Social Choice without the Pareto Principle under Weak Independence

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Abstract

We show that the class of social welfare functions that satisfy a weak independence condition identified by Campbell (1976) and Baigent (1987) is fairly rich and freed of a power concentration on a single individual. This positive result prevails when a weak Pareto condition is imposed. Hence, we can overcome the impossibility of Arrow (1951) by simultaneously weakening the independence and Pareto conditions. Moreover, under weak independence, an impossibility of the Wilson (1972) type vanishes.

1 Introduction

We consider the preference aggregation problem in a society which confronts at least three alternatives. A *Social Welfare Function* (SWF) is a mapping which assigns a social ranking to any logically possible profile of individual rankings. A SWF is *independent of irrelevant alternatives* (IIA) if the social ranking of any pair of alternatives depends only on individuals' preferences over that pair. We know, since the seminal work of Arrow (1951), that IIA and Pareto optimality are incompatible, unless one is ready to admit dictatorial SWFs.

The Arrovian impossibility is remarkably robust against weakenings of IIA.[1] For example, letting k stand for the number of alternatives that the society confronts, Blau (1971) proposes the concept of *m-ary independence* for any integer between 2 and k. A SWF is *m-ary independent* if the social ranking of any set of alternatives with cardinality m depends only on individuals' preferences over that set. Clearly, when $m = 2$, m-ary independence coincides with IIA. Moreover, every SWF trivially satisfies m-ary independence when $m = k$. It is also straighforward to see that m-ary independence implies n-ary independence when $m < n$. Nevertheless, Blau (1971) shows that m-ary independence implies n-ary independence when $n < m < k$ as well. Thus, weakening IIA by imposing independence over sets with cardinality more than two does not allow to escape the Arrovian impossibility, unless independence is imposed over the whole set of alternatives - a condition which is satisfied by the definition of a SWF.

Campbell and Kelly (2000a, 2007) further weaken m-ary independence by requiring that the social preference over a pair of alternatives depends only on individuals' preferences over some proper subset of the set of available alternatives. This condition, which they call *independence of some alternatives* (ISA) is considerably weak. As a result, non-dictatorial SWF that satisfy Pareto optimality and ISA -such as the "gateau rules" identified by Campbell and Kelly (2000a)- do exist. On the other hand, "gateau rules" fail neutrality and as Campbell and Kelly (2007) later show, within the Arrovian framework, an extremely weaker version of ISA disallows both anonymity and neutrality.

Denicolo (1998) identifies a condition called *relational independent deciseveness* (RID). He shows that although IIA implies RID, the Arrovian impossibility prevails when IIA is replaced by RID.

[1] In fact, it is robust against weakenings of other conditions as well: Wilson (1972) shows that the Arrovian impossibility essentially prevails when the Pareto condition is not used. Ozdemir and Sanver (2007) identify severely restricted domains which exhibit the Arrovian impossibility.

Campbell (1976) proposes a weakening of IIA which requires that the social decision between a pair of alternatives cannot be reversed at two distinct preference profiles that admit the same individual preferences over that pair. We refer to this condition as *quasi IIA*.[2] Baigent (1987) shows that every Pareto optimal and quasi IIA SWF must be dictatorial in a sense which is close to the Arrovian meaning of the concept - hence a version of the Arrovian impossibility.[3]

In brief, the literature which explores the effects of weakening IIA on the Arrovian impossibility presents results of a negative nature. We revisit this literature in order to contribute by a positive result. We show that under the weakening proposed by Baigent (1987), the Arrovian impossibility can be surpassed by dropping the Pareto condition: We characterize the class of quasi IIA SWFs and show that this is a fairly large class which is not restricted to SWFs where the decision power is concentrated on one given individual. In fact, this class contains SWFs that are both anonymous and neutral. This positive result prevails when a weak version of the Pareto condition is imposed.

Our findings pave the way to surpass the impossibility of Arrow (1951). Moreover, we establish that there is no tension between quasi IIA and the transitivity of the social outcome. Thus, we also contrast the results of Wilson (1972) and Barberà (2003) who show that the Pareto condition has little impact on the Arrovian impossibility which is essentially a tension between IIA and the range restriction imposed over SWFs.

