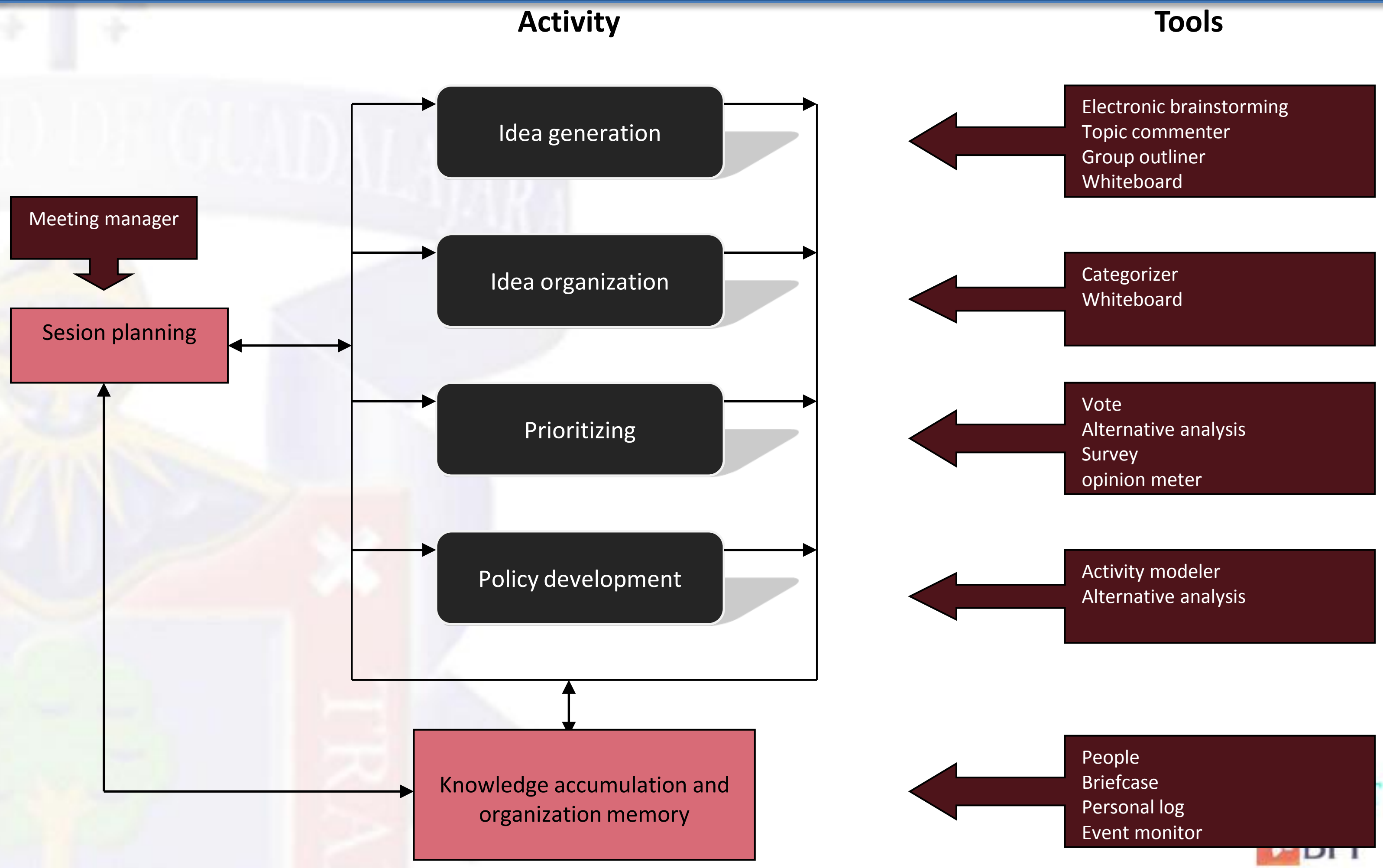
GDSS architecture example (by Group Systems)



