# Llull and Copeland Voting Computationally Resist Bribery and Control[1]

Piotr Faliszewski, Edith Hemaspaandra, Lane A. Hemaspaandra, Jörg Rothe

**Abstract**

Control and bribery are settings in which an external agent seeks to influence the outcome of an election. Constructive control of elections refers to attempts by an agent to, via such actions as addition/deletion/partition of candidates or voters, ensure that a given candidate wins [6]. Destructive control refers to attempts by an agent to, via the same actions, preclude a given candidate's victory [26]. An election system in which an agent can affect the result and in which recognizing the inputs on which the agent can succeed is NP-hard (polynomial-time solvable) is said to be resistant (vulnerable) to the given type of control. Aside from election systems with an NP-hard winner problem, the only systems previously known to be resistant to all the standard control types are highly artificial election systems created by hybridization [27].

We study a parameterized version of Copeland voting, denoted by Copeland$^\alpha$, where the parameter $\alpha$ is a rational number between 0 and 1 that specifies how ties are valued in the pairwise comparisons of candidates. In every previously studied constructive or destructive control scenario, we determine which of resistance or vulnerability holds for Copeland$^\alpha$ for each rational $\alpha$, $0 \leq \alpha \leq 1$. In particular, we prove that Copeland$^{0.5}$, the system commonly referred to as "Copeland voting," provides full resistance to constructive control, and we prove the same for Copeland$^\alpha$, for all rational $\alpha$, $0 < \alpha < 1$. Among systems with a polynomial-time winner problem, Copeland voting is the first natural election system proven to have full resistance to constructive control. In addition, we prove that both Copeland$^0$ and Copeland$^1$ (interestingly, Copeland$^1$ is an election system developed by the thirteenth-century mystic Ramon Llull) are resistant to all standard types of constructive control other than one variant of addition of candidates. Moreover, we show that for each rational $\alpha$, $0 \leq \alpha \leq 1$, Copeland$^\alpha$ voting is fully resistant to bribery attacks, and we establish fixed-parameter tractability of bounded-case control for Copeland$^\alpha$.

We also study Copeland$^\alpha$ elections under more flexible models such as microbribery and extended control, we integrate the potential irrationality of voter preferences into many of our results, and we prove our results in both the unique-winner and the nonunique-winner model. Our vulnerability results for microbribery are proven via a novel technique involving min-cost network flow.

## 1 Introduction

Elections have played an important role in human societies for thousands of years. For example, elections were of central importance in the democracy of ancient Athens. There citizens typically could only agree (vote *yes*) or disagree (vote *no*) with the speaker, and simple majority-rule was in effect. The mathematical study of elections, give or take a

[1]Supported in part by DFG grants RO-1202/{9-3, 11-1, 12-1}, NSF grants CCR-0311021, CCF-0426761, and IIS-0713061, the Alexander von Humboldt Foundation's TransCoop program, the European Science Foundation's EUROCORES program LogICCC, and two Friedrich Wilhelm Bessel Research Awards. Work done in part while the first three authors were visiting Heinrich-Heine-Universität Düsseldorf and while the fourth author was visiting the University of Rochester. Some results have been presented at the 22nd AAAI Conference on Artificial Intelligence (AAAI-07) [20], and at the 4th International Conference on Algorithmic Aspects in Information and Management (AAIM-08) [21].

few discussions by the ancient Greeks and Romans, was until recently thought to have been initiated only a few centuries ago, namely in the breakthrough work of Borda and Condorcet—later in part reinvented by Dodgson (see, e.g., [31] for reprints of their classic papers). One of the most interesting results of this early work is Condorcet's observation [7] that if one conducts elections with more than two alternatives then even if all voters have rational (i.e., transitive) preferences, the society in aggregate can be irrational (indeed, can have cycles of strict preference). Based on his observations, Condorcet suggested that if there exists a candidate $c$ such that $c$ defeats any other candidate in a head-to-head contest then that candidate should win the election. Such a candidate is called a Condorcet winner. Clearly, there can be at most one Condorcet winner in any election and there might be none.

This understanding of history has been reconsidered during the past few decades, as it has been rediscovered that the study of elections was in fact considered deeply as early as the thirteenth century (see Hägele and Pukelsheim [24] and the citations therein regarding Ramon Llull and the fifteenth-century figure Cusanus, especially the citations that there are numbered 3, 5, and 24–27). Ramon Llull (b. 1232, d. 1315), a Catalan mystic, missionary, and philosopher developed an election system that (a) has an efficient winner-determination procedure and (b) elects a Condorcet winner whenever one exists and otherwise elects candidates that are, in some sense, closest to being Condorcet winners. Llull's motivation for developing an election system was to obtain a method of choosing the abbesses, abbots, bishops, and perhaps even the pope. His election ideas never gained public acceptance in medieval Europe and were long forgotten.

It is interesting to note that Llull allowed voters to have *irrational* preferences. Given three candidates, $c$, $d$, and $e$, it was perfectly acceptable for a voter to prefer $c$ to $d$, $d$ to $e$, and $e$ to $c$. On the other hand, in modern studies of voting and election systems each voter's preferences are most typically modeled as a linear order over all candidates. (In this paper, as is standard, "linear order" implies strictness, i.e., no tie in the ordering.) Yet irrationality is a very tempting and natural concept.

