A Liberal Impossibility of Abstract Argumentation

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Abstract

In abstract argumentation, where arguments are viewed as abstract entities with a binary defeat relation among them, a set of agents may assign individual members the right to determine the collective defeat relation on some pairs of arguments. I prove that even under a minimal condition of rationality, the assignment of rights to two or more agents is inconsistent with the unanimity principle, whereby unanimously accepted defeat or defend relation among arguments are collectively accepted. This result expands the domain of liberal impossibility beyond preference aggregation and judgment aggregation, and highlights this impossibility as an inherent tension between individual rights and collective consensus.

1 Introduction

Liberal impossibility captures an inherent tension between individual rights and collective consensus. This paper explores whether this impossibility exists in abstract argumentation, a domain different from preference aggregation and judgment aggregation.

In abstract argumentation, a landmark framework introduced by Dung [1995], arguments are viewed as abstract entities with a binary defeat relation among them. Even ignoring the evaluation of the true/false of each argument, there are multiple ways in which an agent may evaluate defeat relations among arguments. Following Sen's [1970] accounts of rights,[1] a set of agents may assign some individual members the right to determine the collective defeat relation on some pairs of arguments.[2] I prove that when only binary evaluation, *i.e.*, true/false, of each argument is permitted, even under a minimal rationality condition, the assignment of rights to two or more agents is inconsistent with the unanimity principle, whereby unanimously accepted defeat or defend relation among arguments, no matter directly or indirectly, are collectively accepted. Thus, liberal impossibility holds.

The discussion on liberal impossibility, or liberal paradox, was ignited by Sen's [1970] seminal paper in the domain of preference aggregation. Outside this domain, Dietrich and List [2008; hereinafter DL] found that this impossibility also exists in the domain of judgment aggregation, and Sen's impossibility can be regarded as a corollary in their framework.

The current work contributes to the classical but in general stagnated debate about individual rights and collective consensus. I prove a liberal impossibility theorem in argumentation, a vast domain but ignored so far by economists, by introducing abstract argumentation into our perspective. I also show that this result is not a corollary of DL's finding, and hence constitutes a complementary work with Sen and DL. In a new domain this result confirms a vague conjecture of Gaertner *et al.* [1992] that "[i]t is our *belief* that this problem[3] persists under virtually every plausible concept of individual rights that we can think of."

The rest of the paper is structured as follows. In Section 2 I provide a very preliminary background of abstract argumentation. I describe the model in Section 3, and prove the impossibility theorem in Section 4. In the last section I briefly present the result of DL [2008], and show that although their work can incorporate Sen's theorem in an extended framework, it fails to do so for the work in the current paper.

2 Abstract Argumentation: Preliminaries

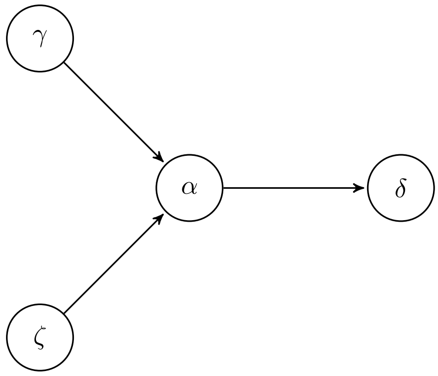
Dung [1995] presented one of the most influential computational models of argumentation. In his model, the internal structure being ignored, arguments are viewed as abstract entities, with a binary defeat relation among them. Formally,

Definition 1 *An* **argumentation framework** *is a pair* $AF = \langle \mathcal{A}, \rightharpoonup \rangle$ *where* $\mathcal{A}$ *is a set of arguments and* $\rightharpoonup$ *is a defeat relation over* $\mathcal{A}$*. We say that an argument* α **defeats** *an argument* β *if* $(\alpha, \beta) \in \rightharpoonup$*, or written as* $\alpha \rightharpoonup \beta$*, and* α *is a* **defeater** *of* β.

