A Deontic Logic for Socially Optimal Norms

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Outline

1. A Model of Interaction
   - Interactions as Games
   - Violation as Inefficiency
2. The Logic
   - Language and Models
   - Semantics
   - Properties
3. Conclusion and Future Work

The Tragedy of Commons

Mr.R \ Mr.C	Fair	Aggr
Fair	(3, 3)	(0, 4)
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- The owners of two broadcasting companies have to choose an advertising strategy to face the competitor.
- An aggressive campaign is individually rational but it ultimately over-exploits the common resource.

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- Conflicting interests;
- What is an *optimal* outcome?

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Definition

Given a set of outcomes W, a set of agents Agt and a partial order $\geq_i$ over W, $x \in W$ is *Strongly Pareto Efficient* (or Optimal) if there is no $y \in W$ for which $y \geq_i x$ for all $i \in Agt$ and $y >_i x$ for some.

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Constructing efficient policies

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But no agent is both able and willing to avoid such outcome.

Effectivity in games

- Pareto Efficiency is independent of agents abilities;
- We need to consider:
  - What agents can do together;
  - What collective choices are the optimal ones.

John Horty,
Deontic Logic and Agency.
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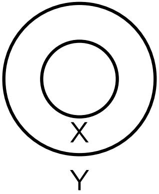
Effectivity in games

Definition (Dynamic Effectivity Function)

Given a finite set of agents Agt and a set of states W, a *dynamic effectivity function* is a function $E : W \to (2^{Agt} \to 2^{2^W})$.

E is outcome monotonic



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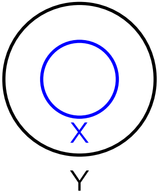

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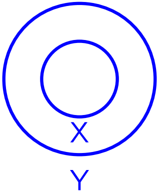

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Lifting Preferences

$$X \geq_i Y \Leftrightarrow x \geq_i y \text{ for } x \in X, y \in Y$$

$$X \geq_C Y \Leftrightarrow X \geq_i Y \text{ for } i \in C$$

$$X >_C Y \Leftrightarrow X \geq_C Y \text{ and not } Y \geq_C X$$

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Pareto Efficient Choices

Definition (Pareto Efficient Choice)

Given a choice set $\mathcal{X} \subseteq \wp(W)$, a choice $X \in \mathcal{X}$ is *Pareto Efficient for coalition* C if, and only if, for no $Y \in \mathcal{X}$, $Y \geq_i X$ for all $i \in C$ and $Y >_i X$ for some. When $C = Agt$ we speak of *Pareto Efficiency*.

Domination

Definition (Subchoice set)

If $X \in E(w)(\overline{C})$, then the *X-subchoice set* for C in w is given by $E^X(w)(C) = \{X \cap Y \mid Y \in E(w)(C)\}$.

Back to the game

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Fair	(3, 3)	(0, 4)
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- $E^{(Aggr_C)}(w)(R) = \{(Aggr_C \wedge Aggr_R),(Aggr_C \wedge Fair_R)\}$
- $E^{(Fair_C)}(w)(R) = \{(Fair_C \wedge Aggr_R),(Fair_C \wedge Fair_R)\}$

- $(Aggr_R) \subseteq [[Aggr_R]]^{PD}$
- $[[Aggr_R]]^{PD} = \{w | PD, w \models Aggr_R\}$

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Definition (Domination)

Given an effectivity function E, X is *undominated* for C in w (abbr. $X \triangleright_{C,w}$) if, and only if,
(i) $X \in E(w)(C)$ and $X'(\subset X) \neq E(w)(C)$
(ii) for all $Y \in E(w)(\overline{C})$, $(X \cap Y)$ is Pareto Efficient in $E^{Y}(w)(C)$ for C.

- 'inwardly' Pareto-like, 'outwardly' strategic.

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Violation

Definition (Violation)

If $C \subseteq C'$, then the choice $X \in E(w)(C)$ is a violation by C towards C' in w ($X \in VIOL_{C,C',w}$) iff it is not undominated for C' in w.

We indicate with $VIOL_{C,w}$ the set $\mathcal{X}$ of violations by C at w towards Agt.

Violation as Inefficiency

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- $VIOL_{Agt,w} = (Aggr_C \wedge Aggr_R)$

Row and Column can cooperate to avoid inefficiency.

