From preferences to judgments and back

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Preferences and judgments

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$(x, y) \in \preceq, \ (x, y) \notin \preceq$

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$\mathcal{I} : \mathcal{L}_i \longrightarrow \{1, 0\}$

$\mathcal{I} \models \phi, \ \mathcal{I} \not\models \phi$

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- Can we view preferences as judgments?
- ... and judgments as preferences?

PA as JA ... axiomatic way

List & Pettit, 03 and Dietrich & List, 07

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$\mathcal{L}_{\texttt{Iss}} \subseteq \mathcal{L}$

$\mathcal{I}^{\sigma} : \mathcal{L}_{\texttt{Iss}} \longrightarrow \{1, 0\}$

$\mathcal{I}^{\sigma} \models bPa, \ \mathcal{I}^{\sigma} \not\models bPa$

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$(a, b) \in \prec, \ (a, b) \notin \prec$

$\forall x, y (xPy \rightarrow \neg yPx)$

$\forall x, y, z ((xPy \wedge yPz) \rightarrow xPz)$

$\forall x, y (x \neq y \rightarrow (xPy \vee yPx))$

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- Preferences are FOL formulae
- Rationality conditions are FOL formulae

Dietrich & List, 07

- “Preference agendas” exhibit characteristic structural properties (e.g., strong connectedness)
- ... which are sufficient to yield impossibility results under Arrow’s conditions for aggregation functions
- ... hence Arrow’s theorem can be obtained as a corollary of more general JA impossibility results

PA as JA ... semantic way

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Rankings as Truth Values

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(Debreu, 1954) Let $\preceq$ be a total preorder on a finite set $\mathtt{Iss}$. There exists a ranking function $u : \mathtt{Iss} \longrightarrow [0, 1]$ such that $\forall x, y \in \mathtt{Iss}$: $x \preceq y$ iff $u(x) \leq u(y)$. Such a function is unique up to ordinal transformations.

Rankings as Truth Values

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$(a, b) \in \preceq, \ (a, b) \notin \preceq$

$$a \preceq b \quad \text{iff} \quad u(a) \leq u(b) \quad \text{iff} \quad u \models a \rightarrow b$$

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$\mathcal{L}\ (\mathbf{P} = \mathtt{Iss})$

$u : \mathcal{L} \longrightarrow [0, 1]$

$u \models a \rightarrow b, \ u \not\models a \rightarrow b$

Rankings as Truth Values

$\mathtt{Iss}$	$\mathcal{L}\left(\mathbf{P} = \mathtt{Iss}\right)$
$\preceq$: total preorder on $\mathtt{Iss}$	$u : \mathcal{L} \longrightarrow [0,1]$
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- Preferences are *implications* in many-valued logic

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- Preferences are *implications* in many-valued logic
- Rationality conditions are captured by the ranking/interpretation function

Importing impossibilities

- "Preference agendas" exhibit characteristic structural properties (e.g., minimal connectedness)
- ... which are sufficient to yield impossibility results
- ... hence PA impossibilities can be obtained as a corollary of more general JA impossibility results

JA as PA

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JA as PA
... the semantic way

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Boolean preferences

$$\mathcal{L}_i \subseteq \mathcal{L}$$

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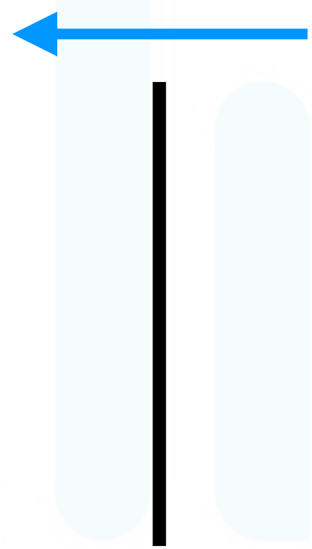
Boolean preferences



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- ... preserving the meaning of Boolean connectives

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- What kind of impossibilities still hold assuming only the set of Boolean Preference profiles (hence giving up Universal Domains)?

Importing impossibilities

- Impossibilities on Boolean preference domains yield JA impossibilities as immediate corollaries:

Importing impossibilities

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For any JA structure $\mathfrak{S}^J$ with a set of issues $\{p, q, p \rightarrow q\} \subseteq \mathcal{L}_i$ (where $\rightarrow$ can be substituted by $\vee$ or $\wedge$), there exists no aggregation function which satisfies **U**, **Sys** and **NoDict**.

Conclusions

Back and forth

PA ↔ JA
- many-valued
- implications

PA ↔ **JA**
- compound issues
- Boolean domains

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PA ⟷ JA
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Future agenda

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2. ... and further possible generalizations (e.g. po-sets)
3. Translate (im)possibilities between the two frameworks
4. Relate many-valued logics on [0,1] to more standard (modal) logics of preferences