The Random Pairs Voting Rule: Introduction and Evaluation With a Large Dataset

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Abstract

This paper introduces and evaluates the Random Pairs voting rule to show that it significantly improves upon the communication complexity of balloting methods requiring complete orderings of candidates while maintaining a high Condorcet efficiency. To this end, Random Pairs is compared to plurality, 2-approval, Borda, and range voting on elections constructed from Yahoo! users' ratings of musical artists. These elections feature seven candidates and two orders of magnitude more voters than previous analyses of voting rules. In this data set, there are few instances of elections without a single Condorcet winner, and empirical results on the other voting rules agree strongly with Merrill's 1984 simulation results. Upper bounds on the communication complexity of the Random Pairs voting rule and the range voting rule are also provided.

1 Introduction

In this United States, most elections are decided using ballots that give voters the ability to provide only a trivial amount of information about their preferences. In races with a single winner, voters are typically asked to indicate the one candidate they prefer above all the others. After all ballots are cast, the candidate who has received a plurality of the votes is declared the winner.

Because most real elections require voters to share only a small portion of their preferences, the outcomes of elections are almost always based upon incomplete information. The amount of information required to be communicated (by the voters and by the election officials) for each voting rule to select a winner is measured in complexity-theoretic terms of upper and lower bounds, with the number of voters and the number of candidates as the variables. This *communication complexity* allows for comparisons between different voting rules, and may inform future protocols that require feedback from multiple users to make a decision. Prior research has evaluated some common voting rules in these terms [3, 14]. Though plurality has a very low communication complexity, the ballots cast are not terribly expressive in that the outcomes of such elections may not reflect the voters' sentiments accurately and plurality may not reliably select the Condorcet winner[1] [6, 11]. Other voting rules select the Condorcet winner more often than plurality, but have a higher communication complexity.

This work was prompted in part by an interest in constructing single-round[2] voting rules that have low communication complexity but that still frequently elect Condorcet winners. Several reasons support the need for voting rules with lower communication complexity. The most obvious is that recording a complete set of preferences for every user would take up too

[1] As defined in Section 2.1

[2] That is, a voting rule that does not rely on preference elicitation or a separate runoff election. All voters communicate whichever preferences they have all at once, and the winner is determined using only those preferences. Plenty of research has gone into preference elicitation, which requires a central authority to iteratively ask questions of each voter about their preferences, often in the form of a pairwise comparison of candidates [2, 17]. The new voting rule described here follows the convention of real-world elections where the central authority requests that each voter complete a single ballot, then applies a decision rule based upon the ballots collected.

FOR PRESIDENT OF THE UNITED STATES Vote for not more than ONE	
ROSS "ROCKY" C. ANDERSON, Justice	⬭
GARY JOHNSON, Libertarian	⬭
PETA LINDSAY, Socialism and Liberation	⬭
BARACK OBAMA, Democratic	⬭
MITT ROMNEY, Republican	⬭

Figure 1: A ballot to elect the President of the United States based on the American state of Vermont's 2012 General Election ballot, in the conventional plurality voting rule.

much space. Second, voters must spend more time than they may be willing to construct these complete rankings, and indeed may not have enough information about the candidates to do so. Last, voters may not want to disclose their complete ranking–an individual could conceivably be identified uniquely if the number of voters is low relative to the number of candidates.

With this in mind, I propose a randomized partial-order voting rule called "Random Pairs" (RP hereafter), which seems to be nearly as expressive as those rules requiring full preference orderings while having a communication complexity not much higher than plurality. While it resembles somewhat the greedy iterative voting procedure in [5], RP requires the user to communicate only once, and with single-bit preferences between two candidates rather than with score-based preferences for individual candidates like range or Borda.

Moving beyond the proposed voting rule, it appears that there is no earlier work evaluating voting rules on elections not derived from synthetic distributions with as many candidates or as many voters as evaluated here. Mattei et al were the first to take a large set of ratings and convert them into three- and four-candidate elections [8], which were then made available online [9]. Their work is extended here to much larger elections both in terms of candidates and of voters.

