

Approximate Judgement Aggregation

(for the case of the doctrinal paradox)

Ilan Nehama

Center for the Study of Rationality
The Selim and Rachel Benin School of Computer Science and Engineering
The Hebrew University of Jerusalem, Israel

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Agenda

- Doctrinal Paradox
- Research Question : Approximate Aggregation
- Approximate Aggregation Results
 - for The Doctrinal Paradox
 - for Other Agendas
 - for a Class of Agendas
- Conclusion

Doctrinal Paradox (Unpacking the court/ Kornhauser and Sager 1986)

Suppose a defendant is accused in court of murder. In order to prove his guiltiness, one should convince the judge of two independent issues:

(A) The defendant killed the victim

(B) The defendant is sane

Conviction is defined to be the conjunction of the first two issues

$(A \wedge B)$ The defendant is guilty.

Doctrinal Paradox (Unpacking the court/ Kornhauser and Sager 1986)

A (Killed)	B (Sane)	$A \wedge B$ (Guilty)
0	1	0
1	0	0
1	1	1
0	0	0

Doctrinal Paradox (Unpacking the court/ Kornhauser and Sager 1986)

Agenda



A (Killed)	B (Sane)	$A \wedge B$ (Guilty)	
0	1	0	
1	0	0	
1	1	1	
0	0	0	
0	1	1	← <i>inconsistent</i>
1	0	1	← <i>inconsistent</i>
1	1	0	← <i>inconsistent</i>
0	0	1	← <i>inconsistent</i>

Doctrinal Paradox (Unpacking the court/ Kornhauser and Sager 1986)

	A (Killed)	B (Sane)	$A \wedge B$ (Guilty)
Judge 1	1	0	0
Judge 2	1	1	1
Judge 3	0	1	0
Majority	1	1	0

Notations

A profile $X \in \{0, 1\}^{n \times m}$ $\left(\begin{array}{l} n : \text{Number of voters} \\ m = 3 : \text{Number of issues} \end{array} \right)$

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_i^3 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 \end{array}$$

The opinion of the i^{th}
voter on the 2^{nd} issue

Notations

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array}$$

The i^{th} row X_i represents the **consistent** opinion of the i^{th} voter

Notations

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The j^{th} column X^j represents the opinions of all voters on the j^{th} issue

$$F \left(\begin{array}{ccc} X_1^1 & X_1^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array} \right) = (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$$

An **aggregation mechanism** returns for every profile an **aggregated opinion**

$$F : \{\{0, 1\}^m\}^n \rightarrow \{0, 1\}^m$$

$$F \left(\begin{array}{ccc} X_1^1 & X_1^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array} \right) = (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$$

Definition (Consistency)

F is **consistent** if it returns a consistent result whenever all voters voted consistently

$$a_3 = a_1 \wedge a_2$$



$$F \begin{pmatrix} X_1^1 & X_1^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{pmatrix} = (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$$

Definition (Independence)

F is **independent** if the aggregated opinion of the j^{th} issue depends solely on the votes for the j^{th} issue

$$F \left(\begin{array}{ccc} \mathbf{X}_1^1 & X_1^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ \mathbf{X}_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ \mathbf{X}_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array} \right) = (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$$

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F is **independent** if the aggregated opinion of the j^{th} issue depends solely on the votes for the j^{th} issue

Aggregation Mechanism - Examples

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_1^3 = X_1^1 \wedge X_1^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array}$$

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Issue-wise Majority : $Maj(X^1)$ $Maj(X^2)$ $Maj(X^3)$

Independence: ✓

Consistency: ✗

Aggregation Mechanism - Examples

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_1^3 = X_1^1 \wedge X_1^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array}$$

Premise
Majority : $Maj(X^1) \quad Maj(X^2) \quad Maj(X^1) \wedge Maj(X^2)$

Independence: X

Consistency: ✓

Aggregation Mechanism - Examples

X_1^1	X_1^2	$X_1^3 = X_1^1 \wedge X_1^2$
\vdots	\vdots	\vdots
X_i^1	X_i^2	$X_i^3 = X_i^1 \wedge X_i^2$
\vdots	\vdots	\vdots
X_n^1	X_n^2	$X_n^3 = X_n^1 \wedge X_n^2$
Constant:	0	$g(X^2)$
	0	0

Independence: ✓

Consistency: ✓

Aggregation Mechanism - Examples

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_1^3 = X_1^1 \wedge X_1^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array}$$

Oligarchy: $\bigwedge X^1$ $\bigwedge X^2$ $\bigwedge X^3$

Independence: ✓

Consistency: ✓

Aggregation Mechanism - Examples

$$\begin{array}{ccc} X_1^1 & X_1^2 & X_1^3 = X_1^1 \wedge X_1^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2 \end{array}$$

Independence:

Consistency:

Are there any other consistent and independent aggregation mechanisms?