[1] Sen's paper, especially his formulation of the notion of rights, has encountered different contentions since its publication. For some representative work, see [Nozick, 1974; Gaertner *et al.*, 1992] among others. For recent development, see [Deb *et al.*, 1997; Dowding and van Hees, 2003]. It is not my interest to clarify the notation of rights in the current paper.

[2] For example, some individual members may have expert knowledge on the defeat relation of some pairs of arguments.

[3] That is, the conflict between individual rights and Pareto optimality, a similar concept with unanimity principle here; emphasis and footnote added.



δ: The suspect is innocent according to the presumption of innocence. α: There is evidence that he was at the crime scene *one hour before* the crime. γ: He was witnessed at a nearby town *at the same time* of the crime. ζ: The police obtained evidence that *at that time* he was on the telephone at that town.

Figure 1: *A Murder Case*

For a fixed set of $\mathcal{A}$, in the following section sometimes I use argumentation framework and defeat relation over $\mathcal{A}$ interchangeably to express the same thing if there is no ambiguity.

An argumentation framework can also be represented as a directed graph, *i.e.*, digraph, in which vertices are arguments and the directed arc denotes defeat relation between arguments. An argumentation and its digraph is shown in the following.

Example 2 (A MURDER CASE) *A murder case is under investigation. Initially argument δ states that the suspect is innocent according to the presumption of innocence. But, argument α claims that there is evidence that he was at the crime scene* one hour before *the crime. However, argument γ declares that he was witnessed at a nearby town* at the same time *of the crime. Also, argument ζ asserts that the police obtained evidence that* at that time *he was on the telephone at that town. Argumentation framework $AF = \langle\{\delta,\alpha,\gamma,\zeta\},\{(\alpha,\delta),(\gamma,\alpha),(\zeta,\alpha)\}\rangle$ corresponds to the digraph in Figure 1.*

In the current work we don't require the defeat relation to be antisymmetric because in real argumentation it is a common phenomenon that two arguments defeat with each other. This is especially usual when we face debates concerning moral value.

Then, when we face an argumentation framework, to determine which arguments are justified and which ones are not is a crucial problem.

For dealing with the reinstatement of arguments, Caminada [2006] introduced the notion of argument labeling, which specifies a particular outcome of argumentation. But for the reason I will mention in the following, here I only adopt his labels `in` and `out`, but not `undec` (undecided).

Definition 3 *Let $\langle\mathcal{A},\rightharpoonup\rangle$ be an argumentation framework. A* **stable labeling** *is a function $\mathcal{L}:\mathcal{A}\to\{\texttt{in},\texttt{out}\}$ such that:*

- *$\forall\alpha\in\mathcal{A}$, $\mathcal{L}(\alpha)=$`in` if $\mathcal{L}(\beta)=$`out` for all β (if any) where $\beta\rightharpoonup\alpha$; and*
- *$\forall\alpha\in\mathcal{A}$, $\mathcal{L}(\alpha)=$`out` if there is a β such that $\beta\rightharpoonup\alpha$ and $\mathcal{L}(\beta)=$`in`.*

With this language, the label `in` means the argument is accepted/justified, the label `out` means the argument is rejected/not justified.

This definition works well for simple cases where we can see clearly which arguments should emerge victoriously. For example, in the argumentation framework $\alpha\rightharpoonup\beta\rightharpoonup\gamma$, α is `in` since it is not defeated by any argument. Consequently β is `out`, and γ is `in`. Even so, however, in some cases the definition above is ambiguous. The Liar Paradox is a famous example that concerns the problem of self-defeat, which makes any determination on which arguments are `in` or `out` impossible based on Definition 3. Thus, if we accept Definition 3, then we impose a constraint on the original definition of argumentation framework, *i.e.*, there is no self-defeating argument. Put in another way, defeat relation $\rightharpoonup$ is irreflexive.

Notice that Definition 3 can actually be seen as a postulate, as it specifies a restriction on both a labeling and an argumentation framework. The meaning of the latter statement will be clear in the following sections.