Section 2 presents the basic notions. Section 3 states our results. Section 4 makes some concluding remarks.

2 Basic Notions

We consider a finite set of individuals N with $\#N \geq 2$, confronting a finite set of alternatives A with $\#A \geq 3$. An *aggregation rule* is a mapping $f : \Pi^N \rightarrow \Theta$ where Π is the set of complete, transitive and antisymmetric binary relations over A while Θ is the set of complete binary relations over A. We conceive $P_i \in \Pi$ as the *preference* of $i \in N$ over A.[4] We write $P = (P_1, ..., P_{\#N}) \in \Pi^N$ for a *preference profile* and $f(P) \in \Theta$ reflects the social preference obtained by the aggregation of P through f. Note that $f(P)$ need not be transitive. Moreover, as $f(P)$ need not be antisymmetric, we write $f^*(P)$ for its strict counterpart.[5]

An aggregation rule f is *independent of irrelevant alternatives* (IIA) iff given any distinct $x, y \in A$ and any $P, P' \in \Pi^N$ with $x \; P_i \; y \Longleftrightarrow x \; P'_i \; y \; \forall i \in N$, we have $x \; f(P) \; y \Longleftrightarrow x \; f(P') \; y$. We write Φ for the set of aggregation rules which satisfy IIA. For any distinct $x, y \in A$, let $\{{x \atop y}, {y \atop x}, xy\}$ be the set of complete and transitive preferences over $\{x, y\}$.[6] An *elementary aggregation rule* is a mapping $f_{\{x,y\}} : \{{x \atop y}, {y \atop x}\}^N \rightarrow \{{x \atop y}, {y \atop x}, xy\}$. Any family $f = \{f_{\{x,y\}}\}$ of elementary aggregation rules indexed over all possible distinct pairs $x, y \in A$ induces an aggregation rule as follows: For each $P \in \Pi^N$ and each $x, y \in A$, let $x \; f(P)$

[2] See Campbell (1976) for a discussion of the computational advantages of quasi IIA. Note that when social indifference is not allowed, IIA and quasi IIA are equivalent.

[3] Baigent (1987) claims this impossibility in an environment with at least three alternatives. Nevertheless, Campbell and Kelly (2000b) show the existence of Pareto optimal and quasi IIA SWF when there are precisely three alternatives. They also show that the impossibility announced by Baigent (1987) prevails when there are at least four alternatives and even under restricted domains.

[4] As usual, for any distinct $x, y \in A$, we intepret $x \; P_i \; y$ as x being preferred to y in view of i.

[5] So for any distinct $x, y \in A$, we have $x \; f^*(P) \; y$ whenever $x \; f(P) \; y$ and not $y \; f(P) \; x$.

[6] We interpret ${x \atop y}$ as x being preferred to y; ${y \atop x}$ as y being preferred to x; and xy as indifference between x and y.

$y \iff f_{\{x,y\}}(P^{\{x,y\}}) \in \{\substack{x\\y}, xy\}$ where $P^{\{x,y\}} \in \{\substack{x\\y}, \substack{y\\x}\}^N$ is the restriction of $P \in \Pi^N$ over $\{x,y\}$.[7] Note that $f = \{f_{\{x,y\}}\} \in \Phi$. Moreover, any $f \in \Phi$ can be expressed in terms of a family $\{f_{\{x,y\}}\} = f$ of elementary aggregation rules.

Let $\Re$ be the set of complete and transitive binary relations over A. A *Social Welfare Function* (SWF) is an aggregation rule whose range is restricted to $\Re$. A SWF $\alpha : \Pi^N \to \Re$ is *Pareto optimal* iff given any distinct $x, y \in A$ and any $P \in \Pi^N$ with $x\ P_i\ y\ \forall i \in N$, we have $x\ \alpha^*(P)\ y$. A SWF $\alpha : \Pi^N \to \Re$ is *dictatorial* iff $\exists i \in N$ such that $x\ P_i\ y$ implies $x\ \alpha^*(P)\ y\ \forall P \in \Pi^N, \forall x, y \in A$. The Arrovian impossibility, as we consider, announces that a SWF $\alpha : \Pi^N \to \Re$ is Pareto optimal and IIA if and only if is α dictatorial.