2 Random Pairs

In an election using the plurality rule, each voter chooses a single "best" candidate by marking that candidate on a ballot, like the one shown in Figure 1. Using the RP voting rule, each voter gets a ballot with a predetermined number of random candidate pairs (defined as p), like the one shown in Figure 2. These pairs are selected uniformly at random before the election by election officials or the party responsible for tabulating and reporting the results. The voter then indicates which candidate of the pair they prefer more, if they have a preference. Because each ballot is provided with only a subset of the possible pairings, the voter will not have the opportunity to compare all candidates, and two different voters will almost certainly receive two different ballots.[3]

[3] I do not evaluate the real-world implications of applying RP to real-world elections. In particular, I envision the challenge of dealing with a voter arriving at a polling place and receiving a ballot that does not contain his or her preferred candidate. I suspect that telling the voter "trust me, the math works out in the end" would do little to console them.

FOR PRESIDENT OF THE UNITED STATES In EACH line below, vote for ONE candidate	
ROSS "ROCKY" C. ANDERSON, Justice ◯	PETA LINDSAY, Socialism and Liberation ◯
PETA LINDSAY, Socialism and Liberation ◯	MITT ROMNEY, Republican ◯
ROSS "ROCKY" C. ANDERSON, Justice ◯	GARY JOHNSON, Libertarian ◯
BARACK OBAMA, Democratic ◯	ROSS "ROCKY" C. ANDERSON, Justice ◯
GARY JOHNSON, Libertarian ◯	BARACK OBAMA, Democratic ◯

Figure 2: A ballot to elect the President of the United States, using the Random Pairs voting rule with five pairs.

In order to fully express a voter's preferences and let them weigh in on every pair of candidates, an RP ballot would need to have $p = \binom{\alpha}{2}$ entries, where α is the number of candidates. With p as low as $\alpha - 1$, a full linear order could be recovered, though it would be unlikely that the voter would receive the exact pairs necessary. In order to drive down the communications complexity further still, this work fixes p to $\lceil \log_2 \alpha \rceil$. Even with this extremely low p, the results below show that RP still selects the Condorcet winner more often than the other voting rules.

The drawback to the RP rule is that it can violate the Pareto and unanimity conditions [15]. Pareto requires that a voting rule must not select a winner when some other candidate is preferred over it by every voter. Unanimity requires that a voting rule must select as a winner the candidate strictly that is preferred to all other candidates by every individual voter, respectively. I assert without proof that these violations of Pareto and unanimity occur with very low probability when voters are presented with too few ballots containing the unanimously-preferred candidate than, say, the second-most-preferred candidate. This probability decreases as the number of voters increases.

2.1 Definitions

A is defined as the set of all candidates and N as the set of voters with $|N| = n$ and $|A| = \alpha$.

Definition 1. The *plurality* voting rule allows each voter to vote for no more than a single candidate. After all ballots are cast, the candidate who receives the most total votes is chosen as the winner.

Definition 2. The *2-approval* voting rule allows each voter to vote for up to two candidates. After all ballots are cast, the candidate who receives the most total votes is chosen as the winner.

Definition 3. The *Borda* voting rule method asks voters to construct a strict linear ordering of the candidates and assign each one points such that the most-desirable candidate receives $\alpha - 1$ points, the second-most-desirable candidate receives $\alpha - 2$ points, and so on, until the least-desirable candidate receives zero points. After all ballots are cast, the candidate with the most total points is chosen as the winner.

Definition 4. The *range* balloting method asks voters to provide a numeric score in a range between zero and some constant r. Candidates receiving a score of zero are the least desirable and those receiving a score of r are the most desirable. It is possible for all candidates to receive duplicate (tie) scores. After all ballots are cast, the candidate with the highest aggregate score is chosen as the winner.