Definition 4 *Let $\langle\mathcal{A},\rightharpoonup\rangle$ be an argumentation framework, and $\mathcal{L}$ a labeling over it. We define:*

- *$\texttt{in}(\mathcal{L})=\{\alpha\in\mathcal{A}|\mathcal{L}(\alpha)=\texttt{in}\}$;*
- *$\texttt{out}(\mathcal{L})=\{\alpha\in\mathcal{A}|\mathcal{L}(\alpha)=\texttt{out}\}$.*

Here an explanation is in order. In the literature of artificial intelligence, starting from the paper of Caminada [2006], many scholars, besides the notion of `in` and `out`, also adopt `undec` to denote the labeling of an argument whose status, *i.e.*, justified or not justified, could not be decided. In real life, *e.g.*, judicial practice, however, an undecided argument is not acceptable. Just as we only call an argument justified or not justified, the labeling of `undec` also is not adopted in the current work. In Section 4 we will see that this refusal is crucial in our impossibility theorem.

We notice that although some argumentation frameworks can only accommodate one stable labeling, say, *e.g.*, argumentation framework $\alpha\rightharpoonup\beta\rightharpoonup\gamma$ with the only stable labeling $\mathcal{L}$ such that $\texttt{in}(\mathcal{L})=\{\alpha,\gamma\}$, there are many argumentation frameworks which accommodate multiple binary labelings. In fact, suppose there are four arguments where $\alpha\rightharpoonup\beta\rightharpoonup\gamma\rightharpoonup\delta$, and $\delta\rightharpoonup\alpha$, then we see that there exist two binary labelings $\mathcal{L}_1$ and $\mathcal{L}_2$ such that $\texttt{in}(\mathcal{L}_1)=\{\alpha,\gamma\}$ and $\texttt{in}(\mathcal{L}_2)=\{\beta,\delta\}$.

But there exist argumentation frameworks which cannot accommodate at least one stable labeling.

Example 5 *Suppose $\mathcal{A}=\{\alpha,\beta,\delta\}$, and the argumentation digraph is shown as Figure 2. Then, we find that it cannot determine which argument is `in` or `out`. In fact,* e.g.*, if we deem that argument α `in`, then according to Definition 3, argument β is `out`, and argument δ `in`. Consequently, α should not be `out`, a contradiction. The same problem arises when we initially deem that argument α `out`.*

Definition 6 *We call an argumentation framework* **admissible** *if it can accommodate at least one stable labeling; otherwise it is* **inadmissible**.

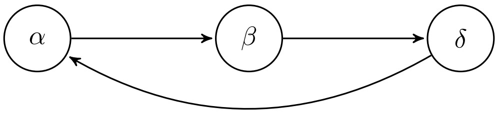


Figure 2: *An Argumentation Framework with Odd Cycle*

For a fixed set of $\mathcal{A}$, we also call the defeat relation over $\mathcal{A}$ admissible or inadmissible depending on the underlying nature of the framework. Just as Definition 3 implies, it does specify a restriction on an argumentation framework. In this paper, I only consider admissible argumentation framework, which captures a minimal condition of rationality, as a reasonable point of view of an agent.

At last, for the definitions that follow, we need to introduce two notations. For any $S \subseteq \mathcal{A}$ and $\alpha \in \mathcal{A}$, let $S^+ = \{\gamma \in \mathcal{A} | \beta \rightharpoonup \gamma$ for some $\beta \in S\}$, and $\alpha^- = \{\beta \in \mathcal{A} | \beta \rightharpoonup \alpha\}$.

Definition 7 *Let $\langle \mathcal{A}, \rightharpoonup \rangle$ be an argumentation framework, and let $S \subseteq \mathcal{A}$ and $\alpha \in \mathcal{A}$. We call S* **defends** *argument α if $\alpha^- \subseteq S^+$. We also say that argument α is* **acceptable with respect to** S.

Intuitively, a set of arguments defends a given argument if it defeats all its defeaters.[4]

3 The Model: Aggregating Argumentation Framework

In the above section I provide a very preliminary introduction to the element of abstract argumentation, focusing on stable labeling instead of argument labeling with `undec`, or more general one.[5] The choice of contents depends on whether they are relevant to the current research, where we analyze the problem of aggregating different individual argumentation frameworks over a common set of arguments to get a social argumentation framework, and to discuss the inconsistency among some desirable properties.[6] In this section, I introduce the model and define three properties.

