Sincere-Strategy Preference-Based Approval Voting Fully Resists Constructive Control and Broadly Resists Destructive Control

Gábor Erdélyi   Markus Nowak   Jörg Rothe

Institut für Informatik, Heinrich-Heine-Universität Düsseldorf, Germany

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Outline

1. Introduction
2. Approval Voting and its Versions
   - Approval Voting (AV)
   - Sincere-Strategy Preference-Based Approval Voting (SP-AV)
3. Results

Parts of this paper were presented at MFCS 2008.

Introduction

- Artificial systems:
  - Hybridization - Fully resists control (Hemaspaandra, Hemaspaandra, Rothe - IJCAI 2007)

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- Natural systems:

Condorcet	$3 \times R$	$4 \times I$	$7 \times V$
Approval	$4 \times R$	$9 \times I$	$9 \times V$
Llull	$14 \times R$	$0 \times I$	$8 \times V$
Copeland	$15 \times R$	$0 \times I$	$7 \times V$
Plurality	$16 \times R$	$0 \times I$	$6 \times V$

Introduction

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- Can we do better?

Voting Systems

- Set of candidates and voters:
  - $C = \{c_1, \ldots, c_m\}$
  - $V = \{v_1, \ldots, v_n\}$
- Voter preferences over C can be represented as
  - preference lists (rankings)
  - approval/disapproval vectors
- Voting rule aggregates the preferences and outputs the set of winners
  - unique winner
  - nonunique winner

Approval Voting

- Introduced by Brams and Fishburn
- Each voter specifies his or her $0 - 1$ approval vector:
  - 1 represents approval
  - 0 represents disapproval
- Ignores preference rankings
- The winners are the candidates with the highest score

Example for Approval Voting

Example

- Set of voters:
  - $V = \{v_1, \ldots, v_{10}\}$
- Set of candidates:
  - $c_1 =$ chicken
  - $c_2 =$ fish
  - $c_3 =$ pork
  - $c_4 =$ rump steak
  - $c_5 =$ tofu

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  - $c_4 =$ rump steak
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- The votes:
  - $v_1 = v_2 = 00010$
  - $v_3 = 00110$
  - $v_4 = 11110$
  - $v_5 = v_6 = v_7 = v_8 = 11011$
  - $v_9 = 01001$
  - $v_{10} = 00001$

Example for Approval Voting

Example

The result of the voting:

- $score(c_1) = 5$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 8$
- $score(c_5) = 6$

- The votes:
  - $v_1 = v_2 = 00010$
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Rules

- Proposed by Brams and Sanver
- Each voter has a preference ranking, a tie free linear ordering of all candidates:

$$c_4 > c_1 > c_3 > c_5 > c_2 > c_6$$

- Line between acceptable and inacceptable candidates:

$$c_4 \quad c_1 \mid c_3 \quad c_5 \quad c_2 \quad c_6$$

- The winners are the candidates with the highest score

Notations

- The set of candidates S_v that voter v approves of is an AV strategy
- The list of all strategies is an AV strategy profile

Conventions

- Admissibility:
  - v's most preferred candidate $\in S_v$
  - v's least preferred candidate $\notin S_v$
- Sincerity: no gaps in the approval strategies
- Sincere strategy is always admissible for at least 2 candidates if $\emptyset \neq S_v \neq C$

Example for SP-AV

Example

- $v_1 = v_2 = \quad c_4 \mid c_1 \; c_3 \; c_5 \; c_2$
- $v_3 = \quad c_3 \; c_4 \mid c_2 \; c_5 \; c_1$
- $v_4 = \quad c_3 \; c_1 \; c_2 \; c_4 \mid c_5$
- $v_5 = v_6 = \quad c_5 \; c_4 \; c_1 \; c_2 \mid c_3$
- $v_7 = v_8 = \quad c_1 \; c_5 \; c_2 \; c_4 \mid c_3$
- $v_9 = \quad c_2 \; c_5 \mid c_1 \; c_3 \; c_4$
- $v_{10} = \quad c_5 \mid c_1 \; c_4 \; c_3 \; c_2$