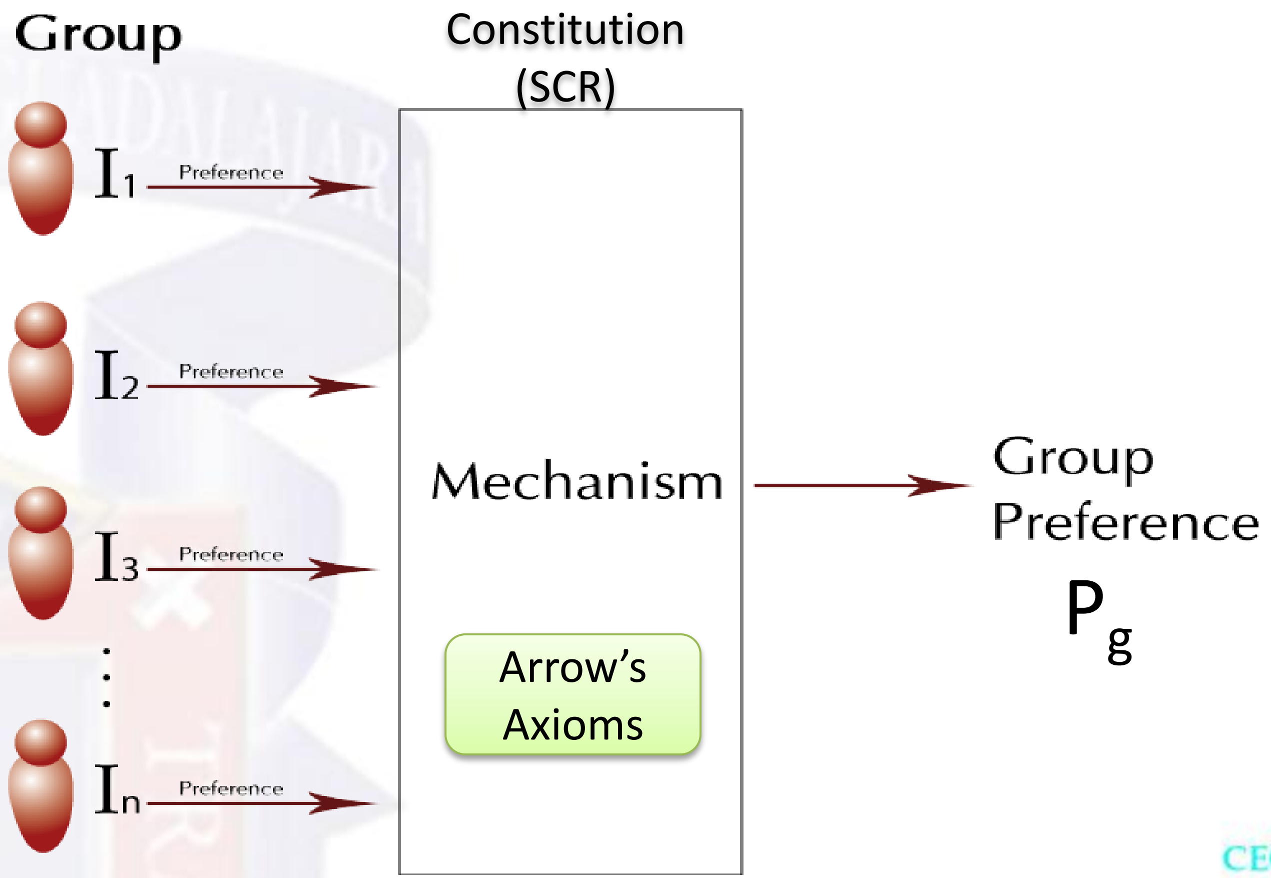
Based on: Turban and Aronson, 2001

GDSS design problems found:

- Do not take into account Arrow's Axioms in their voting methodology
- Driven by the perspective of a single decision maker instead of a group perspective

(French S., 2007)

Group Decision Making



Arrow's Axioms

- 1) Universal domain
- 2) Unanimity
- 3) Independence of Irrelevant alternatives
- 4) Rationality
- 5) No dictatorship

(Arrow K. J. , 1951)

Arrow's Impossibility Theorem

It is impossible to formulate a social preference ordering (P_g) that holds axioms 1, 2, 3,4 and 5

(Arrow K. J. , 1951)

Research paths to try to avoid the impossibility

- Restrict the domain of the constitution
- Diminish the rational conditions of counter-domain of the constitution

- Using more information(to allow group members to express not only a preference ranking but also their strength of preference)