[4]Trivially, for any argument α which has no defeater, since $\alpha^- = \emptyset \subseteq \{\alpha\}^+$, $\{\alpha\}$ defends α. In this case, for simplicity we also say that α defends itself.

[5]For an overall summary of the state-of-the-art achievement of this theory, see Rahwan and Simari [2009]. For the relationship between labeling-based approach and extension-based approach when defining argumentation semantics, see Baroni *et al*. [2004].

[6]Different from my concern in the current work, Bodanza and Auday [2009] analyzed the problem of aggregating individual argumentation frameworks over a common set of arguments in order to *obtain a unique socially justified set of arguments* (emphasis added). They articulated the difference of aggregation methods involved. That is, their work "can be done in two different ways: a social attack relation is built up from the individual ones, and then is used to produce a set of justified arguments, or this set is directly obtained from the sets of individually justified arguments." What we do in this research starts from the first step of the first way, although with totally different destination. In contrast, their "main concern here is whether these two procedures can coincide or under what conditions this could happen."

Conventionally, we use $\mathbb{N}$ to denote the set of natural numbers. For an integer k, $[k]$ denotes the set $\{1, 2, \ldots, k\}$.

We consider a group of agents $N = [n]$ $(n \geq 2)$, and a finite set of arguments $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ $(m \geq 3)$. For each agent $i \in N$, she has her own argumentation framework $AF_i = \langle \mathcal{A}, \rightharpoonup_i \rangle$, build up from her defeat relation $\rightharpoonup_i$. Given a pair of arguments $\alpha, \beta \in \mathcal{A}$, each agent can express her defeat relation by choosing one of the four alternatives: 1) both arguments are perfectly compatible; 2) α defeats β; 3) β defeats α; or 4) they defeat each other (expressing that they are in conflict but have the same power of argumentation, or are indifferent). If we let $[\alpha, \beta]$ denote any ordered pair of arguments α and β, *i.e.*, $[\alpha, \beta]$ is either (α, β) or (β, α), then in the language of digraph, the four alternatives are: 1) $[\alpha, \beta] \notin \rightharpoonup$; 2) $\alpha \rightharpoonup \beta$; 3) $\beta \rightharpoonup \alpha$; or 4) $\alpha \rightleftharpoons \beta$, respectively.

Bodanza and Auday [2009] provided the following two definitions.[7]

Definition 8 *A* **social defeat function** *is a mapping $f : \rightharpoonup_1 \times \ldots \times \rightharpoonup_n \rightarrow \mathcal{A} \times \mathcal{A}$. We call the relation produced by f for each profile of individual defeat relations* **social defeat relation**.

Definition 9 *A* **social argumentation framework** *is a structure $SAF = \langle \mathcal{A}, \{AF_i\}_{i \in N}, \rightharpoonup_f \rangle$, where $\rightharpoonup_f$ is the social defeat relation of SAF produced from social defeat function f.*

Example 10 (SOCIAL ARGUMENTATION FRAMEWORK WITH MAJORITY RULE) *Suppose there are three agents facing a set of three arguments α, β, and γ. Their individual defeat relations are*

$$\rightharpoonup_1: \ \alpha \rightharpoonup \beta \rightharpoonup \gamma,$$

$$\rightharpoonup_2: \ \alpha \leftharpoondown \beta \rightharpoonup \gamma,$$

$$\rightharpoonup_3: \ \alpha \rightharpoonup \beta \leftharpoondown \gamma,$$

respectively. If this society adopts majority rule m as their social defeat function, then the social defeat relation is $\rightharpoonup_m$: $\alpha \rightharpoonup \beta \rightharpoonup \gamma$ $(= \rightharpoonup_1)$.

We can describe the behaviors of defending and defeating with more nuances.

