Impartial decision making among peers

Herve Moulin, *Rice University*

COMSOC 2010, University of Dusseldorf
September 15, 2010

conflict of interest in collective decision making:

my selfish interest *corrupts* the report of my subjective opinion

non corrupted information is more valuable: it produces an *impartial evaluation* *conflict of interests pervasive in collective decisions by and about peers*

example: evaluate the merit of a peer's work, choose a winner among us, a ranking of us all

a necessary condition for the possibility of an impartial process:

- separate aspects of the decision related to *self interest* versus *opinions/views*

then a decision rule creates no conflict of interest if it only elicits opinions, and an agent's report **does not affect** her self interest examples where the separation is plausible

	self-interest	*opinions*
division of a dollar	my share	division of the remainder
award of a prize	do I win?	who wins if not me?
ranking by peers	what is my rank?	ranking of the others
biased jury	does one of mine win?	who wins among mine/others?

- *Impartial division of a dollar,* G. de Clippel, H. Moulin and N. Tideman, *Journal of Economic Theory, 2008.*

- *Impartial award of a prize,* R. Holzman and H. Moulin, mimeo September 2010

- strategyproof and efficient allocation of private goods: Kato and Ohseto (building on the work of Hurwicz, Zhou, Serizawa and Weymark,..)

model 1: award of a prize

$i \in N = \{1, 2, \cdots, n\}$

i's message $m_i \in M_i$

award rule: $M_N \ni m \rightarrow f(m) \in N$

$\rightarrow$ **Impartiality**: $f(m|^i m_i) = i \Leftrightarrow f(m|^i m_i') = i$, for all i, m_i, m_i'

additional requirements:

- **No Discrimination**: $\forall i\ \exists m\ f(m) = i$

- **No Dummy**: $\forall i\ \exists m_i, m'_i, m_{-i} : f(m|^i m_i) \neq f(m|^i m'_i)$

both are (very) weak forms of symmetry among participants

note: full Anonymity impossible

Lemma (easy):

For $n \leq 3$ Impartiality $\cap$ No Discrimination $=$ Impartiality $\cap$ No Dummy $= \varnothing$

For $n = 4$, assume binary messages $m_i = 0, 1$

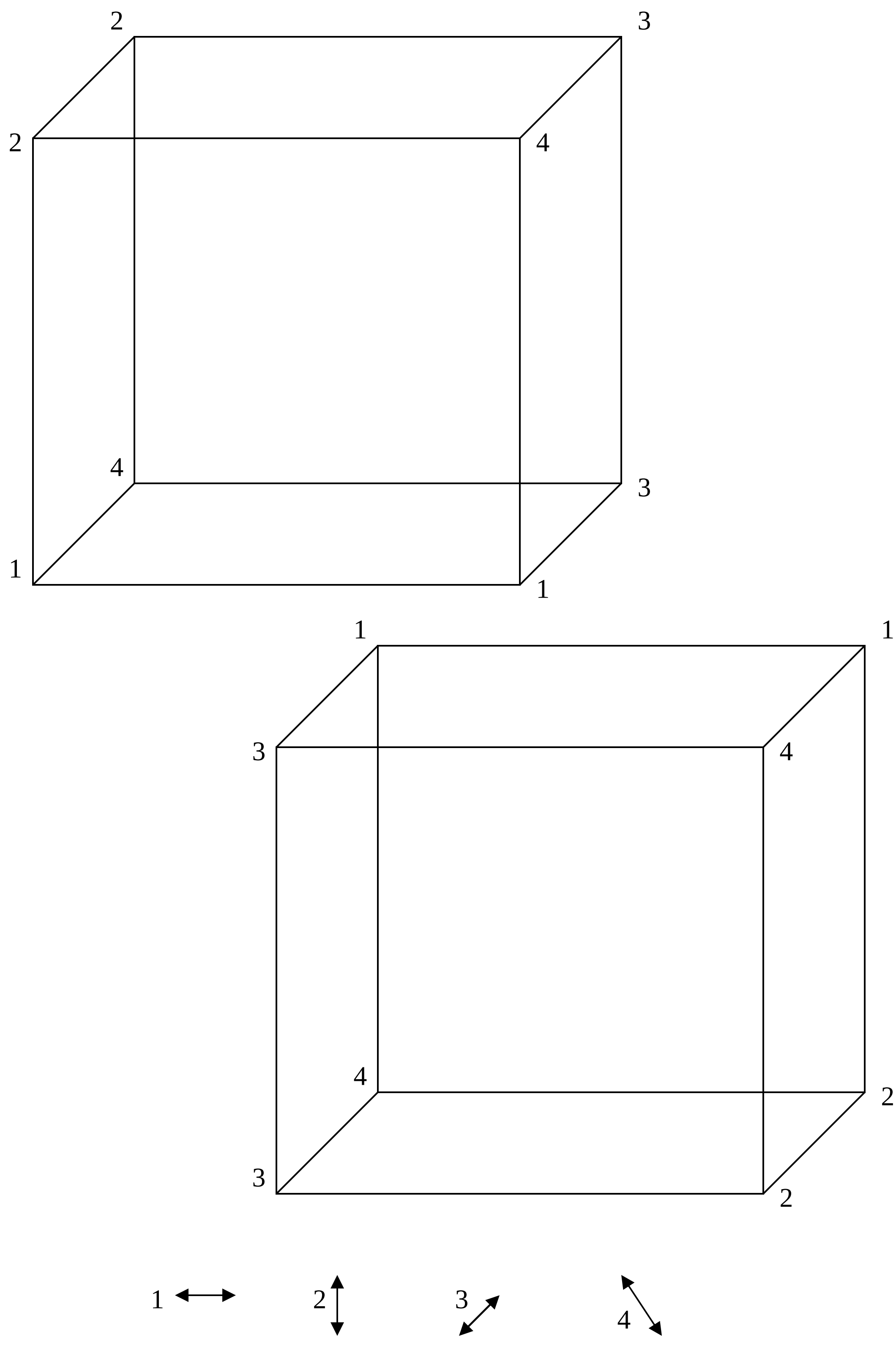
$$\text{Impartiality} \cap \text{No Discrimination} \cap \text{No Dummy} = \{f^4\}$$

