Dependence in Games & Dependence Games

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Part I

Background & outline

Dependence theory in MAS

"One of the fundamental notions of social interaction is the *dependence* relation among agents. In our opinion, the terminology for describing interaction in a multi-agent world is necessarily based on an analytic description of this relation. Starting from such a terminology, it is possible to devise a calculus to obtain predictions and make choices that simulate human behavior" [Castelfranchi 1991]

Outline

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- ☐ *Dependence* in strategic games

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- ☐ *Reciprocity* in strategic games

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- ☐ *Dependence* in strategic games
- ☐ *Reciprocity* in strategic games
- ☐ Reciprocity and game-transformations

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Outline

- ☐ *Dependence* in strategic games
- ☐ *Reciprocity* in strategic games
- ☐ Reciprocity and game-transformations
- ☐ Dependence games: *reciprocity-based coalitional games*

Part II

Dependence in Games

Dependence as *"need for a favor"* (i)

	L	R
U	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

Prisoner's dilemma

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- *Who's benefiting from whom (in a given outcome)?*

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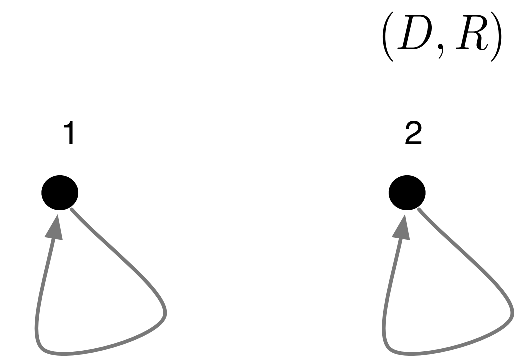

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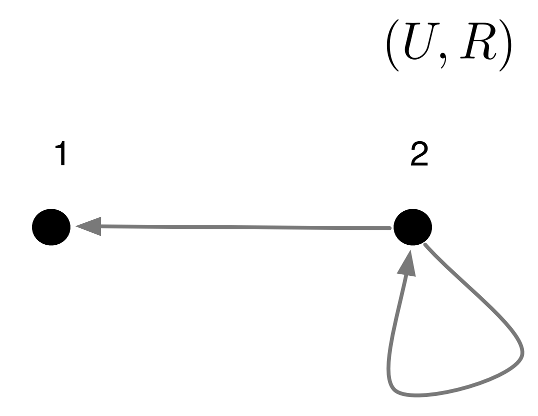

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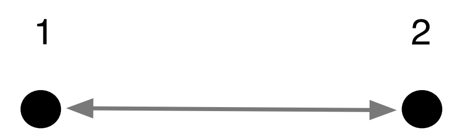
(U, L)



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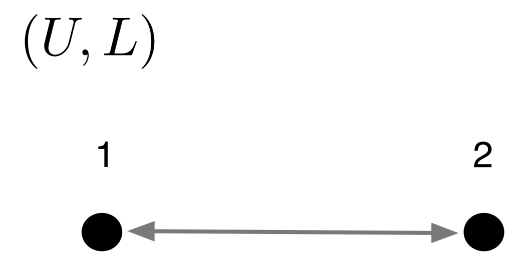

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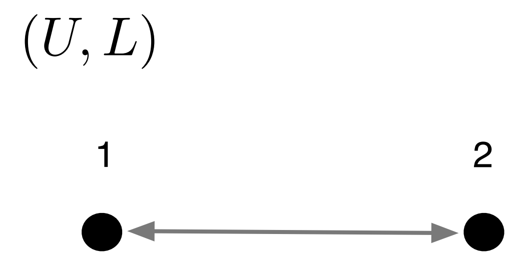

i depends on j for outcome $o(\sigma)$ iff σ_j is a strategy that *favors* i

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Dependence as *"need for a favor"* (ii)

Definition 1 (Best for someone else) *Assume a game $\mathcal{G} = (N, S, \Sigma_i, \succeq_i, o)$ and let $i, j \in N$.*

1. *j's strategy in σ is a* best response for *i iff $\forall \sigma'$, $o(\sigma) \succeq_i o(\sigma'_j, \sigma_{-j})$.*
2. *j's strategy in σ is a* dominant strategy for *i iff $\forall \sigma'$, $o(\sigma_j, \sigma'_{-j}) \succeq_i o(\sigma')$.*