3 Results

Baigent (1987) proves a version of the Arrovian impossibility where IIA and dictatoriality are replaced by their following weaker versions: A SWF α is *quasi IIA* iff given any distinct $x, y \in A$ and any $P, P' \in \Pi^N$ with $x\ P_i\ y \iff x\ P'_i\ y\ \forall i \in N$, we have $x\ \alpha^*(P)\ y \Rightarrow x\ \alpha(P')\ y$. Note that quasi IIA and IIA coincide when indifferences are ruled out from the social preference. A SWF α is *weakly dictatorial* iff $\exists i \in N$ such that $x\ P_i\ y$ implies $x\ \alpha(P)\ y\ \forall P \in \Pi^N, \forall x, y \in A$. Baigent (1987) establishes that every Pareto optimal and quasi IIA SWF is a weak dictatorship. Nevertheless, we remark that, unlike the original version of the Arrovian impossibility, the converse statement is not true: Although every weak dictatorship is quasi IIA, there exists weak dictatorships that are not Pareto optimal.[8] Following this remark, we allow ourselves to the state a slight generalization of this theorem of Baigent (1987), corrected by Campbell and Kelly (2000b)[9]:

Theorem 3.1 *Let $\#A \geq 4$. Within the family of Pareto optimal SWFs, a SWF $\alpha : \Pi^N \to \Re$ is quasi IIA iff a is weakly dictatorial.*

We now explore the effect of being confined to the class of Pareto optimal SWFs. The strict counterpart of $T \in \Theta$ is denoted T^*. Let $\rho : \Theta \longrightarrow 2^{\Re}$ stand for the correspondence which transforms each $T \in \Theta$ over A into a non-empty subset of $\Re$ such that $\rho(T) = \{R \in \Re : xTy \Longrightarrow xRy, \forall x, y \in A\}$. To have a clearer understanding of ρ, we recall that every $T \in \Theta$ induces an ordered list of "cycles".[10] A set $Y \in 2^A \backslash \{\emptyset\}$ is a *cycle* (with respect to $T \in \Theta$) iff Y can be written as $Y = \{y_1, ..., y_{\#Y}\}$ such that $y_i\ T\ y_{i+1}\ \forall i \in \{1, ..., \#Y - 1\}$ and $y_{\#Y}\ T\ y_1$. The *top-cycle* of $X \in 2^A \backslash \{\emptyset\}$ with respect to $T \in \Theta$ is a cycle $C(X,T) \subseteq X$ such that $y\ T^*x\ \forall y \in C(X,T)$, $\forall x \in X \backslash C(X,T)$.[11] Now let $A_1 = C(A,T)$ and recursively define $A_i = C(A \backslash \bigcup_{k=1}^{i-1} A_k, T), \forall i \geq 2$. Given the finiteness of A, there exists an integer k such that $A_{k+1} = \emptyset$. So every $T \in \Theta$ induces a unique ordered partition $(A_1, A_2, ..., A_k)$ of A. It follows from the definition of the top-cycle that whenever $i < j$, we have $xT^*y\ \forall x \in A_i$, $\forall y \in A_j$.

Lemma 3.1 *Take any $T \in \Theta$ which induces the ordered partition $(A_1, A_2, ..., A_k)$. Given any A_i and any $x, y \in A_i$, we have $x\ R\ y$ and $y\ R\ x$, $\forall R \in \rho(T)$.*

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[7] So for any $i \in N$, we have $P_i^{\{x,y\}} = \substack{x\\y} \iff x\ P_i\ y$.

[8] For example the SWF α where $x\ \alpha(P)\ y\ \forall x, y \in A$ and $\forall P \in \Pi^N$ is a weak dictatorship but not Pareto optimal.

[9] See Footnote 3.

[10] we use the definition of "cycle" as stated by Peris and Subiza (1999).

[11] The top-cycle, introduced by Good (1971) and Schwartz (1972), has been explored in details. Moreover, Peris and Subiza (1999) extend this concept to weak tournaments. In their setting, as $C(X,T)$ is a cycle, $\nexists Y \subset C(X,T)$ with $y\ T^*\ x\ \forall y \in Y$, $\forall x \in C(X,T) \backslash Y$.