Definition 11 *For any $\alpha, \beta \in \mathcal{A}$, we call α* **indirectly defeats** *β if there exists an $(\alpha, \beta)-$path with length[8] $k = 2l+1$, where $l \in \mathbb{N}$. If not specified explicitly, we write $\alpha \hookrightarrow \beta$*

[7]Bodanza and Auday [2009] call *social attack relation* instead of *social defeat relation*, and do not define explicitly social defeat function. Instead, they call the aggregation of individual argumentation frameworks "according to some specified mechanism M."

[8]In digraph D, a *path* is an alternating sequences $P = x_1 a_1 x_2 a_2 x_3 \ldots x_{k-1} a_{k-1} x_k$ of vertices x_i and arcs a_j from D such that the tail of a_i is x_i and the head of a_i is x_{i+1} for every $i \in [k-1]$, and $x_i \neq x_j$ if $i \neq j$, $\forall i, j \in [k]$. We say that P is a path from x_1 to x_k or an (x_1, x_k)-path. The length of a path is the number of its arcs. Hence, the path above has length $k - 1$. For P, if $x_1, x_2, \ldots, x_{k-1}$ are distinct, $k \geq 3$ and $x_1 = x_k$, P is a cycle. The length of a cycle is defined in the same way.

no matter α defeats or indirectly defeats β. We call α **indirectly defends**[9] *β if there exists an $(\alpha, \beta)-$path with length $k = 2l + 2$, where $l \in \mathbb{N}$. We write $\alpha \looparrowright \beta$ no matter α defends or indirectly defends β.*

At the same time, we still need to know that there exits another delicate situation defined below, although it will not be incorporated in the desirable properties for a social defeat function.

Definition 12 *For any $\alpha \in \mathcal{A}$, if there exists $\beta \in \mathcal{A}$ such that $\alpha^- \cap \{\beta\}^+ \neq \emptyset$ but not $\alpha^- \subseteq \{\beta\}^+$, we say that β* **partially defends**[10] *α.*

Intuitively, β partially defends α if α has multiple defeaters, and β defeats some (but not all) of them.

Now, suppose we want to find a social defeat function f with the following intuitive properties:

Universal Domain (Condition D): The domain of f is the set of all profiles where each individual defeat relation is admissible, and the range of f is the set of all defeat relations that is admissible.

Unanimity Principle (Condition U): For any $\alpha, \beta \in \mathcal{A}$, $\alpha \hookrightarrow_f \beta$ if $\alpha \hookrightarrow_i \beta$ for all $i \in N$, and $\alpha \looparrowright_f \beta$ if $\alpha \looparrowright_i \beta$ for all $i \in N$.[11]

Minimal Liberalism[12] (Condition L): There are at least *two* agents such that for each of them there is at least one pair of arguments between which she is decisive over the defeat relation. That is, for her there is at least one pair of arguments, say α and β, such that the social defeat relation between these two arguments is the same with her defeat relation between them, *i.e.*, $[\alpha, \beta] \notin \rightharpoonup$, $\alpha \rightharpoonup \beta$, $\beta \rightharpoonup \alpha$, or $\alpha \rightleftharpoons \beta$.

[9] Dung [1995] actually has defined "indirectly defeat" and "indirectly defend". But, using the language here, his called α *indirectly defends* β if there exists an $(\alpha, \beta)-$path with length $k = 2l$, where $l \in \mathbb{N}$. Obviously this definition is not compatible with our definition of "defend" in Definition 7. For example, if there exists an argumentation framework $\beta \rightharpoonup \gamma \rightharpoonup \alpha$, then we see that β *defends* α in our language, but β *indirectly defends* α in Dung's language. Also, in Example 2, if there is another argument β that defeats γ, Dung's definition cannot distinguish the defeat relations among β, γ and α, and $\beta \rightharpoonup \gamma \rightharpoonup \alpha$. Dung would say that in both case β *indirectly defends* α, but I will call β *defends* α in the latter case, and β *partially defends* α in the former case.

[10] Obviously, here $|\alpha^-| > 1$.