Llull's election system is remarkably similar to what is now known as "Copeland elections" [11], a more than half-century old voting procedure that is based on pairwise comparisons of candidates: The winner (by a majority of votes—in this paper "majority" always, as is standard, means strict majority) of each such a head-to-head contest is awarded one point and the loser receives no point; in ties, both parties are (in the most common interpretation of Copeland's meaning) awarded half a point; whoever collects the most points over all these contests (including tie-related points) is the election's winner. In fact, the points awarded for ties in such head-to-head majority-rule contests are treated in two ways, half a point (most common) and zero points (less common), in the literature when speaking of Copeland elections. To provide a framework that can capture both those notions, as well as Llull's system and the whole family of systems created by choices of how we value ties, we introduce a parameterized version of Copeland elections in Definition 1.1 below.

An election is specified by a finite set $C$ of candidates and a finite collection $V$ of voters, where each voter has preferences over the candidates. We consider both rational and irrational voters. The preferences of a rational voter are expressed by a preference list of the form $a > b > c$ (assuming $C = \{a, b, c\}$), where the underlying relation $>$ is a transitive linear order. The preferences of an irrational voter are expressed by a preference table that for any two distinct candidates specifies which of them is preferred to the other by this voter. An election system is a rule that determines the winner(s) of each given election $(C, V)$.

**Definition 1.1** *Let $\alpha$, $0 \leq \alpha \leq 1$, be a fixed rational number. In a* Copeland$^{\alpha}$ *election the voters indicate which among any two distinct candidates they prefer. For each such head-to-head contest, if some candidate is preferred by a majority of voters then he or she obtains one point and the other candidate obtains zero points, and if a tie occurs then both* *candidates obtain $\alpha$ points. Let $E = (C, V)$ be an election. For each $c \in C$, $score_E^\alpha(c)$ is the sum of c's* Copeland$^\alpha$ *points in E. Every candidate c with maximum $score_E^\alpha(c)$ wins.*

Copeland$^\alpha_{\text{Irrational}}$ *denotes the same system but with voters allowed to be irrational.*

So the system widely referred to in the literature as "Copeland elections" is Copeland$^{0.5}$, where tied candidates receive half a point each (see, e.g., Merlin and Saari [36, 32]; the definition used by Conitzer et al. [10] can be scaled to be equivalent to Copeland$^{0.5}$). Copeland$^0$, where tied candidates come away empty-handed, has sometimes also been referred to as "Copeland elections" (see, e.g., Procaccia, Rosenschein, and Kaminka [34] and an early version of this paper [20]). The above-mentioned election system by Ramon Llull is in this notation nothing other than Copeland$^1$, where tied candidates are awarded one point each, just like winners of head-to-head contests.[2] The group stage of FIFA World Cup finals is in essence a collection of Copeland$^\alpha$ tournaments with $\alpha = 1/3$.

At first glance, one might be tempted to think that the definitional perturbation due to the parameter $\alpha$ in Copeland$^\alpha$ elections is negligible. However, it in fact can make the dynamics of Llull's system quite different from those of, for instance, Copeland$^{0.5}$ or Copeland$^0$. We also mention that a probabilistic variant of Copeland voting was defined already in 1929 by Zermelo [38] and later on was reintroduced by several other researchers.

In general it is impossible to design a perfect election system. In the 1950s Arrow [2] famously showed that there is no social choice system that satisfies a certain small set of reasonable requirements, and later Gibbard [23], Satterthwaite [37], and Duggan and Schwartz [13] showed that any natural election system can be manipulated by strategic voting, i.e., by a voter who reveals different preferences than his or her true ones in order to affect an election's result in his or her favor. Also, no natural election system with a polynomial-time winner-determination procedure has yet been shown to be resistant to all types of control via procedural changes. Control refers to attempts by an external agent (called "the chair") to, via such actions as addition/deletion/partition of candidates or voters, make a given candidate win the election (in the case of constructive control [6]) or preclude a given candidate's victory (in the case of destructive control [26]).

These obstacles are very discouraging, but the field of computational social choice theory grew in part from the realization that computational complexity provides a tool to partially circumvent these obstacles. In particular, around 1990 Bartholdi, Tovey, and Trick [4, 6] and Bartholdi and Orlin [3] brilliantly observed that while we might not be able to make manipulation (i.e., strategic voting) and control of elections impossible, we can at least try to make such manipulation and control so computationally difficult that neither voters nor election organizers will attempt it. For example, if there is a way for a committee's chair to set up an election within the committee in such a way that his or her favorite option is guaranteed to win but the chair's computational task would take a million years, then for all practical purposes we may assume that the chair is prevented from finding such a set-up.

Since the seminal work of Bartholdi, Orlin, Tovey, and Trick a large body of research has been dedicated to the study of computational properties of election systems. Some topics that have received much attention are the complexity of manipulating elections [8, 9, 10, 14, 25, 33, 35] and of controlling elections via procedural changes [26, 27, 35, 17]. Recently,

[2] Page 23 of Hägele and Pukelsheim [24] indicates in a way we find deeply convincing (namely by a direct quote of Llull's in-this-case-very-clear words from his *Artifitium Electionis Personarum*—which was rediscovered by those authors in the year 2000) that at least one of Llull's election systems was Copeland$^1$, and so in this paper we refer to the both-candidates-score-a-point-on-a-tie variant as Llull voting.