- Generalization of *best response* and *dominant strategy*

Dependence as *"need for a favor"* (iii)

Definition 2 (Dependence) *Let* $\mathcal{G} = (N, S, \Sigma_i, \succeq_i, o)$ *be a game and* $i, j \in N$*.*

1. i BR-depends *on* j *for profile* σ*—in symbols,* $iR^{BR}_{\sigma}j$*—if and only if* σ_j *is a best response for* i *in* σ*.*

2. i DS-depends *on* j *for profile* σ*—in symbols,* $iR^{DS}_{\sigma}j$*—if and only if* σ_j *is a dominant strategy for* i*.*

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- Each outcome of a game encodes a dependence graph
- Every game univocally determines a set of dependence graphs

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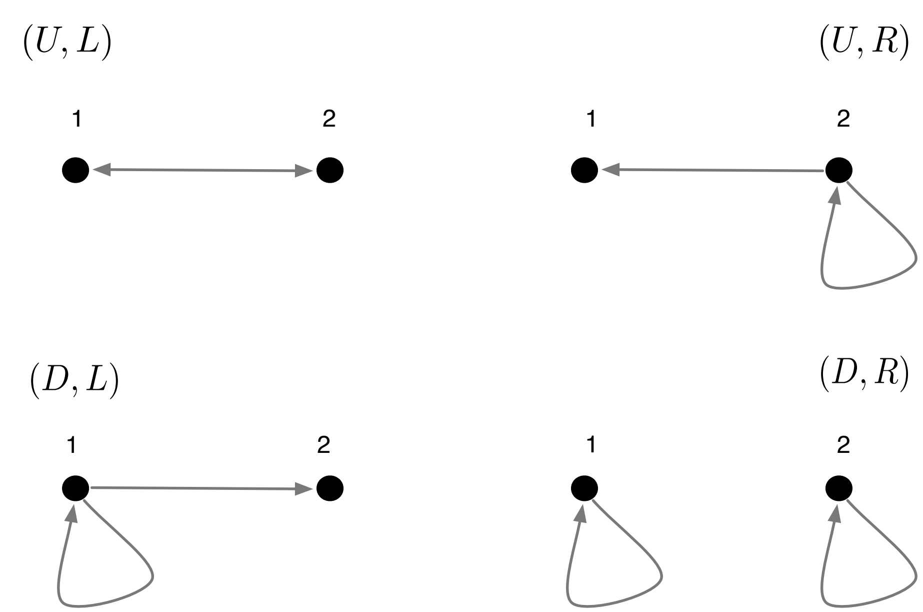

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Part III

Cycles and reciprocity

Cycles and cooperation

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- The existence of dependence cycles signals the existence of parallel interests (*reciprocity*)
- Reciprocity suggests the possibility of cooperation via a *quid pro quod:* I do something for you, you do something for me
- The possibility of such cooperation is characterizable via a very simple kind of game transformation: *game permutation*

Reciprocity

Definition 3 (Reciprocity) *A profile* σ *is* BR-reciprocal *(resp.* DS-reciprocal*) if and only if there exists a partition* $P(N)$ *of* N *such that each element* p *of the partition is the orbit of some* R_σ^{BS}*-cycle (resp., a* R_σ^{DS}*-cycle).*

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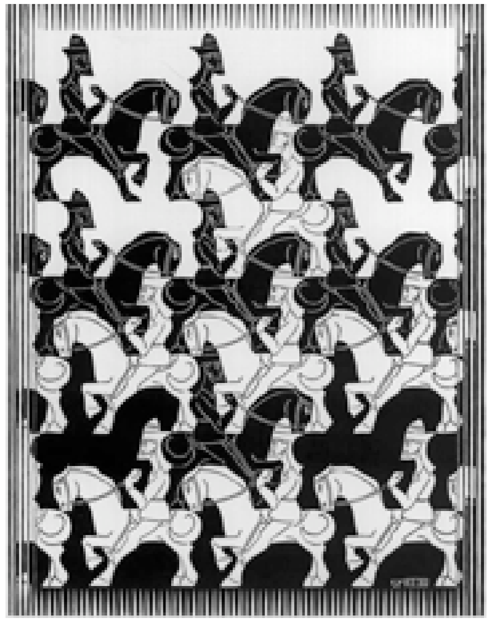
- A profile is reciprocal iff it is partitioned by dependence cycles
- What does the existence of cycles mean from a game-theoretic point of view?