Definition 5. For every pair of candidates i and j such that $i, j \in A$, the *Condorcet* voting rule compares the number of voters that prefer i over j to the number of voters who prefer j over i. There is a winner (the *Condorcet winner*) only if one candidate beats every other candidate in such pairwise match-ups.

Definition 6. The *Copeland* voting rule resembles the Condorcet rule, but selects as the winner the candidate with the best record in pairwise matchups, calculated as the number of pairwise wins minus the number of pairwise losses.

Definition 7. The *Random Pairs* voting rule[4] presents each voter a ballot constructed by selecting p pairs of candidates i_k and j_k uniformly at random such that $i_k \neq j_k$ and $i_k, j_k \in A \; \forall \; k \in \{0, 1, \ldots, p-1\}$. The voter casts a vote for at most one candidate in each pair. RP selects as the winner the candidate with the best record in such pairwise matchups, calculated as the number of pairwise wins minus the number of pairwise losses. As p grows, RP converges to the Copeland rule.

In this work, voters always cast a vote for one candidate for the plurality voting rule and for two candidates in 2-approval. That is, there is no abstention. In the event that more than one winner is selected by any voting rule, ties are broken by selecting at random one of the tied winning candidates as the single winner.

3 Communication Complexity

Conitzer and Sandholm established upper and lower bounds for a number of common voting rules [3], including many of those currently under consideration. While the definitions above do not include quite the same formulations of the various rules as Conitzer and Sandholm, the proofs below follow their lead.

Theorem 1. *The deterministic communication complexity of the* 2*-approval rule has an upper bound of* $O(n \log \alpha)$.

Proof. Much like plurality, each voter must communicate a vote for some candidate with $\log\alpha$ bits. Because there are two votes per voter, each voter must communicate $2 \cdot \log\alpha$ bits, for a total of $2n \cdot \log\alpha$, or $O(n \log \alpha)$. □

Theorem 2. *The deterministic communication complexity of the range rule has an upper bound of* $O(n\alpha)$.

Proof. Providing a rating between zero and some constant r requires $\log r$ bits per candidate. For all voters to provide a rating for each candidate requires the communication of $n\alpha \log r$ bits, but since r is a constant and does not grow with the number of candidates or voters, the communication complexity is $O(n\alpha)$. □

Theorem 3. *The nondeterministic communication complexity of the RP rule is* $O(n(\log \alpha)^2)$.

Proof. Because each voter's ballot is generated at random, the election official producing the ballot must first communicate the ballot to each voter. To communicate one pair requires $2{\cdot}\log \alpha$ bits, so to communicate p pairs to each of n voters requires $2np{\cdot}\log \alpha$ bits, or an upper bound of $O(np \log \alpha)$. Each voter then indicates a preferred candidate for each of $\lceil\log \alpha\rceil$ random pairs of candidates. This requires $\log \alpha$ bits per user, for a total of $O(n \log \alpha)$.

The complexity of communicating the ballots *to* the voters dominates the complexity of the voters returning their ballots. When p is set to $\log \alpha$ (as suggested earlier), the complexity of the RP rule is $O(n(\log \alpha)^2)$. □

[4]As one reviewer noted, RP could be reasonably referred to as "approximate Copeland".

Voting Rule	Upper Bound
Plurality	$O(n \log \alpha)^*$
2-approval	$O(n \log \alpha)$
Random Pairs	$O(n(\log \alpha)^2)$
Range	$O(n\alpha)$
Borda	$O(n\alpha \log \alpha)^*$

Table 1: Communication complexities of the voting rules under consideration where the number of voters is n, the number of candidate is α, and $p = \log \alpha$. The complexities indicated with an asterisk are from [3].