up to relabeling agents and messages

$f^4(\cdot, 0, 0, 0) = f^4(\cdot, 1, 1, 1) = 1$; $f^4(0, \cdot, 1, 0) = f^4(1, \cdot, 0, 1) = 2$

$f^4(1, 1, \cdot, 0) = f^4(0, 0, \cdot, 1) = 3$; $f^4(0, 1, 0, \cdot) = f^4(1, 0, 1, \cdot) = 4$

for $n \geq 5$, there are many more rules


2
3
2
4
4
3
1
1
1
1
3
4
4
2
3
2
1
2
3
4

quota rules

everyone but the incumbent nominates someone (no self nomination)

$q > \frac{n}{2}$: *absolute* quota rule $I^{ab}(q)$: i wins if score(i) $\geq q$

$2 \leq q \leq \frac{n}{2}$ *relative* quota rule $I^{r}(q)$: i wins if score(i) $\geq$ score($j|N\diagdown\{i\}$) $+q$ for all $j \neq i$

if no such winner, the incumbent wins

$\rightarrow$ Impartial, No Discrimination, but the incumbent is a *dummy*

combine two of these rules

partition $N = N_1 \cup N_2$; choose q_1, q_2

*step 1:*run $I^{\varepsilon_1}(q_1)$ in N_1; stop if there is a winner

otherwise go to

step 2: N_1 vote to choose the incumbent $j \in N_2$, then run $I^{\varepsilon_2}(q_2)$ in N_2

$\Rightarrow$ Impartial, No Discrimination, No Dummy

critique: unequal influence of N_1 versus N_2

a more precise description of an agent's decision power:

i influences $j \overset{def}{\Leftrightarrow} \exists\, m \in M^N, m_i' \in M^i :\ f(m|^i m_i) = j \neq f(m|^i m_i')$

Full mutual Influence: $\forall i, j \in N$: i influences j

Full Influence $\Rightarrow$ No Dummy and No Discrimination

nomination rules

simple and natural messages: $M_i = N \diagdown \{i\}$ agent i *nominates* j

Monotonicity: $\forall i, j, i \neq j \ \forall m \in M_N$:$f(m) = j \Rightarrow f(m|^i j) = j$