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Syntax

The syntax of the Logic is defined as follows:

$$\phi ::= p|\neg\phi|\phi \vee \phi|[C]\phi|P(C,\phi)|F(C,\phi)|O(C,\phi)|[rational_C]\phi$$

The informal reading of the modalities is:

- "Coalition C can choose ϕ",
- "It is permitted (/forbidden/obligated) for coalition C to choose ϕ",
- "It is rational for coalition C to choose ϕ".

Structures

Definition (Models)

A *model* is a quadruple

$$(W, E, \{\geq_i\}_{i \in Agt}, V)$$

where:

- W is a nonempty set of states;
- $E : W \longrightarrow (2^{Agt} \longrightarrow 2^{2^W})$ is an outcome monotonic effectivity function.
- $\geq_i \subseteq W \times W$ for each $i \in Agt$, is the preference relation.
- $V : W \longrightarrow 2^{Prop}$ is the valuation function.

Semantics

$$
\begin{array}{rcl}
M, w \models p & \text{iff} & p \in V(w) \\
M, w \models \neg\phi & \text{iff} & M, w \not\models \phi \\
M, w \models \phi \wedge \psi & \text{iff} & M, w \models \phi \text{ and } M, w \models \psi \\
M, w \models [C]\phi & \text{iff} & [[\phi]]^M \in E(w)(C)
\end{array}
$$

$[[\phi]]^M =_{def} \{w \in W \mid M, w \models \phi\}$

Marc Pauly,
A Logic for Social Software.
PhD thesis, 2001.

Semantics

$$M, w \models [rational_C]\phi \quad \text{iff} \quad \forall X (X \triangleright_{C,w} \;\Rightarrow\; X \subseteq [[\phi]]^M)$$

Semantics

$$M, w \models P(C, \phi) \quad \text{iff} \quad \exists X \in E(w)(C) \text{ s.t. } X \in \overline{VIOL}_{C,w} \text{ and } X \subseteq [[\phi]]^M$$
$$M, w \models F(C, \phi) \quad \text{iff} \quad \forall X \in E(w)(C)(X \subseteq [[\phi]]^M \Rightarrow X \in VIOL_{C,w})$$
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Deontic Operators

- $P(C, \phi)$ iff $\exists X \in E(w)(C)$ s.t. $X \in \overline{VIOL}_{C,w}$ and $X \subseteq [[\phi]]^M$ is a *socially safe* permission;
- $O(C, \phi)$ iff $\forall X \in E(w)(C)(X \in \overline{VIOL}_{C,w} \Rightarrow X \subseteq [[\phi]]^M)$ tells a Coalition how to behave to avoid social inefficiency.

Validities

Some Validities	
1	$P(C,\phi) \rightarrow \neg O(C,\neg\phi)$
2	$F(C,\phi) \leftrightarrow \neg P(C,\neg\phi)$
3	$P(C,\phi) \vee P(C,\psi) \rightarrow P(C,\phi \vee \psi)$
4	$O(C,\phi) \wedge [C]\phi \rightarrow P(C,\phi)$
5	$[rational_{Agt}]\phi \wedge [rational_C]\neg\phi \rightarrow F(C,\neg\phi)$

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Non-Validities

Some **non**-Validities	
6	$\neg O(C, \neg\phi) \rightarrow P(C, \phi)$
7	$O(C, \phi) \leftrightarrow \neg O(C, \neg\phi)$
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9	$[rational_C]\phi \leftrightarrow [rational_{Agt}]\phi$

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- $\models_{PD} F(R, Aggr_R) \wedge F(C, Aggr_C)$ (by 5)

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Coalitionally Optimal Norms

$$M, w \models O^{C'}(C, \phi) \quad \text{iff} \quad \forall X \in E(w)(C)(X \in \overline{VIOL}_{C,C',w} \Rightarrow X \subseteq [[\phi]]^M)$$

$$\models_{\mathcal{M}} O^{C}(C, \phi) \leftrightarrow [rational_C]\phi$$

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Conclusion

- We defined the concept of optimality as Pareto Efficiency over the possible system choices;
- We studied the interaction between coalitionally rational and socially rational choice;
- We provided a Cooperative Game Theoretical semantics of Deontic Logic.

Further Developments

- Dynamics: what happens to efficient outcomes when preferences and choices change?
- Regulation: forcing properties that are not socially desirable;

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Further Developments

Mr.B \ Mr.B	Fair	Aggr
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