Proof. Take any $T \in \Theta$ which induces the ordered partition $(A_1, A_2, ..., A_k)$. Take any A_i, any for $x, y \in A_i$ and any $R \in \rho(T)$.If $\#A_i = 1$, then x and y coincide, hence $x\ R\ y$ and $y\ R\ x$ holds by the completeness of R. If $\#A_i = 2$, then $x\ T\ y$ and $y\ T\ x$ since A_i is a cycle, which implies $x\ R\ y$ and $y\ R\ x$ since $R \in \rho(T)$. We complete the proof by considering the case $\#A_i = k \geq 3$. Let $A_i = \{x_1, x_2, ..., x_k\}$. Suppose, without loss of generality, $x_1 R\ x_2$ and *not* $x_2\ R\ x_1$. This implies $x_1\ T^*\ x_2$, as $R \in \rho(T)$. Moreover, as A_i is a cycle, $\exists x \in A_i$ such that $x_2\ T\ x$. Let, without loss of generality, $x_2\ T\ x_3$. Thus $x_2\ R\ x_3$ holds by definition of ρ which implies $x_1 R\ x_3$ and *not* $x_3\ R\ x_1$ by the transitivity of R. Again by definition of ρ, we have $x_1\ T^* x_3$. As A_i is a cycle, $\exists j \in \{4, ..., k-1\}$ such that $x_3\ T\ x_j$. Suppose, without loss of generality, $j = 4$. So $x_3\ T\ x_4$, hence $x_3\ R\ x_4$, implying $x_1 R\ x_4$ and *not* $x_4\ R\ x_1$, which in turn implies $x_1 T^*\ x_4$. So, iteratively, $\forall i \in \{4, ..., k-1\}$, we have $x_i\ T\ x_{i+1}$,which implies $x_i\ R\ x_{i+1}$ and moreover $x_1 R\ x_{i+1}$ and *not* $x_{i+1}\ R\ x_1$. Hence, $x_1\ T^*\ x_{i+1}$. But as A_i is a cycle, we have $x_k\ T\ x_1$. So $x_k\ R\ x_1$ holds by definition of ρ. As we also have $x_i\ R\ x_{i+1}, \forall i \in \{1, ..., k-1\}$, $x_2\ R\ x_1$ holds by transitivity of R, which leads to a contradiction. Therefore, $x\ R\ y$ and $y\ R\ x$ for all $x, y \in A_i, \forall R \in \rho(T)$. ■

Thus for any $T \in \Theta$ which induces the ordered partition $(A_1, A_2, ..., A_k)$ and any $R \in \Re$, we have $R \in \rho(T)$ if and only if for any $x, y \in A$

(i) $x, y \in A_i$ for some $A_i \Longrightarrow xRy$ and yRx

and

(ii) $x \in A_i$ and $y \in A_j$ for some A_i, A_j with $i < j \Longrightarrow xRy$.

We now proceed towards characterizing the family of quasi IIA SWFs. Take any aggregation rule $f \in \Phi$ which satisfies IIA. By composing f with ρ, we get a *social welfare correspondence* $\rho \circ f : \Pi^N \longrightarrow 2^{\Re}$ which assigns to each $P \in \Pi^N$ a non-empty subset $\rho(f(P))$ of $\Re$. Clearly, every singleton-valued selection of $\rho \circ f$ is a SWF.[12] Let $\Sigma^f = \{\alpha : \Pi^N \rightarrow \Re$ $\mid \alpha$ is a singleton-valued selection of $\rho \circ f$ $\}$. We write $\Sigma = \cup_{f \in \Phi} \Sigma^f$. Interestingly, the class of quasi IIA SWFs coincides with Σ.