Permuted games (i): The two Horsemen



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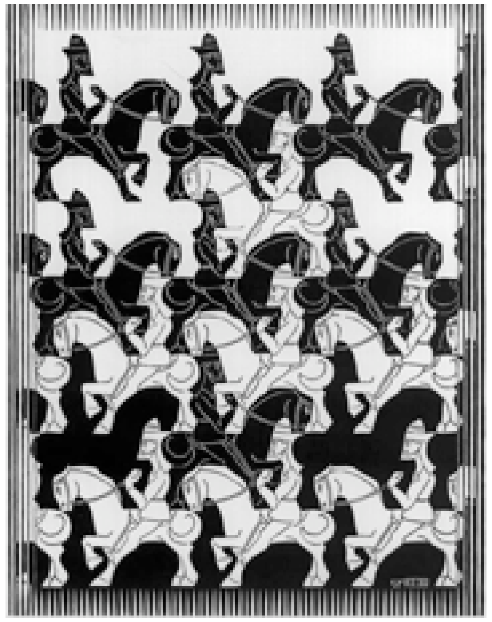

Two horsemen are on a forest path chatting about something. A passerby M, the mischief maker, comes along and having plenty of time and a desire for amusement, suggests that they race against each other to a tree a short distance away and he will give a prize of $100. However, there is an interesting twist. He will give the $100 to the owner of the slower horse. Let us call the two horsemen Bill and Joe. Joe's horse can go at 35 miles per hour, whereas Bill's horse can only go 30 miles per hour. Since Bill has the slower horse, he should get the $100.

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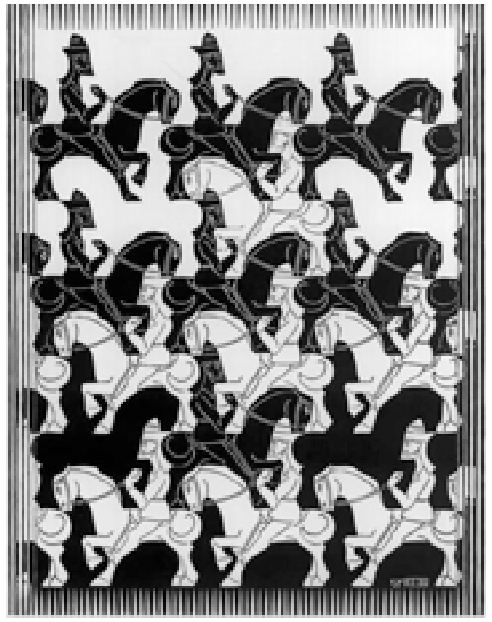

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The two horsemen start, but soon realize that there is a problem. Each one is trying to go slower than the other and it is obvious that the race is not going to finish. [...] Thus they end up [...] with both horses going at 0 miles per hour. [...]

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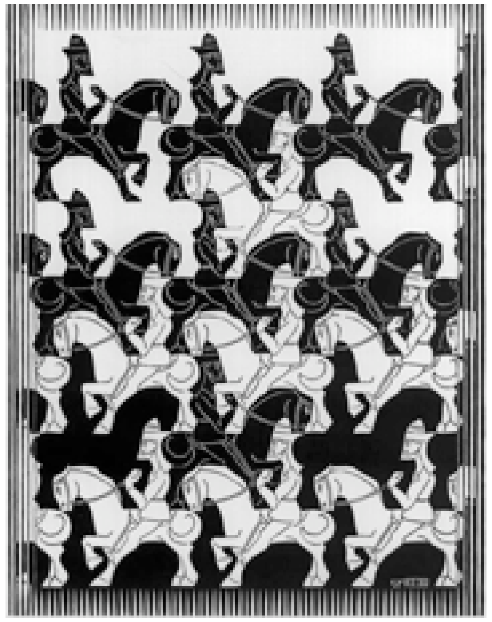

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However, along comes another passerby, let us call her S, the problem solver, and the situation is explained to her. She turns out to have a clever solution. She advises the two men to switch horses. Now each man has an incentive to go fast, because by making his competitor's horse go faster, he is helping his own horse to win!" [Parikh, 2002]