$\langle A, B, A \wedge B \rangle$ - Oligarchy

Definition (Oligarchy)

An oligarchy of S returns 1
iff all the members of S voted 1.

$$u_S(\bar{x}) = \bigwedge_{i \in S} x_i$$

Theorem

Let F be an independent and consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then there exists three boolean functions

$f, g, h : \{0, 1\}^n \rightarrow \{0, 1\}$ s.t. $F(X) = \langle f(X^1), g(X^2), h(X^3) \rangle$

and $f = h \equiv 0$

or $g = h \equiv 0$

or $f = g = h$ and it is an oligarchy.

This theorem is a direct corollary from a series of works in the more general framework of aggregation.

(E.g., Nehring&Puppe 2007, Holzman&Dokow 2008)

Theorem

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then



Research Question

Theorem

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then



Definition (δ -consistent)

F is δ -consistent if the following test fails with probability at most δ :

- Choose a consistent profile X uniformly at random.
- Check whether $F(X)$ is a consistent opinion.

Research Question

Theorem

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then



Definition (δ -independent)

F is δ -independent if the following test fails with probability at most δ :

Choose a consistent profile X uniformly at random.

Choose an issue j uniformly at random.

Choose a random consistent profile Y s.t. $X^j = Y^j$.

Check whether $(F(X))^j$ equals $(F(Y))^j$.



Theorem

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then



Notice that

0 -consistency \equiv Consistency
 0 -independence \equiv Independence

Theorem

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Then



Notice that

0-consistency \equiv Consistency

0-independence \equiv Independence

Moreover, for $\delta < C \cdot 4^{-n} \approx \frac{1}{\text{Number of profiles}}$

δ -consistency \equiv Consistency

δ -independence \equiv Independence

Research Question

Theorem

Let $\delta > \exp(n, \epsilon)$

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

Notice that

0-consistency \equiv Consistency

0-independence \equiv Independence

Moreover, for $\delta < C \cdot 4^{-n} \approx \frac{1}{\text{Number of profiles}}$

δ -consistency \equiv Consistency

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Research Question

Theorem

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

The other direction is trivial

Theorem

Let F and G be two aggregation mechanisms for $\langle A, B, A \wedge B \rangle$ such that

- G is independent and consistent*
- F and G agree on at least $1 - \epsilon$ of the profiles*

then F is ϵ -independent and 6ϵ -consistent.



Main result for $\langle A, B, A \wedge B \rangle$

Theorem

For any $\epsilon > 0$ and $\delta = \text{poly}(\epsilon, n)$: ($\delta \approx C \cdot n^{-2} \epsilon^5$)

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \wedge B \rangle$.

Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

Techniques - How did we get this result?

- Restricting ourself to independent mechanisms.
- Applying an (agenda independent) technique to extend the result to δ -independence and δ -consistency.

Techniques - How did we get this result?

Given an independent δ -consistent aggregation mechanism $F = \langle f, g, h \rangle$

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Definition (Influence (Banzhaf Power Index))

The **influence** of the i^{th} voter on f is the probability he can change the result by changing his vote.

$$I_i(f) = \Pr[f(x) \neq f(x \oplus e_i)]$$

Definition (Ignorability)

The **ignorability** of the i^{th} voter on f is the probability f returns 1 although i voted 0.

$$P_i(f) = \Pr[f(x) = 1 | x_i = 0]$$

Techniques - How did we get this result?

Given an independent δ -consistent aggregation mechanism $F = \langle f, g, h \rangle$

We show that

- f is an oligarchy iff

$$\forall i : I_i(f)P_i(f) = 0$$

Techniques - How did we get this result?

Given an independent δ -consistent aggregation mechanism $F = \langle f, g, h \rangle$

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-

$$\forall i : I_i(f)P_i(g) \leq 4\delta$$

Techniques - How did we get this result?

Given an independent δ -consistent aggregation mechanism $F = \langle f, g, h \rangle$

We show that

- f is an oligarchy iff

$$\forall i : I_i(f)P_i(f) = 0$$



$$\forall i : I_i(f)P_i(g) \leq 4\delta$$

- Let u be the oligarchy of the voters with small ignorability (either $P_i(f)$ or $P_i(g)$) Then,

f and g are close to u
 F is close to $\langle u, u, u \rangle$.