[Sen, A. K. (1979), Van der Veen (1981), Plazola & Guillén (2007)]

Levels of preferential information

We have a set of three alternatives

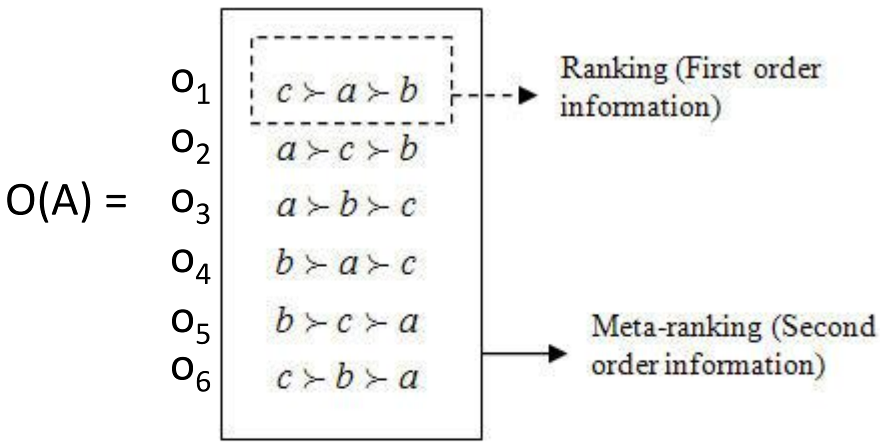
A={a, b, c}

- Zero order Information : choose only one alternative from the set A (i.e. alternative b)
- First order information : rank the alternatives, i.e.

$$b \succ c \succ a$$

Second order information - Meta-ranking:

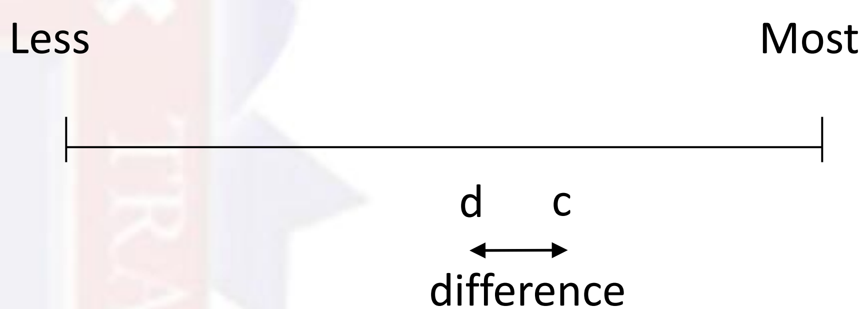
- Orderings of rankings of alternatives on set A
- According to Sen (1979), the use of meta rankings in the problem of social choice can be applied to the problem of finding a meaningful measure of cardinal utility



A.K. Sen. (1979), Interpersonal comparisons on Welfare. *in "Choice, Welfare and Measurement" (H.U. Press, Ed.)*

Preference strength

$$v_i(a) - v_i(b) > v_i(c) - v_i(d)$$



Modeling the preference strength

- The preference strength of each group member can be modeled with an additive value difference function:

$$(a,b) \in P_g \quad \Leftrightarrow \quad \sum_{i \in I(a,b)} (v_i(a) - v_i(b)) \geq \sum_{i \in I(b,a)} (v_i(b) - v_i(a)) \quad \forall a,b \in A$$

$$\textit{where} \quad I(a,b) \subseteq I \quad \textit{is the set of group members that prefer } a \textit{ to } b$$

[Plazola & Guillén (2007)]

Second order constitution

- Each individual of the group expresses his evaluation function in a closed and bounded subset of real numbers Y

$$\text{for each} \quad i \in I \quad v_i : A \rightarrow Y$$

- The preference of the group P_g :

$$P_g \quad \text{is given by}:$$

$$(a,b) \in P_g \quad \Leftrightarrow \quad v_g(a) \geq v_g(b) \quad \forall a,b \in A$$

$$\text{where } v_g \text{ is given by} \quad v_g(a) = \sum_{i \in I} v_i(a)$$

A second order constitution takes into account the preference of each member *i* of the group over the set of weak orders on A, interpreted as possible results P_g of the group choice.
To represent these preferences we take a reference set, common to all member of the group, denoted as O(A), and fashioned by all the posible rankings of the set A in decreasing preference order, each one with the form $a_1 \geq a_2 \geq \ldots \geq a_m$

