Bypassing Combinatorial Protections

Polynomial-Time Algorithms for Single-Peaked Electorates

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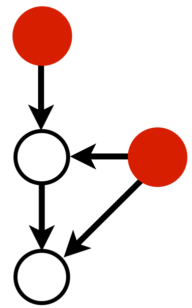
PREFERENCE AGGREGATION IN MULTIAGENT SYSTEMS

Voting Rules

- $C = \{a,b,c,...\}$ is a finite set of candidates or alternatives
- A voting rule f maps a vector $V = (v_1, v_2, \dots, v_n)$ of votes to a non-empty subset $f(V) \subseteq C$ of candidates
  - ignores tie-breaking
- Ranking-based voting rules
  - each vote is a complete ranking of the candidates: $v_i = [b >_i a >_i c]$
- Approval-based voting rules
  - each vote is a set of "approved" candidates: $v_i = \{a,b\}$

Voting rules (2)

- Ranking-based voting rules
  - Many rules are defined via pairwise comparisons (majority graphs)
  - Weak Condorcet winners: all candidates without pairwise defeats
  - A weakCondorcet rule is a rule that precisely returns all weak Condorcet winners whenever at least one exists
  - Llull's rule yields all candidates with minimal number of pairwise defeats
  - Young, Kemeny, Dodgson, Maximin, Fishburn, Schwartz, ...



- Approval voting
  - yields all candidates with maximal number of approvals

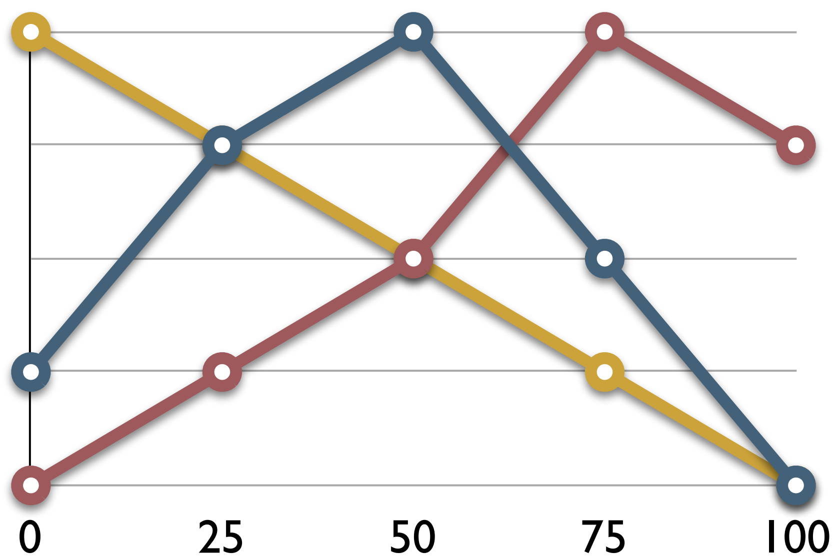
Swaying Elections

- People may try to influence the outcome of an election by
  - manipulating the voters' preferences
    (e.g., bribery, campaigning, strategic manipulation)
  - changing the election's structure
    (e.g., introducing primaries, adding/deleting voters and/or candidates)
- All voting rules are vulnerable to at least some of these attacks
  - But: Attacker's task may be computationally intractable due to combinatorial challenges (e.g., covering or partition problems) [BTT 1989]
- Are such computational protections meaningful in practice?
  - NP-hardness is a worst-case measure
  - heuristics that find successful manipulations in "most" instances
  - approximation algorithms

Single-Peaked Preferences

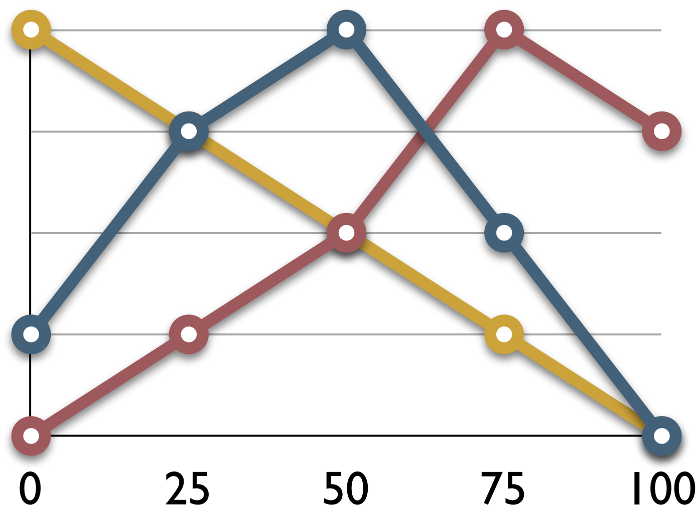
- What should be the registration fee for COMSOC 2010?
  - Candidates: €0, €25, €50, €75, €100

Jörg	Vince	Markus
€50	€75	€0
€25	€100	€25
€75	€50	€50
€0	€25	€75
€100	€0	€100



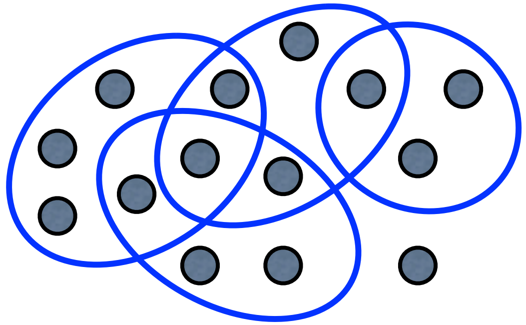
Single-Peaked Preferences (2)