In some settings Llull required the candidate and voter sets to be identical and had an elaborate two-stage tie-breaking rule ending in randomization. We disregard these issues here and cast his system into the modern idiom for election systems. (However, we note in passing that there do exist some modern papers in which the voter and candidate sets are taken to be identical, see for example the work of and references in [1].)

Faliszewski, Hemaspaandra, and Hemaspaandra introduced the study of the complexity of bribery in elections ([19], see also [18]). Bribery shares some features of manipulation and some features of control. In particular, the briber picks the voters he or she wants to affect (as in voter control problems) and asks them to vote as he or she wishes (as in manipulation).

The goal of this paper is to study Copeland$^\alpha$ elections from the point of view of computational social choice theory, in the setting where voters are rational and in the setting where the voters are allowed to have irrational preferences. (Note: When we henceforth say "irrational voters," we mean that the voters may have irrational preferences, not that they each must.) We study the issues of bribery and control and we point the reader to the work of Faliszewski, Hemaspaandra, and Schnoor [22] for work on manipulation.

A standard technique for showing that a particular election-related problem (e.g., the problem of deciding whether the chair can make his or her favorite candidate a winner by influencing at most $k$ voters not to cast their votes) is computationally intractable is to show that it is NP-hard. This approach is taken in almost all of the papers on computational social choice cited above, and it is the approach that we take in this paper. One of the justifications for using NP-hardness as a barrier against manipulation and control of elections is that in multiagent settings any attempts to influence the election's outcome are made by computationally bounded software agents that have neither human intuition nor the computational ability to solve NP-hard problems.

Recently, such papers as [30, 33, 9, 28] have studied the frequency (or sometimes, probability weight) of correctness of heuristics for voting problems. We view worst-case study as a natural prerequisite to a frequency-of-hardness attack: After all, there is no point in seeking frequency-of-hardness results if the problem at hand is in P to begin with. And if one cannot even prove worst-case hardness for a problem, then proving average-case hardness is even more beyond reach. Also, current frequency results have debilitating limitations (for example, being locked into specific distributions; depending on unproven assumptions; adopting "tractability" notions that declare undecidable problems tractable and that are not robust under even linear-time reductions). Although frequency of hardness is a fascinating and important direction, these models are arguably not ready for prime time and, contrary to some people's impression, fail to imply average-case polynomial runtime claims. [15, 28] provide discussion of some of these issues.

We also mention that during our study of Copeland control we have noticed that the proof of an important result of Bartholdi, Tovey, and Trick [6, Theorem 12] (namely, that Condorcet voting is resistant to constructive control by deleting voters) is invalid. The invalidity is due to the proof centrally using nonstrict voters, in violation of Bartholdi, Tovey, and Trick's [6] (and our) model, and the invalidity seems potentially daunting to seek to fix with the proof approach taken there. We noticed also that Theorem 14 of the same paper has a similar flaw, and we have validly reproven their claimed results using our techniques (see [20] and the in-preparation full version of this paper).

Due to space limitations all proofs in this paper are omitted.

# 2 Bribery

In this section we present our results on the complexity of bribery for the Copeland$^\alpha$ election systems, where $\alpha$ is a rational number with $0 \leq \alpha \leq 1$. Our main result, which will be presented in Section 2.1, is that each such system is resistant to bribery, regardless of voters' rationality and of our mode of operation (constructive versus destructive). In Section 2.2, we will provide vulnerability results for Llull and Copeland$^0$ with respect to "microbribery."

## 2.1 Resistance to Bribery

Let $\mathcal{E}$ be an election system. In our case, $\mathcal{E}$ will be either Copeland$^{\alpha}$ or Copeland$^{\alpha}_{\text{Irrational}}$, where $\alpha$, $0 \leq \alpha \leq 1$, is a fixed rational number. The bribery problem for $\mathcal{E}$ with rational voters is defined as follows [19].

**Name:** $\mathcal{E}$-bribery and $\mathcal{E}$-destructive-bribery.

**Given:** A set $C$ of candidates, a collection $V$ of voters specified via their preference lists over $C$, a distinguished candidate $p \in C$, and a nonnegative integer $k$.

**Question (constructive):** Is it possible to make $p$ a winner of the $\mathcal{E}$ election resulting from $(C, V)$ by modifying the preference lists of at most $k$ voters?

**Question (destructive):** Is it possible to ensure that $p$ is not a winner of the $\mathcal{E}$ election resulting from $(C, V)$ by modifying the preference lists of at most $k$ voters?

Our bribery problems are defined above for rational voters only and in the *nonunique-winner* model, i.e., asking whether a given candidate can be made, or prevented from being, *a* winner. Nonetheless, we have proven all our bribery results (and all our control results as well) both for the case of *nonunique winners* and *unique winners*. In the unique-winner model, we ask whether a given candidate can be made, or prevented from being, the sole winner. The versions of these problems for elections with irrational voters allowed is defined exactly as the rational one, with the only difference being that voters are represented via preference tables rather than preference lists, and the briber may completely change a voter's preference table at unit cost.