4 Election Data & Results

In order to concretely compare the voting rules, I considered using synthetic data of the same type as Merrill [10], but elected instead to follow in the footsteps of Mattei et al [8] by constructing elections from existing real-world preference data. Because the RP voting rule provides less of a benefit in races with few candidates,[5] the three- and four-candidate elections derived from the Netflix prize data are unsuitable. I instead used Yahoo's collection of musical artist ratings [18] to construct seven-candidate elections.

4.1 The Data

The Yahoo! Webscope R1 data set provides 11,557,943 ratings from 1,948,882 anonymized users across 98,211 artists sampled over a 30-day period [18]. The provided ratings range from 0 to 100 (inclusive) with the rating 255 as a special case meaning "never play again".[7] The users have an incentive to submit accurate ratings, since, like the Netflix data, accurate ratings result in better future music recommendations. As it is clearly infeasible to exhaustively compute all possible seven-candidate elections,[8] I opted to use some shortcuts to produce usable data. What follows is the rough sequence that led to the final data set.

I began in a similar way as in [7] by selecting three sets of 2000 non-overlapping artists with at least 350 ratings each. For each of these three sets, I then enumerated all collections of three artists that had at least 350 users rating all three of them. Statistical results of this enumeration of the three samples are shown in Table 2. The first sample produced 102,850,561 three-candidate elections, the second produced 90,992,753 elections, and the third produced 79,639,473 elections.

Despite having ten times fewer ratings, four times as many users, and more than five times as many items to rate as the Netflix data set, there was a much greater overlap of user ratings in this data set, as demonstrated by the number of three-candidate elections that were generated. There was so much overlap that extending the 100 million triples to five-artist tuples with more than 350 shared ratings from just *the first* of the sets of triples would have produced on the order of 10^{10} elections, required more than a terabyte of disk space, and taken more than a year of computation. Admittedly, parallelizing the search and

[5] When there are three candidates, there are three possible pairings. In the analysis below, the p pairs presented to each voter are selected *with replacement*, which allows a ballot to have multiple instances of the same matchup.[6] Using the previously-mentioned formula, p is set to 2 and the user returns 2/3 of a full ordering. With seven candidates, there are 21 possible pairings and $p = \lceil \log_2 7 \rceil = 3$, which allows the user to communicate just 1/7 of his or her preferences. That small of a proportion seems suitable to demonstrate that collecting just a few partial preferences from many voters is enough to show the effectiveness of the rule.

[7] For the purposes of this analysis, ratings of 255 are interpreted as ratings of 0.

[8] This would require checking approximately 10^{31} 7-tuples.

Sample #	Min	1st Q	Median	Mean	3rd Q	Max
1	350	418	531	798.7	783	102,700
2	350	412	513	736.7	733	162,000
3	350	408	503	688.3	702	94,400

Table 2: Statistical properties of the number of voters in three-candidate elections produced by the three samples of 2000 artists.

Sample	Min	1st Q	Median	Mean	3rd Q	Max
7-candidate, 60K+ voters	60,000	61,930	65,080	67,610	71,070	118,600

Table 3: Statistical properties of the of the number of voters in seven-candidate elections with more than 60,000 voters produced by sampling the 500 artists with the most ratings.

providing enough disk space could make this feasible, but extending the 5-tuples to all six- and seven-artist tuples would definitely be infeasible.

After concluding that the initial approach would produce too large a search space, I chose from the original data the 500 artists with the most ratings and calculated all the triples that had more than 10,000 shared ratings. This produced nearly 8 million triples. After several tests of various thresholds based on time and space constraints, I settled on enumerating all 7-artist tuples that shared at least 60,000 ratings. That allowed me to produce and evaluate 61,566 elections of 7 candidates, each of which has at least 60,000 voters, in a reasonable amount of time and disk space. Descriptive statistics for this final data set are provided in Table 3.