Anonymous ballots*:* for all $m, m' \in M_N$

$\{\forall i \ |\{j \in M^i | m_j = i\}| = |\{j \in M^i | m'_j = i\}|\} \Rightarrow f(m) = f(m')$

Lemma (easy): *the only impartial nomination rules with anonymous ballots are the constant rules*

eschewing the impossibility: restrict the legitimate ballots $M_i \subseteq N \diagdown \{i\}$

$\Rightarrow$ *positional* nomination rules along a tree *example*

order agents by *seniority*

everyone nominates someone **more senior** *than himself*

the youngest nominated agent wins

- impartial, monotonic, anonymous ballots

- discriminates against the most junior

- the most senior is a dummy

the family of **median nomination rules (**n odd, $n \geq 5$)

the agents are the nodes of a tree Γ

Γ is neither a line nor a simple star

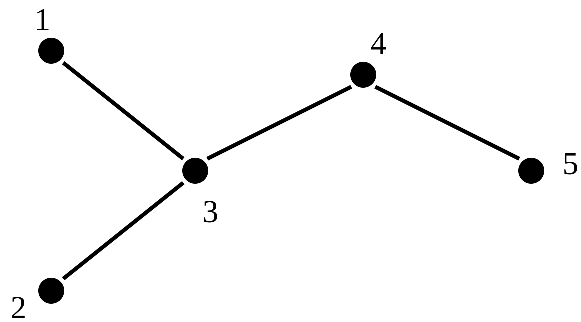
i^* is the median node/agent of Γ

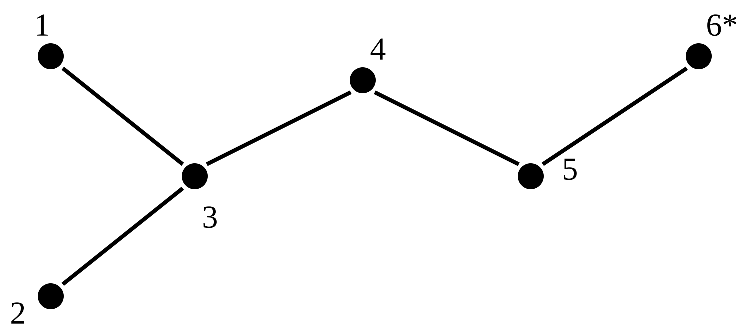
M_i is the largest subtree rooted at j adjacent to i, away from i

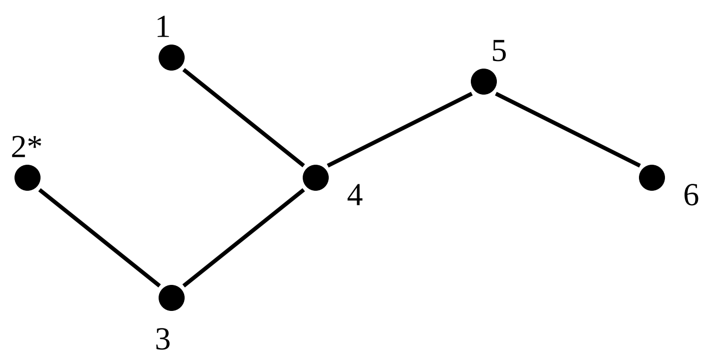
M_{i^*} is *one of* the largest subtrees at j^* adjacent to i^*, away from i^*

$\rightarrow$*winner: the median vote*

n even: add (carefully) a fixed ballot


1
4
5
3
2


1
6*
4
5
3
2


1
5
2*
4
6
3

Theorem:

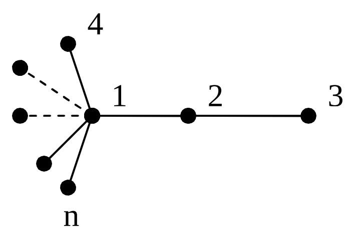
The median nomination rule on Γ *is impartial, monotonic, unanimous and has anonymous ballots; and* i *influences* $j \Leftrightarrow j \in M_i$