**Theorem 2.1** *For each rational $\alpha$, $0 \leq \alpha \leq 1$,* Copeland$^{\alpha}$ *is resistant to both constructive and destructive bribery in both the rational-voters case and the irrational-voters case, in both the nonunique-winner model and in the unique-winner model.*

## 2.2 Vulnerability to Microbribery for Irrational Voters

In this section we explore the problems related to microbribery of irrational voters. In standard bribery problems, which were considered in Section 2.1, we ask whether it is possible to ensure that a designated candidate $p$ is a winner (or, in the destructive case, to ensure that $p$ is not a winner) via modifying the preference tables of at most $k$ voters. That is, we can at unit cost completely redefine the preference table of each voter bribed. Often such an approach is right: We pay for a service (namely, the modification of the reported preference table) and we pay for it in bulk (when we buy a voter, we have complete control over his or her preferences). However, sometimes it may be more reasonable to adopt a more local approach and to pay separately for each preference-table entry flip.

For each rational $\alpha$, $0 \leq \alpha \leq 1$, we define the following two problems.

**Name:** Copeland$^{\alpha}_{\text{Irrational}}$-microbribery and Copeland$^{\alpha}_{\text{Irrational}}$-destructive-microbribery.

**Given:** A set $C$ of candidates, a collection $V$ of voters specified via their preference tables over $C$, a distinguished candidate $p \in C$, and a nonnegative integer $k$.

**Question (constructive):** Is it possible to make $p$ a winner of the election resulting from $(C, V)$ by flipping at most $k$ entries in the preference tables of voters in $V$?

**Question (destructive):** Is it possible to guarantee that $p$ is not a winner of the election resulting from $(C, V)$ by flipping at most $k$ entries in the preference tables of voters in $V$?

Our first result regarding microbribery is that destructive microbribery is easy for $\text{Copeland}^{\alpha}_{\text{Irrational}}$, for each rational $\alpha$, $0 \leq \alpha \leq 1$. We take this opportunity to remind the reader that although the definition of vulnerability requires only that there be a polynomial-time algorithm to determine whether a successful action (in the present case, a destructive microbribe) *exists*, we will in each vulnerability proof provide something far stronger, namely a polynomial-time algorithm that both determines whether a successful action exists and that, when so, explicitly finds a successful action.

**Theorem 2.2** *For each rational $\alpha$, $0 \leq \alpha \leq 1$,* $\text{Copeland}^{\alpha}_{\text{Irrational}}$ *is vulnerable to destructive microbribery.*

Theorem 2.2 is proven via greedy algorithms. The constructive case is more complicated, but we still are able to obtain, for the values $\alpha \in \{0, 1\}$, polynomial-time algorithms via a fairly involved use of flow networks to model how particular $\text{Copeland}^{\alpha}$ points travel between candidates.

**Theorem 2.3** *For $\alpha \in \{0, 1\}$,* $\text{Copeland}^{\alpha}_{\text{Irrational}}$ *is vulnerable to constructive microbribery.*

# 3 Control

## 3.1 Example of one Control Problem and Our Naming Scheme

We now give an example of how to define the control problems we consider, in both the constructive and the destructive version. Let $\mathcal{E}$ be an election system. In our case, $\mathcal{E}$ will be either $\text{Copeland}^{\alpha}$ or $\text{Copeland}^{\alpha}_{\text{Irrational}}$, where $\alpha$, $0 \leq \alpha \leq 1$, is a fixed rational number. The types of control we consider here are well-known from the literature (see, e.g., [6, 20, 26]) and we will content ourselves with the definition of only control via adding candidates. Note that there are two versions of this control type. The *unlimited* version (which, for the constructive case, was introduced by Bartholdi, Tovey, and Trick [6]) asks whether the election chair can add (any number of) candidates from a given pool of spoiler candidates in order to either make his or her favorite candidate win the election (in the constructive case), or prevent his or her despised candidate from winning (in the destructive case):

**Name:** $\mathcal{E}$-$\text{CCAC}_{\text{u}}$ and $\mathcal{E}$-$\text{DCAC}_{\text{u}}$.

**Given:** Disjoint candidate sets $C$ and $D$, a collection $V$ of voters represented via their preference lists (or preference tables in the irrational case) over the candidates in $C \cup D$, and a distinguished candidate $p \in C$.

**Constructive Question ($\mathcal{E}$-$\text{CCAC}_{\text{u}}$):** Does there exist a subset $D'$ of $D$ such that $p$ is a winner of the $\mathcal{E}$ election with candidates $C \cup D'$ and voters $V$?

**Destructive Question ($\mathcal{E}$-$\text{DCAC}_{\text{u}}$):** Does there exist a subset $D'$ of $D$ such that $p$ isn't a winner of the $\mathcal{E}$ election with candidates $C \cup D'$ and voters $V$?

The only difference in the *limited* version of constructive and destructive control via adding candidates ($\mathcal{E}$-CCAC and $\mathcal{E}$-DCAC, for short) is that the chair needs to achieve his or her goal by adding at most $k$ candidates from the given set of spoiler candidates. This version of control by adding candidates was proposed in [20] to synchronize the definition of control by adding candidates with the definitions of control by deleting candidates, adding voters, and deleting voters.

