Sum of Us

Strategyproof Selection from the Selectors

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3rd International Workshop on Computational Social Choice

The Problem

- Approval voting
  - each voter approves of set of candidates (of any size)
  - choose candidate (or committee of desired size) with largest number of votes
- Strategyproof (assuming dichotomous preferences)

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- No longer the case when sets of candidates and voters coincide
  - scientific organizations (GTS, AMS, IEEE, IFAAMAS)
  - web graph, (directed) social networks, reputation systems

Outline

The Model

Deterministic Mechanisms

Randomized Mechanisms

Open Problems

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- Set $N = [n]$ of agents
- Directed graph $G = (N, E) \in \mathcal{G}$, no self-loops
- Ideally: select $S \in \mathcal{S}_k = \{T \subseteq N : |T| = k\}$ to maximize
  $\sum_{i \in S} \deg(i) = \sum_{i \in S} |\{j \in N : (j, i) \in E\}|$
- Mechanism $M : \mathcal{G} \to \Delta(\mathcal{S}_k)$
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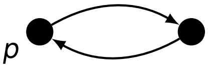

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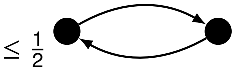

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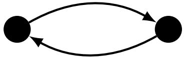

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upper bounds: mechanisms

lower bounds: impossibility results

Bad News

Theorem: Let $n \geq 2$, $k \leq n - 1$. Then there is no strategyproof and α-efficient deterministic mechanism for any finite α.

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Particularly surprising for $k = n - 1$: cannot guarantee to select unique agent receiving any votes

Proof

- Assume for contradiction M was such a mechanism
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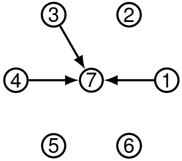

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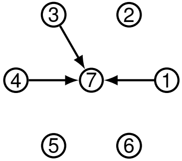

- Isomorphic to $\{0, 1\}^{n-1}$, so we now look at a mechanism $M : \{0, 1\}^{n-1} \rightarrow S_k$

Proof

(1) $n \notin M(\mathbf{0})$ (by assumption)

(2) $n \in M(x)$ for all $x \in \{0,1\}^{n-1} \setminus \{\mathbf{0}\}$ (by α-efficiency for finite α)

(3) $i \in M(x)$ iff $i \in M(x + e_i)$ for all $x \in \{0,1\}^{n-1}$ and $i \in N \setminus \{n\}$ (by strategyproofness)

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Random m-partition (m-RP)

1. assign each agent i.i.d. to one of m sets
2. from each subset, select $\sim k/m$ agents with largest indegree based on edges from *other* subsets

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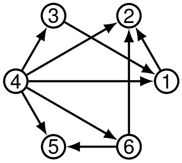

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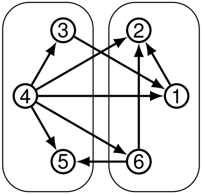

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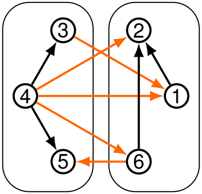

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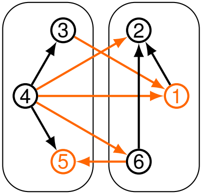

Bounds for Randomized Mechanisms

Theorem: m-RP is (universally) strategyproof for all n, k, m and

- 4-efficient for $m = 2$,
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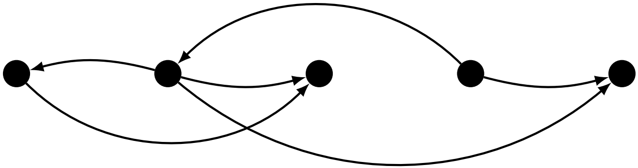

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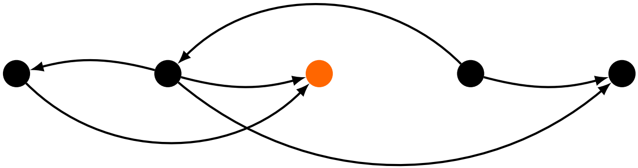

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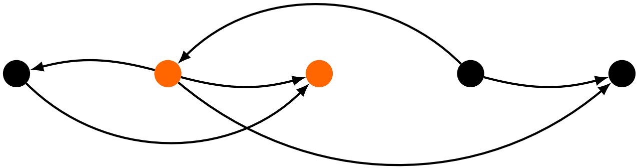

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3. Choose first agent from right to left to receive any votes from its right (or agent 1 if there is no such agent)

Thank you!