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- Approximate Aggregation Results
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 - for a Class of Agendas
- Conclusion

- Doctrinal Paradox
- Research Question : Approximate Aggregation
- Approximate Aggregation Results
 - for The Doctrinal Paradox
 - for Other Agendas
 - Preference Agenda
 - XOR Agenda $\langle A, B, A \oplus B \rangle$
 - for a Class of Agendas
- Conclusion

Other Agendas - Preference Aggregation

$a > b$	$b > c$	$c > a$	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
0	0	0	← <i>inconsistent</i>
1	1	1	← <i>inconsistent</i>

Other Agendas - Preference Aggregation

	: $a > b$	$b > c$	$c > a$
Voter 1	: 1	1	0
Voter 2	: 0	1	1
Voter 3	: 1	0	1
Majority	: 1	1	1

Theorem (Condorcet Paradox)

Pair-wise majority might lead to inconsistent outcome.

Other Agendas - Preference Aggregation

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Theorem (Arrow's Theorem 1950)

So is any other non-dictatorial aggregation mechanism that satisfies independence and Pareto.

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So is any other non-dictatorial aggregation mechanism that satisfies independence and Pareto.

Theorem (Kalai 2002 , Mossel 2009)

For any $\epsilon > 0$:

Let F be an independent, $K\epsilon$ -consistent (and balanced) preference aggregation mechanism.

Then there exists an independent and consistent aggregation mechanism G (i.e., dictatorship) that agrees with F on at least $1 - \epsilon$ of the profiles.



Other Agendas - $\langle A, B, A \oplus B \rangle$

A	B	$A \oplus B$	
0	1	1	
1	0	1	
1	1	0	
0	0	0	
0	1	0	\leftarrow <i>inconsistent</i>
1	0	0	\leftarrow <i>inconsistent</i>
1	1	1	\leftarrow <i>inconsistent</i>
0	0	1	\leftarrow <i>inconsistent</i>

Other Agendas - $\langle A, B, A \oplus B \rangle$

Theorem

For any $\epsilon > 0$ and $\delta = \text{poly}(\epsilon, n)$: ($\delta = C \cdot \epsilon$)

Let F be a δ -independent and δ -consistent aggregation mechanism for $\langle A, B, A \oplus B \rangle$.

Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

Techniques - How did we get this result?

- Restricting ourself to independent mechanisms.
- Applying an (agenda independent) technique to extend the result to δ -independence and δ -consistency.

Techniques - How did we get this result?

Given an independent δ -consistent aggregation mechanism $F = \langle f, g, h \rangle$

We describe f, g , and h using Fourier representation and prove that

$$1 - 2\delta = \sum_{\chi} \hat{f}(\chi) \hat{g}(\chi) \hat{h}(\chi)$$

when

- The summation is over all functions χ s.t. $\langle \chi, \chi, \chi \rangle$ is consistent
- $|\hat{f}(\chi)|$ equals $1 - 2d$ for d being the distance between f and χ .

in order to get that F is 'close to' $\langle \chi, \chi, \chi \rangle$.

Theorem

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = \text{poly}(\frac{1}{n}, \epsilon, m)$:

Let \mathbb{X} be a premise-conclusion agenda over m issues in which each issue is either a premise, or a conclusion of at most two premises.

Let F be a δ -independent and δ -consistent aggregation mechanism for \mathbb{X} .

Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

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Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

For instance: $\langle A, B, A \oplus B \rangle$
 $\langle A, B, A \wedge B, A \vee B \rangle$
 $\langle A, B, C, A \wedge B \vee C \rangle$
 $\langle A, B, C, A \wedge B, B \oplus C, A \wedge C \rangle$
 ~~$\langle A \wedge B, B \wedge C, C \wedge A \rangle$~~

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 $\langle A, B, C, A \wedge B, B \oplus C, A \wedge C \rangle$
 ~~$\langle A \wedge B, B \wedge C, C \wedge A \rangle$~~

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Then there exists an independent and consistent aggregation mechanism G that agrees with F on at least $1 - \epsilon$ of the profiles.

- Technique:
- \wedge and \oplus represent all boolean functions of two arguments.
 - Induction over the number of issues.

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- Open question:

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- We proved approximate aggregation theorems for a class of premise conclusion agendas.
- Open question:
 - Find an agenda and an aggregation mechanism that is δ -independent and δ -consistent but is far from any independent consistent aggregation mechanism.

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- We proved approximate aggregation theorems for a class of premise conclusion agendas.
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Thank You

More information

email: ilan.nehama@mail.huji.ac.il

Homepage: www.cs.huji.ac.il/~ilan_n

Paper: <http://arxiv.org/abs/1008.3829>

Please write me any comments/questions/suggestions you have.



מרכז לחקר
הרציונליות
Center for
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