Example for SP-AV

Example

- $v_1 = v_2 = \; c_4 \mid c_1 \; c_3 \; c_5 \; c_2$
- $v_3 = \; c_3 \; c_4 \mid c_2 \; c_5 \; c_1$
- $v_4 = \; c_3 \; c_1 \; c_2 \; c_4 \mid c_5$
- $v_5 = v_6 = \; c_5 \; c_4 \; c_1 \; c_2 \mid c_3$
- $v_7 = v_8 = \; c_1 \; c_5 \; c_2 \; c_4 \mid c_3$
- $v_9 = \; c_2 \; c_5 \mid c_1 \; c_3 \; c_4$
- $v_{10} = \; c_5 \mid c_1 \; c_4 \; c_3 \; c_2$

The result of the voting:

- $score(c_1) = 5$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 8$
- $score(c_5) = 6$

Violations Against the Conventions

- Violations against admissible AV strategies in control via:
  - Deleting Candidates,
  - Partition of Candidates,
  - Partition of Voters.

Example for Re-Enforcing the Conventions

Example

Nonvegetarian food:

- $v_1 = v_2 = \ c_4 \mid c_1 \ c_3 \ c_2$
- $v_3 = \ c_3 \ c_4 \mid c_2 \ c_1$
- $v_4 = \ c_3 \ c_1 \ c_2 \mid c_4$ $\longleftarrow$ $v_4 = \ c_3 \ c_1 \ c_2 \ c_4 \mid$
- $v_5 = v_6 = \ c_4 \ c_1 \ c_2 \mid c_3$
- $v_7 = v_8 = \ c_1 \ c_2 \ c_4 \mid c_3$
- $v_9 = \ c_2 \mid c_1 \ c_3 \ c_4$
- $v_{10} = \ c_1 \mid c_4 \ c_3 \ c_2$ $\longleftarrow$ $v_{10} = \ \mid c_1 \ c_4 \ c_3 \ c_2$

Example for Re-Enforcing the Conventions

Example

Nonvegetarian food:

- $v_1 = v_2 = \; c_4 \mid c_1 \; c_3 \; c_2$
- $v_3 = \; c_3 \; c_4 \mid c_2 \; c_1$
- $v_4 = \; c_3 \; c_1 \; c_2 \mid c_4$
- $v_5 = v_6 = \; c_4 \; c_1 \; c_2 \mid c_3$
- $v_7 = v_8 = \; c_1 \; c_2 \; c_4 \mid c_3$
- $v_9 = \; c_2 \mid c_1 \; c_3 \; c_4$
- $v_{10} = \; c_1 \mid c_4 \; c_3 \; c_2$

Result of (C_1, V):

- $score(c_1) = 7$
- $score(c_2) = 6$
- $score(c_3) = 2$
- $score(c_4) = 7$

Example for Re-Enforcing the Conventions

Example

- $v_1 = v_2 = \ c_4 \mid c_1 \ c_5$
- $v_3 = \ c_4 \mid c_5 \ c_1$
- $v_4 = \ c_1 \ c_4 \mid c_5$
- $v_5 = v_6 = \ c_5 \ c_4 \mid c_1 \quad \longleftarrow \quad v_5 = v_6 = \ c_5 \ c_4 \ c_1 \mid$
- $v_7 = v_8 = \ c_1 \ c_5 \mid c_4 \quad \longleftarrow \quad v_7 = v_8 = \ c_1 \ c_5 \ c_4 \mid$
- $v_9 = \ c_5 \mid c_1 \ c_4$
- $v_{10} = \ c_5 \mid c_1 \ c_4$