[11] We don't impose any constraint on the social defeat relation between any arguments α and β when all agents deem that they are compatible, *i.e.*, $[\alpha, \beta] \notin \rightharpoonup$. Also, there is no constraint when all agents deem argument α partially defends argument β.

Indirect defeat (or defense) can be obtained through different paths for all the agents. Although the paths, that can be seen as different justifications for the statement, are different, we can still think all individuals share a similar opinion when $\alpha \hookrightarrow_i \beta$ for all $i \in N$. That is, they agree that α defeats β directly or indirectly. We can interpret the case of $\alpha \looparrowright_i \beta$ similarly. I use the term "unanimity" in this sense.

[12] This concept can also accommodate the idea of expert right just as in DL [2008], where some group members may have expert knowledge on certain issues and may therefore be granted the right to be decisive on them. To follow the convention, however, I still use the term here.

4 Impossibility Theorem

The following example provides a good motivation for the current work.

Example 13 (A DEBATE ABOUT MIGRATION OF THE DIRTY INDUSTRIES TO THE LDCS)[13] *Imagine there is a debate in a committee of the World Bank about whether it should encourage more migration of the dirty industries to the LDCs (less developed countries). This committee is constituted of economists Alan and Brenda, who have different opinions about the defeat relations among the following three arguments:*

β: The measurement of the costs of health-impairing pollution depends on the foregone earnings from increased morbidity and mortality. From this point of view a given amount of health-impairing pollution should be done in the country with the lowest cost, which will be the country with the lowest wages. Rational agents in LDCs would accept migration of the dirty industries from developed countries for compensation between the least that agents in LDCs will accept and the most that agents in rich countries will offer. This voluntary agreement is an welfare improvement on both parties.

α: In reality normally LDCs accepts migration of the dirty industries due to their ignorance of the potential danger of pollution.

δ: In reality normally LDCs accepting migration of the dirty industries know the potential danger of pollution. But this voluntary agreement is unfair.

Initially both Alan and Brenda are welfarists who believe that morality is centrally concerned with the welfare or well-being of individuals. Thus, argument β is a counterargument of argument δ. Besides that, Brenda considers that argument α is a counterargument of β, so her argumentation framework is

$$\text{Brenda: } \alpha \rightharpoonup \beta \rightharpoonup \delta.$$

That is, she is not a stubborn welfarist, and realizes that there are hidden stories behind the so-called "voluntary" agreement. Consequently she prefers to give up her support to argument β, and finally justifies arguments α and δ.

On the contrary, Alan considers that argument δ is a counterargument of α. For him, no matter how to evaluate a policy, in reality there are many agreements where one party has to or prefer to sign even all negative influences involved are known; acceptance of dirty industry is one of these cases. So his argumentation framework is:

$$\text{Alan: } \beta \rightharpoonup \delta \rightharpoonup \alpha.$$

That is, since he is a stubborn welfarist, unshakably he justifies argument β, and argument α too with the sacrifice of argument δ.[14]

[13] This example is inspired by a shocking real one, see pp.12-23 of Hausman and McPherson [2006].

[14] This is a special case where, according to Definition 3, once we know the argumentation framework of any member of committee, we know her of his evaluation of justified or not for each argument.

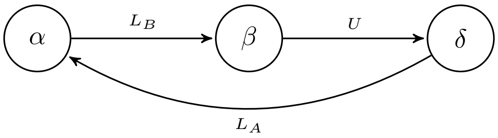


Figure 3: *A Debate about Migration of the Dirty Industries to the LDCs*

Now suppose that Brenda is an expert in dealing with the defeat relation between arguments α and β, so the World Bank assigns her the task. Similarly, Alan is assigned to determine the defeat relation between arguments δ and α. Also, this committee accepts the unanimous defeat relation among any pair of arguments. Under the circumstances, we see that the committee as a whole, its argumentation framework can be depicted as the one in Figure 2. For convenience, we reproduce it in Figure 3.[15]

Then, the committee finds that it cannot determine which argument is justified or not since this is an inadmissible argumentation framework.

