Multivariate Complexity of Swap Bribery

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COMSOC 2010, Düsseldorf

Bribery in elections

spending money to influence the voters' preferences

- pay money to voters/to chair
- campaigning

$\Longrightarrow$ bad/good phenomenon
both hardness and tractability results interesting!

Bribery as a computational problem

Bribery

Input: $\mathcal{E}$-Election $E = (C, V)$, preferred candidate $p \in C$, cost function, budget β.
Question: Is it possible to bribe voters such that p wins, respecting the budget?

In the following: $m = |C| = \#$ candidates
$n = |V| = \#$ votes.

Bribery as a computational problem

Special model considered here:

SWAP BRIBERY [ELKIND, FALISZEWSKI, SLINKO, SAGT 2009]
cost function: every voter assigns certain price for swapping the positions of two *consecutive* candidates in his preference list.

Example: $v : a > b > p$

v's list of costs of swaps:

$c(a \curvearrowleft\!\!\curvearrowright b) = 2 \qquad c(a \curvearrowleft\!\!\curvearrowright p) = 3 \qquad c(b \curvearrowleft\!\!\curvearrowright p) = 1$

briber wants $\tilde{v} : p > b > a$

cost of a set of swaps:

v :	$a > b > p$.	swap $a \curvearrowleft\!\!\curvearrowright b$ at cost 2
$\tilde{v}$:	$b > a > p$.	swap $a \curvearrowleft\!\!\curvearrowright p$ at cost 3
	$b > p > a$.	swap $b \curvearrowleft\!\!\curvearrowright p$ at cost 1
	$p > b > a$.	total cost: 6

Bribery as a computational problem

SWAP BRIBERY

Input: $\mathcal{E}$-Election $E = (C, V)$, preferred candidate $p \in C$, cost functions, budget β.
Question: Is there a set of swaps with total cost $\leq \beta$, such that p wins the bribed election?

for costs in $\{0, \delta > 0\}$, budget $\beta = 0$: POSSIBLE WINNER.

Some known results for Swap Bribery

[Elkind, Faliszewski, Slinko, SAGT 2009]

- hardness results for Borda: **NP-c**

  (from Possible Winner [Xia, Conitzer, AAAI, 2008]),

  Copeland$^{\alpha}$: **NP-c** , Maximin: **NP-c**

- case study for k-approval $(\underbrace{1, 1, \ldots, 1}_{k}, 0, \ldots, 0)$
  - $k = 1$ (Plurality): **P**

    $k = m - 1$ (Veto): **P**
  - $1 \leq k \leq m$, m or n constant: **P**
  - $k = 2$: **NP-c**

    (from Possible Winner, [Betzler, Dorn, J.Comput.Syst.Sci., 2010])
  - $3 \leq k \leq m - 2$, k fixed, costs in $\{0, 1, 2\}$: **NP-c**

- k part of the input: **NP-c** even for 1 voter!

Multivariate complexity analysis of Swap Bribery

so far: complexity measured in size of the input (1-dimensional)

now: complexity measured in size of the input
and certain 'parameters' (multi-dimensional)

e.g.: # candidates
votes
candidates with special property
cost
budget
...

Which parameters have a significant influence on the hardness of the problem?

Multivariate complexity analysis of SWAP BRIBERY

t - parameter

NP-hard problems: presumably cannot avoid exp. running times.

But: Maybe we can restrict exponential part of running time to a certain parameter! E.g. $2^t \cdot |x|^2$
$\Rightarrow$ If value of t ist small in certain settings: efficient algorithm!

fixed-parameter tractability

A problem is *fixed-parameter tractable* if it can be solved in

$$f(t) \cdot \text{poly}(|x|) \text{ time}$$

($|x|$ - size of the input)

corresponding complexity class: **FPT**

What about running time $|x|^t$? **not** in **FPT**!

Multivariate complexity analysis of Swap Bribery

Intractability results

Hardness classes

First level of fixed-parameter intractability: class W[1]

hardness/completeness via parameterized reduction.

Multivariate complexity analysis of SWAP BRIBERY

Goal: Analyze complexity of SWAP BRIBERY from a parameterized/multivariate point of view.

Special focus on k-approval.

Our investigations

Complexity depending on

(1) cost function, budget
(2) combined parameter ($n = \#$ votes, $\beta =$ budget) } k-approval
(3) $m = \#$ candidates

1. Complexity depending on cost function

k-approval

Theorem 1

Costs uniform (every swap has the same cost):
Swap Bribery for *k*-approval is in **P**

$\rightarrow$ network flow problem

Theorem 2

As soon as there are two different costs:
Swap Bribery for *k*-approval is **NP-c**.
Swap Bribery for *k*-approval is **W[1]-hard** with respect to β

$\rightarrow$ (parameterized) reduction from Multicolored Clique

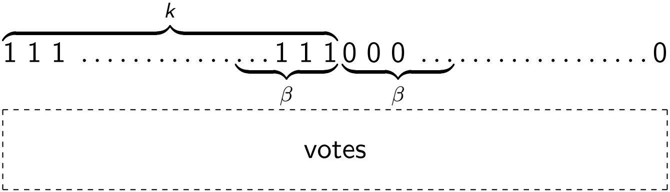
2. Complexity depending on combined parameter (n, β)

k-approval

Theorem 3

If minimum cost of a swap is 1:
Swap Bribery for k-approval is in **FPT** with respect to (n, β)

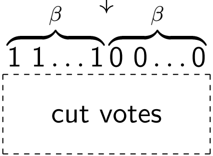
2. Complexity depending on combined parameter (n, β)



minimum cost of a swap $= 1$: only candidates that can be swapped within budget β from 1- to 0-position or vice versa are interesting.

$\Rightarrow$ cut votes (such that only relevant candidates stay)

$\downarrow$



$+$

some more votes that take into account points of ‘lost’ positions

2. Complexity depending on combined parameter (n, β)

$$\overbrace{1\,1\ldots1}^{\beta}\overbrace{0\,0\ldots0}^{\beta}$$

cut votes

$+$

some more votes that take into account points of 'lost' positions

remaining profile is much smaller:

– only $O(n^2\beta^2)$ candidates left

– new votes, but only $O(n^2\beta)$ many of them

$\rightarrow$ brute force on the smaller instance ('problem kernel'), leads to an **FPT** running time

3. Complexity depending on m = number of candidates

Any voting system that can be described by *linear inequalities*, e.g. scoring rules, Maximin, Copeland$^{\alpha}$, Bucklin, Ranked Pairs, ...

Theorem 4

For all voting rules that can be described by linear inequalities: SWAP BRIBERY is in **FPT** with respect to m.

$\rightarrow$ ILP formulation

In a similar way:
Many other problems are in **FPT** with respect to m as well, e.g.

- POSSIBLE WINNER
- MANIPULATION
- CONTROL
- LOBBYING

Summary

Results

Complexity depending on

(1) cost function, budget: **P/NP-c, W[1]-hard (**β**)**
(2) ($n = \#$ votes, $\beta =$ budget): **FPT**
} *k*-approval

(3) $m = \#$ candidates: **FPT**

Summary

What else is interesting?

- different parameters
- different voting systems
- destructive case
- different models?