[Plazola & Guillén (2007)]

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Magnitude of the vote

- A class A Constitution (of additive function) implicitly contains a voting system that includes the choice set O(A)
- The magnitude of the vote $w_i(o)$ of the individual *i* for the element $o \in O(A)$ to be selected as ranking of the group is equal to the sum of the magnitudes of votes that the individual *i* assigns to each one of the ordered pairs belonging to such element $(a,b) \in o$ that is:

$$w_i(o) = \sum_{(a,b) \in o \cap P_i} (v_i(a) - v_i(b) \quad \forall o \in O(A)$$

P_i is a weak order over A is called *first option* of the individual *i* over O(A)

[Plazola & Guillén (2007)]

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Magnitude of the vote against

- The magnitude of the vote (in favor) can be represented instead in terms of "magnitude of the votes against" or the *cost c(o)* given by

$$c(o) = \sum_{(a,b)\in o-P} (v(b) - v(a)) \quad \forall o \in O(A)$$

- Where P is the weak order on A corresponding to his preference over A given by

$$(a,b) \in P \Leftrightarrow v(a) \geq v(b) \quad \forall a,b \in A$$

- And $(a,b) \in o - P$ denote the alternative pairs that are in o but not in P

[Plazola & Guillén (2007)]

How to solve the problem of interpersonal comparisons

- Adding preferential information using a criterion of equity among individuals, in which everybody influences the group ranking to the same degree instead of the comparison of the preference strength among group individuals.

[Plazola & Guillén (2007)]

Outline of the method

Steps:

1. Each member *i* of the group **I** set up his preference ordering over the set A of alternatives
2. Generate the whole set of permutations of the set A of alternatives for each group member.
3. Calculate the magnitude of votes against
4. Calculate the differences between the magnitudes of consecutive votes. The result is a set of algebraic expressions which are the restrictions of a Linear Program.
5. Solve the Linear Program
6. Re-calculate the magnitude of votes against using the values found after solving the linear program.
7. Aggregate the information

Step (1)

1. Each member *i* of the group **I** set up his preference ordering over the set A of alternatives.

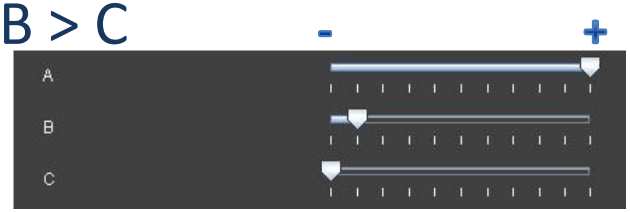
Set of alternatives:
A= {A, B ,C}

First order preferences

Common Reference scale to provide *a-priori* additional information

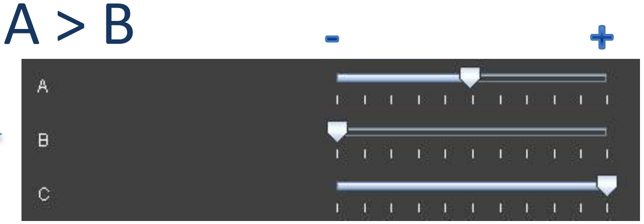
- Member 1:

A > B > C



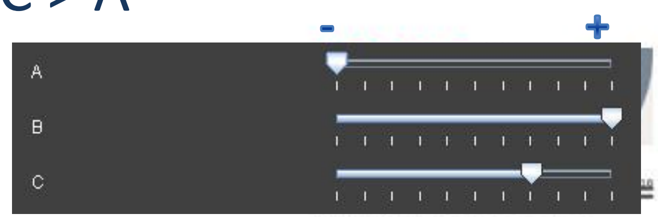
- Member 2:

C > A > B



- Member 3:

B > C > A



Step (2)

2. Generate the whole set of permutations O(A) of the set A of alternatives for each group member

Second order preference information

- Member 1
  - A > B > C
  - A > C > B
  - B > A > C
  - B > C > A
  - C > A > B
  - C > B > A
- Member 2
  - C > A >B
  - C > B > A
  - A > C >B
  - A > B > C
  - B > C > A
  - B > A > C
- Member 3
  - B > C > A
  - B > A > C
  - C > B > A
  - C > A > B
  - A > B > C
  - A > C > B