As seen in the above definition example, we use the following naming conventions for control problems. The name of a control problem starts with the election system used (when clear from context, it may be dropped), followed by CC for "constructive control" or by DC for "destructive control," followed by the acronym of the type of control: AC for "adding (a limited number of) candidates," $AC_u$ for "adding (an unlimited number of) candidates," DC for "deleting candidates," PC for "partition of candidates," RPC for "run-off partition of candidates," AV for "adding voters," DV for "deleting voters," and PV for "partition of voters," and all the partitioning cases (PC, RPC, and PV) are followed by the acronym of the tie-handling rule used in subelections, namely TP for "ties promote" (i.e., all winners of a given subelection are promoted to the final round of the election) and TE for "ties eliminate" (i.e., if there is more than one winner in a given subelection then none of this subelection's winners is promoted to the final round of the election).

Note that our definitions focus on *a winner*, i.e., they are in the *nonunique-winner model*. The *unique-winner* analogs of these problems can be defined by requiring the distinguished candidate $p$ to be the unique winner (or to not be a unique winner in the destructive case).

Let $\mathcal{E}$ be an election system and let $\Phi$ be a control type. We say $\mathcal{E}$ is *immune to $\Phi$-control* if the chair can never reach his or her goal (of making a given candidate win in the constructive case, and of blocking a given candidate from winning in the destructive case) via asserting $\Phi$-control. $\mathcal{E}$ is said to be *susceptible to $\Phi$-control* if $\mathcal{E}$ is not immune to $\Phi$-control. $\mathcal{E}$ is said to be *vulnerable to $\Phi$-control* if it is susceptible to $\Phi$-control and there is a polynomial-time algorithm for solving the control problem associated with $\Phi$. $\mathcal{E}$ is said to be *resistant to $\Phi$-control* if it is susceptible to $\Phi$-control and the control problem associated with $\Phi$ is NP-hard. The above notions were introduced by Bartholdi, Tovey, and Trick [6] (see also, e.g., [26, 35, 27, 20]).

## 3.2 Resistance and Vulnerability to Control

Our main result in this section is Theorem 3.1 below.

**Theorem 3.1** *Let $\alpha$ be a rational number with $0 \leq \alpha \leq 1$.* Copeland$^\alpha$ *elections are resistant and vulnerable to control types as indicated in Table 1. The same results hold for the case of irrational voters and in both the nonunique-winner model and the unique-winner model.*

In particular, Theorem 3.1 says that the notion widely referred to in the literature simply as "Copeland elections," which we here for clarity call Copeland$^{0.5}$, possesses all ten of our basic types of constructive resistance and, in addition, even has constructive $AC_u$ resistance. And the other notion that in the literature is occasionally referred to as "Copeland elections," namely Copeland$^0$, as well as Llull elections, which are here denoted by Copeland$^1$, both possess all ten of our basic types of constructive resistance. However, Copeland$^0$ and Copeland$^1$ are vulnerable to this eleventh type of constructive control, the incongruous but historically resonant notion of constructive control by adding an unlimited number of candidates (i.e., $CCAC_u$).

Note that Copeland$^{0.5}$ has a higher number of constructive resistances, by three, than even plurality, which was before this paper the reigning champ among natural election systems. (Although the results regarding plurality in Table 1 are stated for the unique-winner version of control, for all the table's Copeland$^\alpha$ cases, $0 \leq \alpha \leq 1$, our results hold both in the cases of unique winners and of nonunique winners, thus allowing an apples-to-apples comparison to hold.) Admittedly, plurality does perform better with respect to destructive candidate control problems, but still our study of Copeland$^\alpha$ makes significant steps forward in the quest for a fully control-resistant natural election system with an easy winner problem.

Among the systems with a polynomial-time winner problem, Copeland$^{0.5}$—and indeed all Copeland$^\alpha$, $0 < \alpha < 1$—have the most resistances currently known for any natural election system whose voters vote by giving preference lists. However, we mention that

| | Copeland$^\alpha$ | | | | | | Plurality | |
|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0$ | | $0 < \alpha < 1$ | | $\alpha = 1$ | | | |
| Control type | CC | DC | CC | DC | CC | DC | CC | DC |
| $AC_u$ | V | V | R | V | V | V | R | R |
| AC | R | V | R | V | R | V | R | R |
| DC | R | V | R | V | R | V | R | R |
| RPC-TP | R | V | R | V | R | V | R | R |
| RPC-TE | R | V | R | V | R | V | R | R |
| PC-TP | R | V | R | V | R | V | R | R |
| PC-TE | R | V | R | V | R | V | R | R |
| PV-TE | R | R | R | R | R | R | V | V |
| PV-TP | R | R | R | R | R | R | R | R |
| AV | R | R | R | R | R | R | V | V |
| DV | R | R | R | R | R | R | V | V |

Table 1: Comparison of control results for Copeland$^\alpha$ elections, where $\alpha$ with $0 \leq \alpha \leq 1$ is a rational number, and for plurality-rule elections. R means resistance to a particular control type and V means vulnerability. The results regarding plurality are due to Bartholdi, Tovey, and Trick [6] and Hemaspaandra, Hemaspaandra, and Rothe [26]. (Note that CCAC and CCDC resistance results for plurality, not handled explicitly in [6, 26], follow immediately from the respective $CCAC_u$ and $DCAC_u$ results.)

after our work, Erdélyi, Nowak, and Rothe [17] have shown that a certain rather subtle election system with a richer voter preference type—each voter specifies both a permutation and a set—has nineteen (out of a possible twenty-two) control resistances.