The following theorem reveals an inherent tension between liberal rights and collective consensus in a most general situation of argumentation, where the core concepts are only defeating, defending, and (not) being justified.

Theorem 14 *There is no social defeat function that can simultaneously satisfy Conditions D, U, and L in abstract argumentation.*

Proof. Remember that for any pair of arguments, say α and β, an agent can express one of the four alternatives, 1) $[\alpha, \beta] \notin \rightharpoonup$; 2) $\alpha \rightharpoonup \beta$; 3) $\beta \rightharpoonup \alpha$; or 4) $\alpha \rightleftharpoons \beta$, respectively. For any society which respects liberal right, such an alternative should form the social defeat relation between α and β if this agent is decisive over the defeat relation between these two arguments.

Let the two agents referred to in Condition L be 1 and 2, respectively, and the two pairs of arguments referred to be (α, β) and (δ, γ), respectively. There are no more other arguments in $\mathcal{A}$. If (α, β) and (γ, δ) are the same pair of arguments, then there is a contradiction. Thus, they have at most one argument in common, say $\alpha = \gamma$. Assume now that agent 1 deems that α defeats β, and agent 2 deems that δ defeats γ $(= \alpha)$. And let everyone in the community including agent 1 and 2 deem that β defeats δ. That is, the argumentation frameworks of agent 1 and 2 are

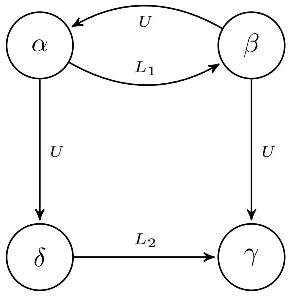
$$\text{agent 1: } \alpha \rightharpoonup \beta \rightharpoonup \delta;$$
$$\text{agent 2: } \beta \rightharpoonup \delta \rightharpoonup \alpha.$$

By Condition D all the frameworks are admissible. But by Condition L, as the society, α must defeat β, and δ must defeat γ $(= \alpha)$, while by Condition U, β must defeat δ. Consequently, we get the same argumentation framework with odd cycle as shown in Figure 3. An argumentation framework with odd cycle, however, obviously is inadmissible, a contradiction.

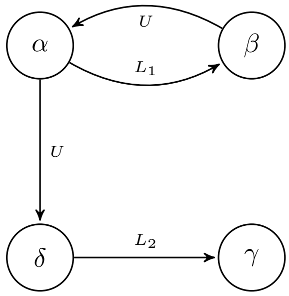
Next, let α, β, γ and δ be all distinct. Besides deeming that α defeats β from her liberal right, suppose that agent 1 also deems that β defeats γ, and γ defeats δ. Let everyone else in the community including agent 2 deem that β defeats α, α defeats δ, and δ defeats γ. That is,

$$\text{agent 1: } \alpha \rightharpoonup \beta \rightharpoonup \gamma \rightharpoonup \delta;$$
$$\text{agent } 2, \ldots, n\text{: } \beta \rightharpoonup \alpha \rightharpoonup \delta \rightharpoonup \gamma.$$

By Condition D all the frameworks are admissible. But by Condition L, as the society, α must defeat β, and remembering the liberal right of agent 2, δ should defeat γ, while by Condition U, α must defeat or indirectly defeat δ. Since we have known that δ defeats γ, it follows that α cannot indirectly defeat δ. Thus, α must defeat δ. Similarly we see that β defeats or indirectly defeats γ. Also, α must defend γ by Condition U, so there is no arc from α to γ. Nevertheless, by Condition U again, β must defend or indirectly defend δ, and since there is only one argument α that defeats δ, consequently β must defeat α. Depending on whether β both defeats and indirectly defeats γ, or β only indirectly defeats γ, the social argumentation frameworks can be shown in Figure 4.[16] No matter in which case, it contradicts with the liberal right of agent 1, who deems that α defeats β. ■



(a) β both defeats and indirectly defeats γ



(b) β only indirectly defeats γ

Figure 4: *A Liberal Paradox of Four Arguments*

From the explanation under Definition 4, however, we should know that this is not universal. In any case, what we are interested in is the aggregation of argumentation frameworks, not the aggregation of individual viewpoint as to the justification of arguments.