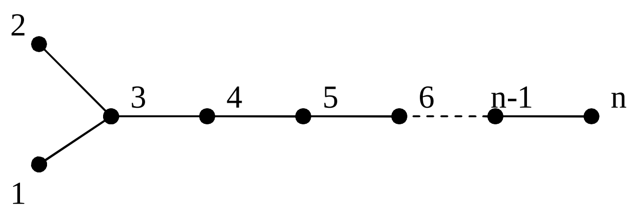
- Unanimity: if all $j \in N \diagdown \{i\}$ such that $i \in M_j$ nominate i, then i wins

the two extreme methods: the quasi-star and the quasi-line

tradeoff: maximize min $|M_i| \leftrightarrow \sum_N |M_i|$

critique: unequal influence


4
1
2
3
n


2
3
4
5
6
n-1
n
1

Open question: can we construct an impartial, monotonic nomination rule meeting No Discrimination and No Dummy?

voting rules

the most natural messages: $M_i = \mathcal{L}(N \diagdown \{i\})$ linear ordering of other agents

- **Monotonicity:** lifting j in i's ranking does not threaten j's win

- **Unanimity:** $\{i =$top$\{m_j\}$ for all $j \in N \diagdown \{i\}\} \Rightarrow i$ wins *the family of* **partition voting rules (**$n \geq 7$)

partition $N = \cup_{k=1}^{K} N_k$ *in* **districts** *s. t.* $|N_1| \geq 4$ *and* $|N_k| \geq 3$ *for* $k \geq 2$

for each k *choose a quota rule* $I^{\varepsilon_k}(N_k, q_k), \varepsilon_k = ab, r$

choose a default agent i^* *in district* 1

two equivalent definitions: direct voting, or two steps voting

Step 1

run $I^{\varepsilon_k}(N_k, q_k)$ *in each district* $k \geq 2$: *call* i *a local winner if she wins*

call i^* *a local winner if he wins in* $I^{\varepsilon_1}(N_1, q_1)$

call $i \in N_1 \diagdown \{i^*\}$ *a local winner if she wins without* i^**'s support*

$$\text{if } \varepsilon_1 = ab : \; s_i(N_1 \diagdown \{i, i^*\}) \geq q_1$$

$$\text{if } \varepsilon_1 = r : \; s_i(N \diagdown \{i, i^*\}) \geq s_j(N \diagdown \{i, j\}) + q_1 \text{ for all } j \in N_1 \diagdown \{i\}$$

If there is no local winner anywhere, i^* *wins*

if there is a single local winner, she wins; otherwise go to

Step 2 All the non local winners use a standard voting rule to award the prize to one of the local winners.

Theorem

A partition voting rule is impartial, unanimous, and has full mutual influence. If it uses an absolute quota in district 1, or if $|N_1| = 4$*, the rule is monotonic.*

under *Impartial Culture* the probability that at least a local winner exists goes to 1 if the district size remains bounded while n increases.

$\Rightarrow$ the advantage of the default agent vanishes *variant:* strengthen Full Influence to Full Pivots:

agent i can be pivotal between j and k, for all i, j, k

$\longrightarrow$ more complex variants of the partition rules

two vague open questions

- what is the *special role* of median rules among anonymous monotonic nomination rules?

- can we find impartial rules *more equitable* than the partition voting rules?

model 2: peer ranking

assign n ranks to n agents

private consumption of one's rank

$i \in N, a \in A$

$\mathbf{\Sigma}(N, A) \ni \sigma$: bijection $N \rightarrow A$

i's message $m_i \in M_i$

assignment mechanism: $M_N \ni m \rightarrow \theta(m) \in \mathbf{\Sigma}(N, A)$

- **Impartiality**: $\theta(m|^i m_i)[i] = \theta(m|^i m_i')[i]$, for all i, m_i, m_i'

- **Full Ranks :** for all $i \in N$, $a \in A$, for some $m \in M_N$: $\theta(m)[i] = a$

- **Full Range:** for all $\sigma \in \boldsymbol{\Sigma}(N, A)$ for some $m \in M_N$: $\sigma = \theta(m)$

Lemma (easy):

For $n = 3$, Impartiality $\cap$ Full Ranks $= \varnothing$

For $n = 4$, Impartiality $\cap$ Full Ranks $\neq \varnothing$

$M^i = \{0, 1\}$ for all i, $A^* = \{1, 2, 3, 4\}$

$\theta^4(0,0,0,0) = 1234$; $\theta^4(1,0,0,0) = 1432$; $\theta^4(0,0,0,1) = 1324$; $\theta^4(1,0,0,1) = 1$