Step (3)

3. Calculate the magnitude of negative vote for each permutation, replacing the intermediate *a-priori* values of the reference scale by unknown values.

$$c(o) = \sum_{(a,b)\in o-P} (v(b) - v(a))$$

- Member 1
  - A > B > C = 0
  - A > C > B = x
  - B > A > C = 10 - x
  - C > A > B = 10 + x
  - B > C > A = 20 - x
  - C > B > A = 20
- Member 2
  - C > A >B = 0
  - C > B > A = x
  - A > C >B = 10 - x
  - A > B > C = 20 - x
  - B > C > A = 10 + x
  - B > A > C = 20
- Member 3
  - B > C > A = 0
  - C > B > A = 10 - x
  - B > A > C = x
  - C > A > B = 20 - x
  - A > B > C = 10 + x
  - A > C > B = 20

Step (4)

4. Calculate the differences between magnitudes of consecutive votes, resulting a set of algebraic expressions which constitute the restrictions of a linear program problem.

- Member 1
  - max: m;
  - C1: m - x <= 0;
  - C2: m + 2x <= 10;
  - C3: m - 2x <= 0;
- Member 2
  - max: m;
  - C1: m - x <= 0;
  - C2: m + 2x <= 10;
  - C3: m <= 10;
  - C4: m - 2x <= -10;
  - C5: m + x <= 10;
- Member 3
  - max: m;
  - C1: m + x <= 10;
  - C2: m - 2x <= -10;
  - C3: m + 2x <= 20;

Step (5)

5. Solve the linear program problem. The result values, are the intermediate values of the reference scale that guarantee that the differences between magnitudes of consecutive votes are equal or that maximizes the minimum difference between consecutive votes.

- Member 1
  Solution: x = 3.3333
- Member 2
  Solution: x = 5
- Member 3
  Solution: x = 6.6667

Step (6)

6. Re-calculate the magnitude of votes against using the values found after solving the linear program problem.

$$c(o) = \sum_{(a,b)\in o-P} (v(b) - v(a))$$

Member 1
- A > B > C = 0
- A > C > B = 3.33333
- B > A > C = 6.66667
- C > A > B = 13.33333
- B > C > A = 16.66667
- C > B > A = 20

Member 2
- C > A >B = 0
- C > B > A = 5
- A > C >B = 5
- A > B > C = 15
- B > C > A = 15
- B > A > C = 20

Member 3
- B > C > A = 0
- C > B > A = 3.333302
- B > A > C = 6.66667
- C > A > B = 13.33333
- A > B > C = 16.66667
- A > C > B = 20

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Step (7)

6. Aggregate the group information.

	orderings	Member 1	Member 2	Member 3		Magnitude of votes against for group
O_1	A > B > C	0	15	16.66667		31.66667
O_2	A > C > B	3.33333	5	20		28.33333
O_3	B > A > C	6.66667	20	6.66667		33.33334
O_4	C > A > B	13.33333	0	13.33333		26.66666
O_5	B > C > A	16.66667	15	0		31.66667
O_6	C > B > A	20	5	3.33333		28.33333

$$w_g(o) = \sum_{i \in I} w_i(o) = \sum_{(a,b) \in o} \sum_{i \in I(a,b)} (v_i(a) - v_i(b)),\ \forall o \in O(A);$$

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Step (8)

8. The group preference is the ordering which magnitude of votes against is the lowest.

	orderings	Member 1	Member 2	Member 3		Magnitude of votes against for group
O_4	C > A > B	13.33333	0	13.33333		26.66666

Group Preference P_g → $C \succ A \succ B$

Conclusions and future work

- A review of the literature shows that many GDSS not take into account the results of Arrow's theorem in the voting procedures they use.
- It is proposed the design of a GDSS, which uses an aggregation method based on the use of more preferential information: meta-rankings and strength of preference. This method take into account the results of Arrow´s Theorem [A proof can be revised in L. Plazola, and S. Guillén. (2007)]
- The method mentioned above considers a new manner of interpersonal comparison of the strength of preference, based on the use of a equity criterion in which each individual influences the group decision on the same degree.
- At the moment only has been considered problems where indifference is not considered. Therefore this is part of a future work.

References

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