### 3.3 FPT Algorithm Schemes for Bounded-Case Control

The study of fixed-parameter complexity (see, e.g., [12]) has been expanding explosively since it was parented as a field by Downey, Fellows, and others in the late 1980s and the 1990s. Although the area has built a rich variety of complexity classes regarding parameterized problems, for the purpose of the current paper we need focus only on one very important class, namely, the class FPT. Briefly put, a problem parameterized by some value $j$ is said to be *fixed-parameter tractable* (equivalently, to belong to the class FPT) if there is an algorithm for the problem whose running time is $f(j)n^{O(1)}$.

**Fixed–Parameter Tractability Results.** In their seminal paper on NP-hard winner-determination problems, Bartholdi, Tovey, and Trick [5] suggested considering hard election problems for the cases of a bounded number of candidates or a bounded number of voters, and they obtained efficient-algorithm results for such cases. Within the study of elections, this same approach—seeking efficient fixed-parameter algorithms—has also been used, for example, within the study of bribery [19].

We obtain for resistant-in-general control problems a broad range of efficient algorithms for the case when the number of candidates or voters is bounded. Our algorithms are not merely polynomial time. Rather, we give algorithms that prove membership in FPT. And we prove that our FPT claims hold even under the succinct input model—in which the voters are input via "(preference-list, binary-integer-giving-frequency-of-that-preference-list)" pairs— and even in the case of irrational voters. We obtain such algorithms for all the voter-control cases, both for bounded numbers of candidates and for bounded numbers of voters, and for all the candidate-control cases with bounded numbers of candidates. On the other hand, we show that for the resistant-in-general irrational-voter, candidate-control cases, resistance still holds even if the number of voters is limited to being at most two.

Let us fix our notation. We consider two parameterizations: bounding the number of candidates and bounding the number of voters. We use the same notations used throughout this paper to describe problems, except we postpend a "-$\text{BV}_j$" to a problem name to state that the number of voters may be at most $j$, and we postpend a "-$\text{BC}_j$" to a problem name to state that the number of candidates may be at most $j$. In each case, the bound applies to the full number of such items involved in the problem. For example, in the case of control by adding voters, the $j$ must bound the total of the number of voters in the election added together with the number of voters in the pool of voters available for adding.

To state our fixed-parameter tractability results concisely, we borrow a notational approach from transformational grammar, and use square brackets as an "independent choice" notation. So, for example, the claim $\begin{bmatrix}\text{It}\\\text{She}\\\text{He}\end{bmatrix}\begin{bmatrix}\text{runs}\\\text{walks}\end{bmatrix}$ is a shorthand for six assertions: It runs; She runs; He runs; It walks; She walks; and He walks. A special case is the symbol "$\emptyset$" which, when it appears in such a bracket, means that when unwound it should be viewed as no text at all. For example, "$\begin{bmatrix}\text{Succinct}\\\emptyset\end{bmatrix}$ Copeland is fun" asserts both "Succinct Copeland is fun" and "Copeland is fun."

**Theorem 3.2** *For each rational $\alpha$, $0 \leq \alpha \leq 1$, and each choice from the independent choice brackets below, the specified parameterized (as $j$ varies over $\mathbb{N}$) problem is in* FPT*:*

$$\begin{bmatrix}\text{succinct}\\\emptyset\end{bmatrix}\text{-}\begin{bmatrix}\text{Copeland}^{\alpha}\\\text{Copeland}^{\alpha}_{\text{Irrational}}\end{bmatrix}\text{-}\begin{bmatrix}\text{C}\\\text{D}\end{bmatrix}\text{C}\begin{bmatrix}\text{AV}\\\text{DV}\\\text{PV-TE}\\\text{PV-TP}\end{bmatrix}\text{-}\begin{bmatrix}\text{BV}_j\\\text{BC}_j\end{bmatrix}.$$

**Theorem 3.3** *For each rational $\alpha$, $0 \leq \alpha \leq 1$, and each choice from the independent choice brackets below, the specified parameterized (as $j$ varies over $\mathbb{N}$) problem is in* FPT*:*

$$\begin{bmatrix}\text{succinct}\\\emptyset\end{bmatrix}\text{-}\begin{bmatrix}\text{Copeland}^{\alpha}\\\text{Copeland}^{\alpha}_{\text{Irrational}}\end{bmatrix}\text{-}\begin{bmatrix}\text{C}\\\text{D}\end{bmatrix}\text{C}\begin{bmatrix}\text{AC}_{\text{u}}\\\text{AC}\\\text{DC}\\\text{PC-TE}\\\text{PC-TP}\\\text{RPC-TE}\\\text{RPC-TP}\end{bmatrix}\text{-BC}_j.$$

The proofs of Theorems 3.2 and 3.3, which in particular employ Lenstra's [29] algorithm for bounded-variable-cardinality integer programming, are omitted here.

**FPT and Extended Control.** We now introduce and look at extended control. By that we do not mean changing the basic control notions of adding/deleting/partitioning candidates/voters. Rather, we mean generalizing past merely looking at the constructive (make a distinguished candidate a winner) and the destructive (prevent a distinguished candidate from being a winner) cases. In particular, we are interested in control where the goal can be far more flexibly specified, for example (though in the partition cases we will be even more flexible than this), we will allow as our goal region any (reasonable—there are some time-related conditions) subcollection of "Copeland output tables" (specifications of who won/lost/tied each head-to-head contest).