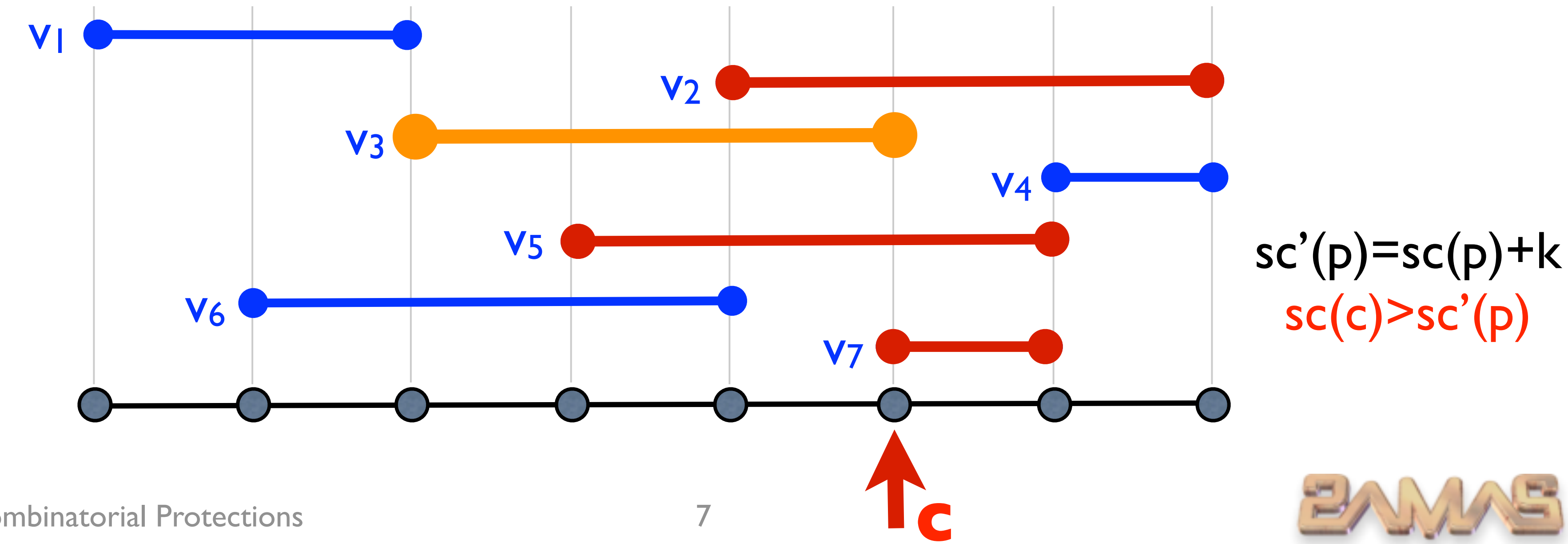
- Preferences are single-peaked iff there exists a linear ordering over C such that if b lies between a and c, then ($a >_i b \Rightarrow b >_i c$) for all voters i
    - natural variant for approval votes: approved candidates form an interval
    - Popular model in political science
        - left-right political spectrum
    - Singled-peakedness can be checked in polynomial time [BT 1986]
    - weak Condorcet winners always exist (nice characterization in terms of median voters)



Bribery

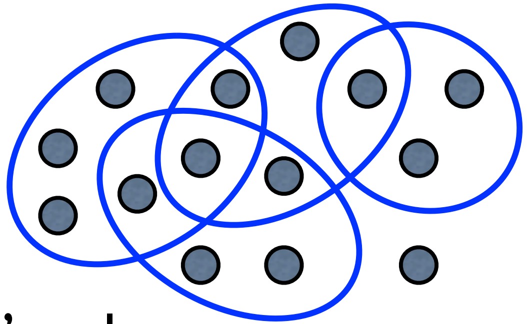
- Is it possible to bribe at most k voters such that p wins?
  - NP-complete for approval voting
    - reduction from X3C [FHH 2009]
  - Theorem: For single-peaked electorates, approval bribery is in P





Bribery

- Is it possible to bribe at most *k* voters such that *p* wins?
  - NP-complete for approval voting
    - reduction from X3C [FHH 2009]
  - Theorem: For single-peaked electorates, approval bribery is in P
  - NP-hard for Llull's rule and [FHH 2009] Kemeny's rule
  - Theorem: For single-peaked electorates, this problem is in P for all weakCondorcet rules
    - e.g., Fishburn, Maximin, Young, Llull, *Kemeny, Schwartz, Nanson*, etc.



Control

- Is it possible to add/delete *k* voters such that *p* wins?
  - NP-hard for Kemeny's rule and Young's rule
  - Theorem: For single-peaked electorates, both problems are in P for all weakCondorcet rules
- Can the set of voters be partitioned into two subsets (primary elections) such that *p* wins the final election?
  - NP-complete for Llull's rule [FHHR 2009]
  - Theorem: For single-peaked electorates, this problem is in P for all weakCondorcet rules

2x	1x	2x	1x
b	a	c	a
a	b	a	c
c	c	b	b

Manipulation

- Constructive coalition weighted manipulation problem: Is it possible to set the preferences of manipulative voters such that p wins?
- We completely characterize all scoring rules where CCWM is in P or NP-complete for single-peaked electorates, respectively

Conclusion

- It has been shown in previous work that various manipulative attacks on voting rules are computationally intractable
- In many realistic settings preferences may be assumed to be single-peaked
- The preference profiles constructed in many hardness proofs are so intricate that they cannot be realized by single-peaked electorates
- Good news: Young, Kemeny, and Dodgson winners can be computed in P for single-peaked electorates