Theorem 3.2 *A SWF $\alpha : \Pi^N \rightarrow \Re$ is quasi IIA iff $\alpha \in \Sigma$.*

Proof. To establish the "only if" part, let $\alpha : \Pi^N \rightarrow \Re$ be a quasi IIA SWF. For any distinct $x, y \in A$, we define $f_{\{x,y\}} : \{\begin{smallmatrix} x \\ y \end{smallmatrix}, \begin{smallmatrix} y \\ x \end{smallmatrix}\}^N \rightarrow \{\begin{smallmatrix} x \\ y \end{smallmatrix}, \begin{smallmatrix} y \\ x \end{smallmatrix}, xy\}$ as follows: For any $r \in \{\begin{smallmatrix} x \\ y \end{smallmatrix}, \begin{smallmatrix} y \\ x \end{smallmatrix}\}^N$,

$$f_{\{x,y\}}(r) = \begin{cases} \begin{smallmatrix} x \\ y \end{smallmatrix} & \text{if} \quad x\ \alpha^*(P)\ y \text{ for some } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \\ \begin{smallmatrix} y \\ x \end{smallmatrix} & \text{if} \quad y\ \alpha^*(P)\ x \text{ for some } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \\ xy & \text{if } x\ \alpha(P)\ y \text{ and } y\ \alpha(P)\ x \text{ for all } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \end{cases}.$$

As α is quasi IIA, $f_{\{x,y\}}$is well-defined. Thus $f = \{f_{\{x,y\}}\} \in \Phi$. We now show $\alpha(P) \in \rho(f(P))$ $\forall P \in \Pi^N$. Take any $P \in \Pi^N$ and any distinct $x, y \in A$. First let $x\ f^*(P)\ y$. So $f_{\{x,y\}}(P^{\{x,y\}}) = \begin{smallmatrix} x \\ y \end{smallmatrix}$. By definition of $f_{\{x,y\}}$, we have $x\ \alpha^*(Q)\ y$ for some $Q \in \Pi^N$ with $Q^{\{x,y\}} = P^{\{x,y\}}$ which implies $x\ \alpha(P)\ y$ as α is quasi IIA. If $y\ f^*(P)\ x$, then one can similarly $y\ \alpha(P)\ x$. Now, let $x\ f(P)\ y$ and $y\ f(P)\ x$. So, $f_{\{x,y\}}(P^{\{x,y\}}) = xy$ which, by definition of $f_{\{x,y\}}$, implies $x\ \alpha(Q)\ y$ and $y\ \alpha(Q)\ x$ for all $Q \in \Pi^N$ with $Q^{\{x,y\}} = P^{\{x,y\}}$, hence $x\ \alpha(P)\ y$ and $y\ \alpha(P)\ x$. Thus, $x\ f(P)\ y \Longrightarrow x\ \alpha(P)\ y$ for any $x, y \in A$, establishing $\alpha(P) \in \rho(f(P))$.

To establish the "if" part, take any $\alpha \in \Sigma$. So there exists $f \in \Phi$ such that $\alpha(P) \in \rho(f(P))$ $\forall P \in \Pi^N$. Suppose α is not quasi IIA. So, $\exists x, y \in A$ and $\exists P, Q \in \Pi^N$ with

[12]We say that $\alpha : \Pi^N \rightarrow \Re$ is a singleton-valued selection of $\rho \circ f$ iff $\alpha(P) \in \rho \circ f(P)$ $\forall P \in \Pi^N$.

$P^{\{x,y\}} = Q^{\{x,y\}}$such that $x\ \alpha^*(P)\ y$ and $y\ \alpha^*(Q)\ x$. By the definition of ρ we have x $f^*(P)\ y$ and $y\ f^*(Q)\ x$ which implies $f_{\{x,y\}}(P^{\{x,y\}}) = \genfrac{}{}{0pt}{}{x}{y}$ and $f_{\{x,y\}}(Q^{\{x,y\}}) = \genfrac{}{}{0pt}{}{y}{x}$, giving a contradiction as $P^{\{x,y\}} = Q^{\{x,y\}}$, thus showing that α is quasi IIA. ■

By juxtaposing Theorems 3.1 and 3.2, one can conclude that removing the Pareto condition has a dramatic impact, as the class Σ of quasi IIA SWFs is fairly large and allows those where the decision power is not concentrated on a single individual. This positive result prevails when the following weak Pareto condition is imposed: A SWF α is *weakly Pareto optimal* iff given any distinct $x, y \in A$ and any $P \in \Pi^N$ with $x\ P_i\ y\ \forall i \in N$, we have $x\ \alpha(P)\ y$. An aggregation rule $f \in \Phi$ is *weakly Pareto optimal* iff for any $x, y \in A$ and any $r \in \{\genfrac{}{}{0pt}{}{x}{y}, \genfrac{}{}{0pt}{}{y}{x}\}^N$ with $r_i = \genfrac{}{}{0pt}{}{x}{y}\ \forall i \in N$, we have $f_{\{x,y\}}(r) \in \{\genfrac{}{}{0pt}{}{x}{y}, xy\}$. Let Φ^* stand for the set of weakly Pareto optimal and IIA aggregation rules and $\Sigma^* = \cup_{f\in\Phi^*}\Sigma^f$.

