Nice, but Are They Relevant? A Political Scientist Looks at Social Choice Results

Hannu Nurmi

The motivation for introducing a new voting system or criticizing an old one is often a counterintuitive or unexpected voting outcome. A case in point is Borda's memoir where he criticized the plurality voting and suggested his own method of marks [2]. With time this approach focusing on a specific flaw of a system has given way to studies dealing with a multitude of systems and their properties. An example of such studies (e.g. [3]) is summarized in Table 1.

Here criterion a denotes the Condorcet winner criterion, b the Condorcet loser one, c strong Condorcet criterion, d monotonicity, e Pareto, f consistency, g Chernoff property, h independence of irrelevant alternatives and i invulnerability to the no-show paradox. A "1" ("0", respectively) in the table means that the system represented by the row satisfies (violates) the criterion represented by the column.

A more "graded" approach to comparing two systems with respect to one criterion has also been suggested [1]. The superiority of system A with respect to system B takes on degrees from strongest to weakest as follows:

1. A satisfies the criterion, while B doesn't, i.e. there are profiles where B violates the criterion, but such profiles do not exist for B.

2. in every profile where A violates the criterion, also B does, but not vice versa.

3. in *practically all profiles* where A violates the criterion, also B does, but not vice versa ("A dominates B almost everywhere").

4. in a plausible probability model B violates the criterion with higher probability than A.

5. in those political cultures that we are interested in, B violates the criterion with higher frequency than A.

Comparing systems with respect to just one criterion is, however, not plausible since criteria tend to be contested not only among the practitioners devising voting systems, but also within the scholarly community. Suppose instead that one takes a more holistic view of Table 1 and gives some consideration to all criteria. A binary relation of dominance could then be defined as follows: A system A (strictly) dominates system B in terms of a set of criteria, if and only if whenever B satisfies a criterion, so does A, but not the other way around.

But all criteria are not of equal importance. Nor are they unrelated. Moreover, Table 1 tells very little – in fact nothing – about the likelihood of criterion violations in those cases where those violations are possible. To find out how often a given system violates a criterion – say, elects a Condorcet loser – one has to know how often various preference profiles occur and how these are mapped into voting strategies by voters. Once we know these two things we can apply the system to the voting strategy n-tuples (if the number of voters is n), determine the outcomes, and, finally, compare these with preference profile to find out whether the choices dictated by the criterion contradict those resulting from the profile, e.g. if an eventual Condorcet loser was chosen. Traditionally, two methods have been resorted in estimating the frequency of criterion violations: (i) probability modeling, and (ii)

	Criterion								
Voting system	a	b	c	d	e	f	g	h	i
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maximin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	1
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0

Table 1: A Comparison of voting procedures

computer simulations. Both are based on generating artificial electorates and calculating how frequently the criterion is violated or some other incompatibility is encountered in these electorates.

A consideration not disclosed by Table 1 is the intuitive difficulty of finding examples demonstrating criterion violations. In some cases such examples are rather straight-forward, while in others one has to work them out. We shall discuss some of these and dwell on their implications for voting system choice.

The mainstream social choice theory is based on the assumption that the individuals are endowed with complete and transitive preference relations over choice alternatives. Since there are circumstances under which non-transitive preferences make perfect sense, it is worthwhile to find out whether plausible alternatives to the ranking assumption exist. Towards the end of the paper we shall briefly outline some of these.

References

[1] E. Lagerspetz. Evaluating voting systems. In M. Wiberg, editor. *Reasoned Choices*, The Finnish Political Science Association, 2004.

[2] I. McLean and A. B. Urken, editors. *Classics of Social Choice*, The University of Michigan Press, 1995.

[3] H. Nurmi. *Voting Procedures under Uncertainty*, Springer Verlag, 2002.

Hannu Nurmi
Public Choice Research Centre
University of Turku
FI-20014 TURKU
Finland
Email: hnurmi@utu.fi