Even with a range of possible values from 0 to 100, users' reviews featured many identical ratings, with nearly 30% of the total ratings as zeroes, about 24% as 90, and about 12% as 100. This produces many ties within the constructed ballots. These ties were broken in the Netflix data by simply selecting the lowest internal identifier [8].[9] In three-candidate races like those constructed from the Netflix data set, this approach may have been feasible, but with more candidates and many more voters in elections, this approach skews the election results so much that during the initial test runs, the plurality winner almost always selected the candidate with the lowest internal identifier. Instead, ties are broken between identically-rated artists randomly, so that every artist with an equally-high rating has an equal chance of winning.[10] Ties in the final tally of votes (or score) in plurality, 2-approval, Borda, range, and RP elections were also broken randomly.

4.2 Condorcet Efficiency

The Condorcet winner in a single-winner election is the candidate that would defeat every other candidate in pairwise elections [1]. Given a complete preference ordering for each voter, this winner is typically considered the best alternative. However, many balloting methods do not collect a complete preference ordering, and therefore might not select as a winner the Condorcet winner.[11] For such methods, a measurement of their effectiveness with wide appeal is *Condorcet efficiency* [11].

The Condorcet efficiency of some voting rule is the fraction of elections where the rule

[9] In the Netflix set, this is a unique numeric identifier given to each movie. In the Yahoo set, this is a unique identifier given to each artist.

[10] Practically speaking, I shuffled the order of the artists on every user's ballot.

[11] Put another way, these voting rules are not *Condorcet consistent.*[12]

Rule	CE, $n \geq 60K$	CE, $n \geq 70K$	CE, $n \geq 80K$	CE, Merrill [11]
Plurality	0.606	0.575	0.600	0.520
2-Approval	0.712	0.680	0.676	0.637[12]
Range	0.774	0.812	0.810	N/A
Borda	0.815	0.772	0.777	0.853
Random Pairs	0.954	0.954	0.951	N/A

Table 4: Condorcet efficiencies (CE) of several voting rules. 61,566 elections were evaluated, each with seven candidates and a minimum of 60,000 voters. Of these elections, 70 did not have a Condorcet winner. Of the remaining 61,496, the above indicates the percentage of elections that the indicated voting rule selected the Condorcet winner. The second and third columns indicate the Condorcet efficiency of each voting rule when only the elections with the given minimum number of voters ($\geq$70,000, and $\geq$80,000, respectively) are considered.

selects a winner that is also the Condorcet winner. When there is no single Condorcet winner, the election is not considered in the calculation of Condorcet efficiency. In the case of the 61,566 elections conducted, there were only 70 elections without single Condorcet winners, which accounts for a paltry 0.11% of all ballots.

Unlike in [8], strict orders are not enforced, and ties are not broken when computing a voter's contribution to the pairwise tallies leading to the selection of the Condorcet winner. As noted previously, this is in part due to the number of ties found in user ratings in this data set. The results here show the various voting rules agreeing far less often than in previous studies that constructed elections from real-world data. Table 4 shows the Condorcet efficiency of each voting rule across all the elections.

It is striking how closely the empirical results match Merrill's "random society"[13] results with seven candidates [11]: Plurality is 8.6% higher, Approval is 7.5% higher, and Borda is 3.8% lower than the Condorcet efficiencies found in the synthetic elections.[14] One major difference, though, is how few elections (0.11%) derived from the Yahoo! data had no Condorcet winners, when "random society" elections of seven candidates have no Condorcet winner 35.7% of the time. When the elections that have fewer than 70,000 voters are removed, the Condorcet efficiencies change slightly, but are otherwise fairly stable. The same holds true when the elections with fewer than 80,000 voters are removed. The only minor result from looking at different numbers of voters is that RP's Condorcet efficiency is much more stable than the others regardless of the number of voters. Though additional simulations with different numbers of candidates and voters would provide a clearer view of its performance, these initial results are encouraging. It is clear, though, that the RP rule performs significantly better than all of the others tested, even with its lower communication complexity.

[12] Admittedly, Merrill uses a slightly different formulation, while voters in this analysis are required to cast exactly two votes.