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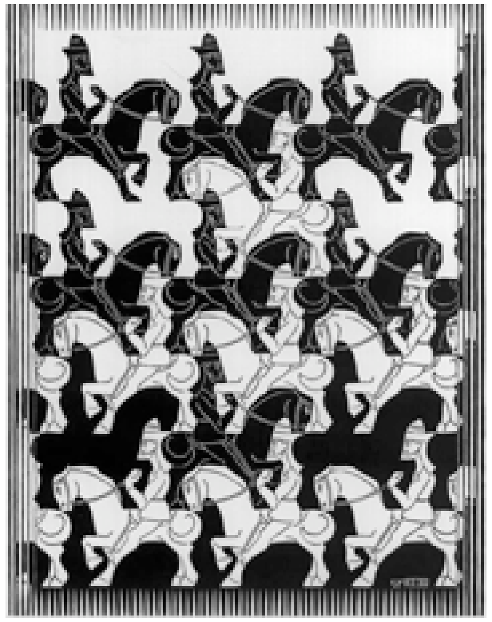

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Permuted games (iii)

Theorem 1 (Reciprocity as equilibrium through permutation) *Let $\mathcal{G}$ be a game and σ a profile. It holds that σ is BR-reciprocal (resp., DS-reciprocal) iff there exists a bijection $\mu : N \mapsto N$ s.t. σ is a BR-equilibrium (resp., DS-equilibrium) in the permuted game $\mathcal{G}^{\mu}$.*

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- Reciprocity is characterized by the existence of equilibria in games permuted in accordance to the existing cycles
- Reciprocity = cooperation *implementable* by game-permutation

Part IV

Dependence games

Dependence games (i)

Definition 4 (Coalitional games from strategic ones) *Let $\mathcal{G}$ be a game. The coalitional game $\mathcal{C}^{\mathcal{G}} = (N, S, E^{\mathcal{G}}, \succeq_i)$ of $\mathcal{G}$ is a coalitional game where the effectivity function $E^{\mathcal{G}}$ is defined as follows:*

$$X \in E^{\mathcal{G}}(C) \Leftrightarrow \exists \sigma_C \forall \sigma_{\overline{C}} \; o(\sigma_C, \sigma_{\overline{C}}) \in X.$$

- Recipe for building a coalitional game from a strategic one

Dependence games (ii)

Definition 5 (Dependence games from strategic ones) *Let $\mathcal{G}$ be a game. The dependence game $\mathcal{C}^{\mathcal{G}}_{DEP} = (N, S, E^{\mathcal{G}}_{DEP}, \succeq_i)$ of $\mathcal{G}$ is a coalitional game where the effectivity function $E^{\mathcal{G}}_{DEP}$ is defined as follows:*

$$
\begin{aligned}
X \in E^{\mathcal{G}}_{DEP}(C) \quad \Leftrightarrow \quad & \exists \sigma_C, \mu_C \text{ S.T.} \\
& \exists \sigma_{\overline{C}}, \mu_{\overline{C}} : [((\sigma_C, \sigma_{\overline{C}}), (\mu_C, \mu_{\overline{C}})) \in AGR(\mathcal{G})] \\
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where $\mu : N \to N$ is a bijection.

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where $\mu : N \rightarrow N$ is a bijection.

- The effectivity function is restricted so that a set of outcomes can be forced by a coalition only in the presence of reciprocity

Core of dependence games

Theorem 2 (*DEP* vs. *CORE*) *Let $\mathcal{G} = (N, S, \Sigma_i, \geq_i, o)$ be a game. It holds that, for all agreements (σ, μ):*

$$(\sigma, \mu) \in DEP(\mathcal{G}) \Leftrightarrow o(\sigma) \in CORE(C^{\mathcal{G}}_{DEP}).$$

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- For any game in strategic form, the core of its coalitional dependence formulation coincides with the set of *undominated* reciprocal states (agreements).
- NB: there is no relation between the core of the coalitional game and the dependence game of a same strategic game.

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Part V

Conclusions

Results

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- Formal analysis of a notion of dependence between players

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- Characterization of a notion of reciprocity as equilibrium in a permuted game

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- Formal analysis of a notion of dependence between players
- Characterization of a notion of reciprocity as equilibrium in a permuted game
- Characterization of a notion of cooperation via agreement as the core of a dependence game

Thank you!