Allocation via Deferred-Acceptance under Responsive Priorities (with Lars Ehlers)

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Düsseldorf, COMSOC: 15.09.2010

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- *Microeconomic Matching Theory*:
  the allocation or exchange of scarce, heterogeneous, indivisible commodities without monetary transfers.

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- more generally coalition and network formation.

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  This led to *inefficiency*.
- 1945 – 1952: shorter decision times at a later time lead to chaotic recontracting (*exploding offers*).

The National Resident Matching Program (NRMP)

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  The new algorithm is a successful example of market design.[1]

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- So without the use of design, the evolution of the market converged towards a good mechanism.
- In 1998, using theory, the new NRMP mechanism (a generalized *applicant proposing deferred acceptance algorithm*) was developed by Roth and Peranson for the NRMP.[2]

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- (R, q) determines an *allocation problem with capacity constraints*.
- An allocation problem where at most one copy of each object type is available is called a house allocation problem.

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- An *allocation rule* φ is a systematic way (a function) to assign an allocation to each problem (R, q).
- We call $\varphi_i(R, q)$ the *allotment of agent i* at allocation $\varphi(R, q)$.

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Definition (Weak Non-Wastefulness)

No agent receives the null object while he would prefer an available object that is not assigned.

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If an agent truncates her preference in a way such that her allotment remains acceptable under the truncated preference, then the allocation does not change.

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Definition (Resource-Monotonicity)

The availability of more real objects ($q \leq q'$) has a (weakly) positive effect on all agents.

Priority Structures

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for example,

$$\succ_x : 1\ 2\ \ldots\ n$$

means that

agent 1 has a higher priority for object type x than agent 2,

agent 2 has a higher priority for object type x than agent 3,

etc.

Responsive Deferred-Acceptance or responsive DA-Rules

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Now, the corresponding *responsive deferred-acceptance* or *responsive DA-rule* always allocates the student/agent-optimal allocation that is obtained by using Gale and Shapley's (1962) student/agent-proposing deferred-acceptance algorithm (as explained on the next slide).

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The final matching is the "agent-optimal" (stable) allocation obtained for $(R, \succ, q)$.

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Definition (Two-Agent Consistent Conflict Resolution)

If in two maximal conflict situations between two agents (comparing $((R^x, R^x, R_{-i,j}), q)$ with $((R^x, R^x, R_{-i,j}), q'))$ one of them receives the object, the conflict is resolved consistently in that it has to be the same agent in both problems who "wins the conflict" and receives the object.

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- Independence of properties (was very tough!).

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Despite the importance of deferred acceptance rules in both theory and practice, few axiomatization have yet been obtained in an object allocation setting with unspecified priorities.

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  They use two new monotonicity properties (*individually rational monotonicity* and *weak Maskin monotonicity*) together with *non-wastefulness* and *population-monotonicity*.

Final Remarks

Despite the importance of deferred acceptance rules in both theory and practice, few axiomatization have yet been obtained in an object allocation setting with unspecified priorities.

- Most papers deal with house allocation problems & efficiency (Ehlers, 02, Ehlers & Klaus 03, 06, 07, 09, Kesten 09, Pápai 00).
- Only other general result: Kojima & Manea (2009): "Axioms for Deferred Acceptance," *Econometrica*, forthcoming. They characterize DA-rules with substitutable priorities (a larger class of rules!).
  They use two new monotonicity properties (*individually rational monotonicity* and *weak Maskin monotonicity*) together with *non-wastefulness* and *population-monotonicity*.
- *The "advantage" of our result*: we characterize the "classic" (= responsive) DA-rules based on priorities that are defined per object type using basic and intuitive properties.