Example for Re-Enforcing the Conventions

Example

- $v_1 = v_2 = \; c_4 \mid c_1 \; c_5$
- $v_3 = \; c_4 \mid c_5 \; c_1$
- $v_4 = \; c_1 \; c_4 \mid c_5$
- $v_5 = v_6 = \; c_5 \; c_4 \mid c_1$
- $v_7 = v_8 = \; c_1 \; c_5 \mid c_4$
- $v_9 = \; c_5 \mid c_1 \; c_4$
- $v_{10} = \; c_5 \mid c_1 \; c_4$

The result of the second stage:

- $score(c_1) = 3$
- $score(c_4) = 6$
- $score(c_5) = 6$

Plurality and Approval

Theorem

Control by	*Plurality*		*AV*	
	Constr.	*Destr.*	*Constr.*	*Destr.*
Adding an Unlimited Number of Candidates	*R*	*R*	*I*	*V*
Adding a Limited Number of Candidates	*R*	*R*	*I*	*V*
Deleting Candidates	*R*	*R*	*V*	*I*
Partition of Candidates	*TE: R* *TP: R*	*TE: R* *TP: R*	*TE: V* *TP: I*	*TE: I* *TP: I*
Run-off Partition of Candidates	*TE: R* *TP: R*	*TE: R* *TP: R*	*TE: V* *TP: I*	*TE: I* *TP: I*
Adding Voters	*V*	*V*	*R*	*V*
Deleting Voters	*V*	*V*	*R*	*V*
Partition of Voters	*TE: R* *TP: V*	*TE: R* *TP: V*	*TE: R* *TP: R*	*TE: V* *TP: V*

Results

Theorem

Control by	*SP-AV*		*AV*		*Plurality*	
	Constr.	*Destr.*	*Constr.*	*Destr.*	*Constr.*	*Destr.*
Adding an Unlimited Number of Candidates	**R**	**R**	*I*	*V*	*R*	*R*
Adding a Limited Number of Candidates	**R**	**R**	*I*	*V*	*R*	*R*
Deleting Candidates	**R**	**R**	*V*	*I*	*R*	*R*
Partition of Candidates	*TE:* **R.** *TP:* **R**	*TE:* **R** *TP:* **R**	*TE: V* *TP: I*	*TE: I* *TP: I*	*TE: R* *TP: R*	*TE: R* *TP: R*
Run-off Partition of Candidates	*TE:* **R** *TP:* **R**	*TE:* **R** *TP:* **R**	*TE: V* *TP: I*	*TE: I* *TP: I*	*TE: R* *TP: R*	*TE: R* *TP: R*
Adding Voters	**R**	**V**	*R*	*V*	*V*	*V*
Deleting Voters	**R**	**V**	*R*	*V*	*V*	*V*
Partition of Voters	*TE:* **R** *TP:* **R**	*TE:* **V** *TP:* **R**	*TE: R* *TP: R*	*TE: V* *TP: V*	*TE: R* *TP: V*	*TE: R* *TP: V*

Proof Technique

- Resistancy results follow via reduction from Hitting Set and Exact Cover by 3-Sets
- Vulnerability results follow via polynomial time Algorithm
- Some results are straightforward modifications of results and constructions from Hemaspaandra, Hemaspaandra, Rothe - Anyone but him
- But some results require new constructions

Contrast

Table

Number of resistances, immunities, and vulnerabilities to our 22 control types.

Number of	*Condorcet*	*Approval*	*Llull*	*Copeland*	*Plurality*	*SP-AV*
resistances	3	4	14	15	16	19
immunities	4	9	0	0	0	0
vulnerabilities	7	9	8	7	6	3

Summary

- SP-AV offers:
  - Full resistance to constructive control
  - Full resistance to candidate control
  - More resistances than is currently known for any other natural voting system with a polynomial-time winner problem
  - Fewer vulnerabilities than is currently known for any other natural voting system with a polynomial-time winner problem

Thank you very much!