$\theta^4(0,0,1,0) = 2134$; $\theta^4(0,1,1,0) = 2143$; $\theta^4(0,0,1,1) = 2314$; $\theta^4(0,1,1,1) = 2$

$\theta^4(1,1,0,0) = 3412$; $\theta^4(1,1,1,0) = 3142$; $\theta^4(1,1,0,1) = 3421$; $\theta^4(1,1,1,1) = 3$

$\theta^4(0,1,0,0) = 4213$; $\theta^4(0,1,0,1) = 4321$; $\theta^4(1,0,1,0) = 4132$; $\theta^4(1,0,1,1) = 4$

fairly symmetric treatment of the agents

range is not full (15 assignments)

use $\theta^4 \longrightarrow$ **an impartial mechanism with full ranks** for any n divisible by 4

fix a partition $N = N_1 \cup N_2 \cup N_3 \cup N_4$ with $|N_i| = \frac{n}{4}$ and an order τ of A

play θ^4 with agents in N_i jointly playing the 1st coordinate 0 or 1

N_i gets rank/object 1 $\Rightarrow$ the first $|N_i|$ ranks in τ go to N_i; etc..

agents in $N \diagdown N_i$ jointly choose the assignment of these $|N_i|$ ranks inside N_i

construct **an impartial mechanism with full range**

$\rightarrow$*separating family* in A : $\mathcal{S} \subset 2^A$ such that

for all $a, b \in A, a \neq b$, there exists $S \in \mathcal{S}$: $a \in S, b \notin \mathcal{S}$

$\rightarrow$separating family of size k: for all $S \in \mathcal{S}$: $|S| = k$

Lemma:

For $n = |A| \geq 6$, we can find three **pairwise disjoint** separating families in A, all of identical size.

For $n \leq 5$, we can find at most two such disjoint families.

$A = \{a, b, c, d, e, f\}$

$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$
abc	abd	abe
bcd	bce	bcf
cde	cdf	acd
def	ade	bde
aef	bef	cef
abf	acf	adf

$|A| \geq 7, A = \{1, 2, \cdots, n\} \Rightarrow$ for $1 \leq t < \frac{n}{2}$ $\mathcal{S}_t = \{(a, a+t) | a \in A\}$ are separating and pairwise disjoint *choose three "leaders' agents* $1, 2, 3$

step 1: the leaders choose impartially three ranks for themselves

key: all assignments of $\{1, 2, 3\}$ *to* A *are in the range*

step 2: the leaders choose $i \in N \diagdown \{1, 2, 3\}$ and assign her one of the free ranks;

agent i chooses $j \in N \diagdown \{1, 2, 3, i\}$ and assign him one of the free ranks;

etc...

step 1 explained:

choose three separating families $\mathcal{S}_i, i = 1, 2, 3$, of identical size, pairwise disjoint

each leader chooses $S_i \in \mathcal{S}_i$; given $(S_1, S_2, S_3) \in \mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{S}_3$

assign 1 to a rank in $S_3 \cap S_2^c \neq \varnothing$

assign 2 to a rank in $S_1 \cap S_3^c \neq \varnothing$

assign 3 to a rank in $S_2 \cap S_1^c \neq \varnothing$

break ties in $S_3 \cap S_2^c$ by an onto vote of leaders 2 and 3

break ties in $S_1 \cap S_3^c$ by an onto vote of leaders 1 and 3

break ties in $S_1 \cap S_3^c$ by an onto vote of leaders 1 and 3

- many variants in step 2

- critique: the three leaders influence the rest of the agents, but not vice versa

Mutual Influence:

$$\forall i, j \in N \ \exists \ m_i, m_i' \in M^i, m_{-i} \in M^{N \setminus i} : \ \theta(m|^i m_i)[j] \neq \theta(m|^i m_i')[j]$$

we can find an impartial assignment mechanism with full range, satisfying Mutual Influence

its definition is more complex

Open question: in the ranking interpretation (as opposed to assignment), the natural message space is $M_i = \mathcal{L}(N \diagdown \{i\})$. Can we achieve the same properties in that format? and Unanimity?