Theorem 3.3 *A SWF $\alpha : \Pi^N \to \Re$ is weakly Pareto optimal and quasi IIA iff $\alpha \in \Sigma^*$.*

Proof. To show the "only if" part, take any SWF $\alpha : \Pi^N \to \Re$ which is weakly Pareto optimal and quasi IIA. For any distinct $x, y \in A$, we define $f_{\{x,y\}} : \{\genfrac{}{}{0pt}{}{x}{y}, \genfrac{}{}{0pt}{}{y}{x}\}^N \to \{\genfrac{}{}{0pt}{}{x}{y}, \genfrac{}{}{0pt}{}{y}{x}, xy\}$ as follows: For any $r \in \{\genfrac{}{}{0pt}{}{x}{y}, \genfrac{}{}{0pt}{}{y}{x}\}^N$,

$$f_{\{x,y\}}(r) = \begin{cases} \genfrac{}{}{0pt}{}{x}{y} \text{ if } & x\ \alpha^*(P)\ y \text{ for some } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \\ \genfrac{}{}{0pt}{}{y}{x} \text{ if } & y\ \alpha^*(P)\ x \text{ for some } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \\ xy \text{ if } x\ \alpha(P)\ y \text{ and } y\ \alpha(P)\ x \text{ for all } P \in \Pi^N \text{ with } P^{\{x,y\}} = r \end{cases}.$$

As α is quasi IIA, $f_{\{x,y\}}$is well-defined. Thus $f = \{f_{\{x,y\}}\} \in \Phi$. Suppose, f is not weakly Pareto optimal. So, $\exists x, y \in A$ and $\exists P \in \Pi^N$ with $x\ P_i\ y\ \forall i \in N$ such that $y\ f^*(P)\ x$, implying $f_{\{x,y\}}(P^{\{x,y\}}) = \genfrac{}{}{0pt}{}{y}{x}$. By definition of $f_{\{x,y\}}$, we have $y\ \alpha^*(Q)\ x$ for some $Q \in \Pi^N$ with $Q^{\{x,y\}} = P^{\{x,y\}}$, contradicting that α is weakly Pareto optimal, which establishes $f = \{f_{\{x,y\}}\} \in \Phi^*$. We now show $\alpha(P) \in \rho(f(P))\ \forall P \in \Pi^N$. Take any $P \in \Pi^N$ and any distinct $x, y \in A$. First let $x\ f^*(P)\ y$. So $f_{\{x,y\}}(P^{\{x,y\}}) = \genfrac{}{}{0pt}{}{x}{y}$. By definition of $f_{\{x,y\}}$, we have $x\ \alpha^*(Q)\ y$ for some $Q \in \Pi^N$ with $Q^{\{x,y\}} = P^{\{x,y\}}$ which implies $x\ \alpha(P)\ y$ as α is quasi IIA. If $y\ f^*(P)\ x$, then one can similarly $y\ \alpha(P)\ x$. Now, let $x\ f(P)\ y$ and $y\ f(P)\ x$. So, $f_{\{x,y\}}(P^{\{x,y\}}) = xy$ which, by definition of $f_{\{x,y\}}$, implies $x\ \alpha(Q)\ y$ and $y\ \alpha(Q)\ x$ for all $Q \in \Pi^N$ with $Q^{\{x,y\}} = P^{\{x,y\}}$, hence $x\ \alpha(P)\ y$ and $y\ \alpha(P)\ x$. Thus, $x\ f(P)\ y \Longrightarrow x\ \alpha(P)$ y for any $x, y \in A$, establishing $\alpha(P) \in \rho(f(P))$.