Since from a Copeland output table, in concert with the current $\alpha$, one can read off the $\text{Copeland}^{\alpha}_{\text{Irrational}}$ scores of the candidates, this allows us a tremendous range of descriptive flexibility in specifying our control goals, e.g., we can specify a linear order desired for the candidates with respect to their $\text{Copeland}^{\alpha}_{\text{Irrational}}$ scores, we can specify a linear-order-with-ties desired for the candidates with respect to their $\text{Copeland}^{\alpha}_{\text{Irrational}}$ scores, we can specify the exact desired $\text{Copeland}^{\alpha}_{\text{Irrational}}$ scores for one or more candidates, we can specify that we want to ensure that no candidate from a certain subgroup has a $\text{Copeland}^{\alpha}_{\text{Irrational}}$ score that ties or beats the $\text{Copeland}^{\alpha}_{\text{Irrational}}$ score of any candidate from a certain other subgroup, etc.

All the FPT algorithms given in Theorems 3.2 and 3.3 regard, on their surface, the standard control problem, which tests whether a given candidate can be made a winner (constructive case) or can be precluded from being a winner (destructive case). We note that the general approaches used to prove those results in fact yield FPT schemes even for the far more flexible notions of control that we just mentioned.

**Resistance Results.** In contrast with the FPT results in Theorems 3.2 and 3.3, we show that for each rational $\alpha$, $0 \leq \alpha \leq 1$, for $\text{Copeland}^{\alpha}_{\text{Irrational}}$ all the candidate-control cases that we showed earlier in this paper (i.e., without bounds on the number of voters) to be resistant remain resistant even for the case of bounded voters. This resistance holds even when the input is not in succinct format, and so it certainly also holds when the input is in succinct format.

It remains open whether Table 1's resistant, rational-voter, candidate-control cases remain resistant for the bounded-voter case.

# 4 Conclusions

We have shown that from the computational point of view the election systems of Llull and Copeland (i.e., $\text{Copeland}^{0.5}$) are broadly resistant to bribery and procedural control, regardless of whether the voters are required to have rational preferences. It is rather charming that Llull's 700-year-old system shows perfect resistance to bribery and more resistances to (constructive) control than any other natural system (even far more modern ones) with an easy winner-determination procedure—other than $\text{Copeland}^{\alpha}$, $0 < \alpha < 1$—is known to possess, and this is even more remarkable when one considers that Llull's system was defined long before control of elections was even explicitly studied. Copeland voting matches Llull's perfect resistance to bribery and in addition has perfect resistance to (constructive) control.

A natural open direction would be to study the complexity of control for additional election systems. Particularly interesting would be to seek existing, natural voting systems that have polynomial-time winner determination procedures but that are resistant to all standard types of both constructive *and destructive* control. Also extremely interesting would be to find single results that classify, for broad families of election systems, precisely what it is that makes control easy or hard, i.e., to obtain dichotomy meta-results for control (see Hemaspaandra and Hemaspaandra [25] for some discussion regarding work of that flavor for manipulation).

**Acknowledgments:** We thank Felix Brandt for pointing us to the work of Zermelo, and we thank the anonymous AAAI-07, AAIM-08, and COMSOC-08 referees for their helpful comments on earlier versions of this paper.

# References

[1] A. Altman and M. Tennenholtz. An axiomatic approach to personalized ranking systems. In *Proc. IJCAI'07*, pages 1187–1192. AAAI Press, 2007.

[2] K. Arrow. *Social Choice and Individual Values.* John Wiley and Sons, 1951 (revised editon, 1963).

[3] J. Bartholdi, III and J. Orlin. Single transferable vote resists strategic voting. *Social Choice and Welfare*, 8(4):341–354, 1991.

[4] J. Bartholdi, III, C. Tovey, and M. Trick. The computational difficulty of manipulating an election. *Social Choice and Welfare*, 6(3):227–241, 1989.

[5] J. Bartholdi, III, C. Tovey, and M. Trick. Voting schemes for which it can be difficult to tell who won the election. *Social Choice and Welfare*, 6(2):157–165, 1989.

[6] J. Bartholdi, III, C. Tovey, and M. Trick. How hard is it to control an election? *Mathematical and Computer Modeling*, 16(8/9):27–40, 1992.

[7] J.-A.-N. de Caritat, Marquis de Condorcet. *Essai sur l'Application de L'Analyse à la Probabilité des Décisions Rendues à la Pluralité des Voix.* 1785. Facsimile reprint of original published in Paris, 1972, by the Imprimerie Royale.

[8] V. Conitzer and T. Sandholm. Universal voting protocol tweaks to make manipulation hard. In *Proc. IJCAI'03*, pages 781–788. Morgan Kaufmann, August 2003.

[9] V. Conitzer and T. Sandholm. Nonexistence of voting rules that are usually hard to manipulate. In *Proc. AAAI'06*, pages 627–634. AAAI Press, July 2006.

[10] V. Conitzer, T. Sandholm, and J. Lang. When are elections with few candidates hard to manipulate? *Journal of the ACM*, 54(3):Article 14, 2007.