[15] In the following digraphs we sometimes label the force determining the social defeat relation between two arguments besides the corresponding arrow, where U denotes Condition U, and L with a subscript denotes the liberty of corresponding economist (agent).

[16] Which argumentation framework is the final one depends on more details about the social defeat function. But this is not the interest of the current research.

5 Discussion: beyond Judgment Aggregation

Liberal impossibility not only haunts preference aggregation, it also appears in judgment aggregation, an emerging active multidisciplinary field. In a recent paper, Dietrich and List (2008) identified a problem that generalizes Sen's liberal paradox. Under plausible conditions, they proved that the assignment of rights to two or more agents or subgroups is also inconsistent with the unanimity principle.

Simply speaking, there is a group of agents $N = [n]$ $(n \geq 2)$ and an agenda, *i.e.*, a non-empty subset X of logic $\mathbf{L}$ expressed as $X = \{p, \neg p : p \in X_+\}$ for a set $X_+ \subseteq \mathbf{L}$ of unnegated propositions on which binary judgments, *i.e.*, yes or no, are made. They call propositions $p, q \in X$ *conditionally dependent* if there exist $p^* \in \{p, \neg p\}$ and $q^* \in \{q, \neg q\}$ such that $\{p^*, q^*\} \cup Y$ is inconsistent for some $Y \subseteq X$ consistent with each of p^* and q^*. The agenda X is *connected* if any two propositions $p, q \in X$ are conditionally dependent. Their main finding is that if and only if the agenda is connected, there exists no aggregation function F generating consistent collective judgment sets that satisfies universal domain, minimal rights and the unanimity principle.[17]

Moreover, after an easy transformation from the question of whether alternative a is strict better than alternative b to the question of whether proposition "alternative a is strict better than alternative b" is true, they proved that the preference agenda is connected. Consequently, Sen's Liberal Paradox becomes a corollary naturally.

Since judgment aggregation and argumentation share some common interests, and both depend on the toolset of logic in a different sense, especially due to the implied seemingly relationship between "connected" agenda and digraph, it may be conjectured that the result of DL will cover our finding in the current paper. But we can show that it is totally not the case.

In fact, although the easy transformation mentioned above helps DL successfully incorporate the domain of preference aggregation into the one of judgment aggregation, a similar practice fails to do so for the sake of abstract argumentation. In my model, for each pair of arguments what really is aggregated is the defeat relations between them among all agents, instead of `in` or `out` of these two arguments. Thus, for any two arguments α and β we first need to ask if we introduce proposition p to denote $\alpha \rightharpoonup \beta$, then what? In DL's paper, actually in the mainstream research of judgment aggregation until now, any proposition only adopts classical two-value logic, viz yes or no. When we talk about the aggregation of defeat relation, for any pair of arguments α and β, there exist four possibilities, viz $[\alpha, \beta] \notin \rightharpoonup$, $\alpha \rightharpoonup \beta$, $\beta \rightharpoonup \alpha$, or $\alpha \rightleftharpoons \beta$. Thus, if we use the language of logic, p should be a proposition in a four-value logic, for which DL's framework cannot cover.

Dietrich [2007] does tackled Arrowian impossibility in a generalized model. But it is still an open question whether liberal paradox exists in general logic.

Therefore, what we do in the current paper is a complementary work with Sen and DL.

[17] Concretely, they define these three properties as:

Universal Domain: The domain of F is the set of all possible profiles of consistent and complete individual judgment sets.

Minimal Rights: There exist (at least) two agents who are each decisive on (at least) one proposition-negation pair $\{p, \neg p\} \subseteq X$.

Unanimity Principle: For any profile $(A_1, \ldots, A_n)$ in the domain of F and any proposition $p \in X$, if $p \in A_i$ for all agents i, then $p \in F(A_1, \ldots, A_n)$, where A_i is the judgment set of agent i.

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