To show the "if" part, take any $\alpha \in \Sigma^*$. So there exists $f \in \Phi^*$ such that $\alpha(P) \in \rho(f(P))$ $\forall P \in \Pi^N$. Take any distinct $x, y \in A$ and any $P \in \Pi^N$ with $x\ P_i\ y\ \forall i \in N$. By the weak Pareto optimality of f, we have $f_{\{x,y\}}(P^{\{x,y\}}) \in \{\genfrac{}{}{0pt}{}{x}{y}, xy\}$, hence $x\ f(P)\ y$, which implies x $\alpha(P)\ y$ by the definition of ρ. Thus, α is weakly Pareto optimal. The "if"part of Theorem 3.2 establishes that α is quasi IIA, completing the proof. ■

4 Concluding Remarks

Within the scope of the preference aggregation problem, we contribute to the understanding of the well-known tension between requiring the pairwise independence of the aggregation rule and the transitivity of the social preference. As Wilson (1972) shows, a SWF $\alpha : \Pi^N \to \Re$ is *non-imposed*[13] and IIA if and only if α is dictatorial or *antidictatorial*[14] or *null*[15]. Thus, aside from these, any aggregation rule which is IIA allows non-transitive social outcomes. In case these outcomes are rendered transitive according to one of the prescriptions made by ρ, we attain a SWF which fails IIA but satisfies quasi IIA. In fact, as Theorem 3.2 states, the class of quasi IIA SWFs coincides with those which can be attained through a selection made out of the social welfare correspondence obtained by the composition of a SWF that is IIA with ρ. This can be interpreted as a positive result, as the class of quasi IIA SWFs is fairly rich and not restricted to those where the decision power is concentrated on one individual. In fact, this class contains SWFs that are both anonymous and neutral.[16] Moreover, as Theorem 3.3 states, this positive result prevails when a weaker version of the Pareto condition is imposed. Thus, we can conclude that the transitivity of the social outcome can be achieved at a cost of reducing IIA to quasi IIA and compromising of the strenght of the Pareto condition - hence an escape from an impossibility of both the Arrow (1951) and Wilson (1972) type.

Another way of looking at the problem is to conceive it as determining the possible "stretchings" of the null rule (which is well-known to be IIA) without violating quasi-IIA. This angle of view advises caution about our optimism on escaping the Arrow/Wilson impossibilities, as this escape imposes indifference in social preference. So it is worth exploring "how far" quasi IIA SWFs are from the null rule. This exploration requires to ask for the minimization of the imposed social indifference. The answer is straightforward for a given aggregation rule $f \in \Phi$: Taking the transitive closure of the social preference is the selection of $\rho \circ f$ which minimizes the imposed social indifference.[17] Nevertheless, the choice of the (non-dictatorial) f that minimizes the imposed social indifference.remains as an interesting open question.[18]

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[13] $\alpha : \Pi^N \to \Re$ is *non-imposed* iff for any $x, y \in A$, there exists $P \in \Pi^N$ with $x\ \alpha(P)\ y$.

[14] α is *anti-dictatorial* iff $\exists i \in N$ such that $x\ P_i\ y$ implies $y\ \alpha^*(P)\ x\ \forall P \in \Pi^N, \forall x, y \in A$.

[15] $\alpha : \Pi^N \to \Re$ is *null* iff $x\ \alpha(P)\ y\ \forall x, y \in A$ and $\forall P \in \Pi^N$.

[16] As a matter of fact, the SWF in Example 2 of Campbell and Kelly (2000b), which shows the failure of Theorem 3.1 for $\#A = 3$, belongs to this class.

[17] By "taking the transitive closure", we mean to replace cycles with indifference classes. Formally speaking, writing $(A_1, A_2, ..., A_k)$ for the ordered partition induced by $f(P) \in \Theta$ at $P \in \Pi^N$, take $\alpha(P) \in \rho(f(P))$ where $x\ \alpha^*(P)\ y\ \forall x \in A_i$ and $\forall y \in A_j$ with $i < j$. One can see Sen (1986) for a general discussion of the "closure methods".

[18] We conjecture, by relying on Dasgupta and Maskin (2008), that this will be the pairwise majority rule.

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