[13] In these synthetic elections, each voter has a *utility* for each candidate that indicates how strongly a he or she prefers each candidate. In a random society, these utilities are selected uniformly (and independently) at random within some interval. Because all preference orders are therefore equally likely, random societies are also *impartial cultures*[4].

[14] A reviewer noted that this result seems to conflict with the findings in [16], which find that impartial cultures are a worst-case scenario, and any alternative should provide a higher Condorcet efficiency. Aside from the reviewer's suggestion that it could be due to the high number of ties, there are no obvious explanations at present.

5 Conclusions & Future Directions

5.1 The RP Voting Rule

In the elections constructed from the Yahoo! artist data, the RP voting rule selects the Condorcet winner more often than any of the others, and its Condorcet efficiency remains fairly stable across some variations in the number of voters. In addition to beating all the other voting rules in terms of Condorcet efficiency, RP also is slightly worst than than the two best rules in terms of communication complexity.

Several future directions are immediately evident. The seven-candidate elections with many voters offer a glimpse into the performance of RP, but it is likely that the measured performance will change when there are (a) fewer voters, (b) different numbers of candidates, and (c) other values of p. Intuitively, RP should perform worse with fewer voters due to its reliance on probability and the law of large numbers. Perhaps 95% is the Condorcet efficiency to which it converges given the current values of p and α.[15] With more candidates, RP should perform worse, too.[16] And finally, increasing p should, by virtue of providing a more complete picture of every user's preferences, increase the Condorcet efficiency at the expense of also increasing the communication complexity. By fixing all other parameters, this approach could allow some measurement of the tradeoffs between the two.

Because of its inherent randomness, the RP rule intuitively seems like it should be resistant to strategic voting, and determining if this is indeed the case seems worthwhile. Re-running the elections and determining how often the Condorcet loser [13] is elected by RP (and the other voting rules, for that matter) could also provide supporting evidence for RP's usefulness.

Finally, one reviewer noted that it may also be the case that generating the pairs uniformly at random (as previously discussed) is not the best method to produce ballots that are most likely to result in the selection of the Condorcet winner. It might be better to "cover" as many candidates as possible given the p pairs that are to be generated, and avoid duplicating information that could be inferred from previously-generated pairs.

5.2 The Constructed Elections

The elections constructed for testing the voting rules feature more candidates and more than two orders of magnitude more voters than similar analyses. These elections produce Condorcet efficiency results for several common voting rules that very closely match the results generated by Merrill's "random society" synthetic model. However, the prevalence of ties in the data set could be affecting the usefulness of the results. A reasonable next step to evaluate how large an effect the ties had would be to eliminate all voters that had any ties at all, and re-evaluate the elections with only those voters that had distinct ratings for each of the seven candidates.

Next steps with the election data include converting it to PrefLib's "Orders with Ties, Complete List" format [9] and making it available to the community at large, since these elections may be useful to other researchers. I also hope to construct elections with more candidates by extending the existing 7-candidate tuples. Producing 10-candidate elections would allow comparison with the Condorcet efficiency of Merrill's largest simulated elections. Similarly, smaller elections (in terms of voters or candidates) may easily be constructed by sampling the election data produced here. It also remains to be seen whether the Condorcet

[15] A reviewer offers the alternative explanation that perhaps the remaining 5% of races illustrate the case when first- and second-place candidates are nearly tied and sampling such a small portion of the voters may not select the Condorcet winner reliably enough.

[16] This would certainly follow the patterns of Condorcet efficiency as the number of candidates increases shown in [10].

efficiencies of other voting rules measured by Merrill (Runoff, Hare/IRV, and Coombs) match between the synthetic data and the Yahoo! data.

Given that the users' original artist ratings are readily available and can be used directly as utilities, this analysis can be extended to compare voting rules' social utility efficiencies [11] and related efficiency measurements using non-synthetic election data.

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