[11] A. Copeland. A "reasonable" social welfare function. Mimeographed notes from a Seminar on Applications of Mathematics to the Social Sciences, University of Michigan, 1951.

[12] R. Downey and M. Fellows. *Parameterized Complexity.* Springer-Verlag, 1999.

[13] J. Duggan and T. Schwartz. Strategic manipulability without resoluteness or shared beliefs: Gibbard–Satterthwaite generalized. *Social Choice and Welfare*, 17(1):85–93, 2000.

[14] E. Elkind and H. Lipmaa. Small coalitions cannot manipulate voting. In *Proc. FC'05*, pages 285–297. Springer-Verlag LNCS #3570, February/March 2005.

[15] G. Erdélyi, L. Hemaspaandra, J. Rothe, and H. Spakowski. On approximating optimal weighted lobbying, and frequency of correctness versus average-case polynomial time. In *Proc. FCT'07*, pages 300–311. Springer-Verlag LNCS #4639, August 2007.

[16] G. Erdélyi, M. Nowak, and J. Rothe. Sincere-strategy preference-based approval voting broadly resists control. In *Proc. MFCS'08*, pages 311–322. Springer-Verlag LNCS #5162, August 2008.

[17] G. Erdélyi, M. Nowak, and J. Rothe. Sincere-strategy preference-based approval voting fully resists constructive control and broadly resists destructive control. Technical Report cs.GT/0806.0535, ACM Computing Research Repository (CoRR), June 2008. A precursor appears as [16].

[18] P. Faliszewski. Nonuniform bribery (short paper). In *Proc. AAMAS'08*, pages 1569–1572, May 2008.

[19] P. Faliszewski, E. Hemaspaandra, and L. Hemaspaandra. The complexity of bribery in elections. In *Proc. AAAI'06*, pages 641–646. AAAI Press, 2006.

[20] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Llull and Copeland voting broadly resist bribery and control. In *Proc. AAAI'07*, pages 724–730. AAAI Press, 2007.

[21] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Copeland voting fully resists constructive control. In *Proc. AAIM'08*, pages 165–176. Springer-Verlag, June 2008. To appear.

[22] P. Faliszewski, E. Hemaspaandra, and H. Schnoor. Copeland voting: Ties matter. In *Proc. AAMAS'08*, pages 983–990, May 2008.

[23] A. Gibbard. Manipulation of voting schemes. *Econometrica*, 41(4):587–601, 1973.

[24] G. Hägele and F. Pukelsheim. The electoral writings of Ramon Llull. *Studia Lulliana*, 41(97):3–38, 2001.

[25] E. Hemaspaandra and L. Hemaspaandra. Dichotomy for voting systems. *Journal of Computer and System Sciences*, 73(1):73–83, 2007.

[26] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Anyone but him: The complexity of precluding an alternative. *Artificial Intelligence*, 171(5-6):255–285, April 2007.

[27] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Hybrid elections broaden complexity-theoretic resistance to control. In *Proc. IJCAI'07*, pages 1308–1314. AAAI Press, 2007.

[28] C. Homan and L. Hemaspaandra. Guarantees for the success frequency of an algorithm for finding Dodgson-election winners. *Journal of Heuristics.* To appear. Full version available as Department of Computer Science, University of Rochester Technical Report TR-881, September 2005, revised June 2007.

[29] H. Lenstra, Jr. Integer programming with a fixed number of variables. *Mathematics of Operations Research*, 8(4):538–548, 1983.

[30] J. McCabe-Dansted, G. Pritchard, and A. Slinko. Approximability of Dodgson's rule. *Social Choice and Welfare*, 2008. DOI 10.1007/s00355-007-0282-8.

[31] I. McLean and A. Urken. *Classics of Social Choice.* University of Michigan Press, 1995.

[32] V. Merlin and D. Saari. Copeland method II: Manipulation, monotonicity, and paradoxes. *Journal of Economic Theory*, 72(1):148–172, 1997.

[33] A. Procaccia and J. Rosenschein. Junta distributions and the average-case complexity of manipulating elections. *Journal of Artificial Intelligence Research*, 28:157–181, February 2007.

[34] A. Procaccia, J. Rosenschein, and G. Kaminka. On the robustness of preference aggregation in noisy environments. In *Proc. AAMAS'07*, pages 416–422. ACM Press, 2007.

[35] A. Procaccia, J. Rosenschein, and A. Zohar. Multi-winner elections: Complexity of manipulation, control, and winner-determination. In *Proc. IJCAI'07*, pages 1476–1481. AAAI Press, 2007.

[36] D. Saari and V. Merlin. The Copeland method I: Relationships and the dictionary. *Economic Theory*, 8(1):51–76, 1996.

[37] M. Satterthwaite. Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217, 1975.

[38] E. Zermelo. Die Berechnung der Turnier-Ergebnisse als ein Maximumproblem der Wahrscheinlichkeitsrechnung. *Mathematische Zeitschrift*, 29(1):436–460, 1929.

Piotr Faliszewski and Lane A. Hemaspaandra
Department of Computer Science
University of Rochester
Rochester, NY 14627, USA

Edith Hemaspaandra
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, USA

Jörg Rothe
Institut für Informatik
Heinrich-Heine-Universität Düsseldorf
40225 Düsseldorf, Germany