## Aggregating Referee Scores: an Algebraic Approach

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## Outline

(1) Introduction

(2) Problem Formulation
(3) The Opinion Calculus

4 Evaluating Referee Scores
(5) Conclusion

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## RUN Research Group



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## Peer Reviewing

- Peer reviewing (or refereeing) is the process of evaluating submitted documents by anonymous experts (referees)
- Widely applied by scientific journals, conferences, and funding agencies
- Submitted documents are typically reviewed by 3-4 referees
- Referee reports typically contain:
- Scores for various criteria (e.g. originality, clarity, etc.)
- Overall score for paper quality (e.g. 1-10)
- Level of expertise (e.g. 1-10)
- Detailed comments
- Papers with highest aggregated scores are accepted $\Rightarrow$ How?


## Demo

A prototype implementation is available at:
http://www.iam.unibe.ch/~run/referee

## Formal Setting

Input:

$$
\begin{aligned}
\mathcal{D}=\{1, \ldots, n\} & \rightarrow \text { submitted documents } \\
\mathcal{R}=\{1, \ldots, m\} & \rightarrow \text { referees } \\
\text { referees }(i) \subseteq \mathcal{R} & \rightarrow \text { referees assigned to document } i \\
s_{i, j}=\left(q_{i, j}, e_{i, j}\right) & \rightarrow \text { referee } j \text { 's score for document } i \\
q_{i, j} \in[0,1] & \rightarrow \text { quality judgement } \\
e_{i, j} \in[0,1] & \rightarrow \text { expertise level }
\end{aligned}
$$

## Formal Setting (cont.)

Output:

$$
\begin{aligned}
s_{i}=\bigotimes_{j \in \text { referees }(i)} s_{i, j} & \rightarrow \text { combined score } s_{i}=\left(q_{i}, e_{i}\right) \text { for document } i \\
q_{i} \in[0,1] & \rightarrow \text { combined quality judgement } \\
e_{i} \in[0,1] & \rightarrow \text { combined expertise level } \\
\mathcal{S}=\left\{s_{1}, \ldots, s_{n}\right\} & \rightarrow \text { set of combined scores } \\
(\mathcal{D}, \preceq) & \rightarrow \text { total preorder over } \mathcal{D} \\
r: \mathcal{D} \rightarrow \mathbb{N} & \rightarrow \text { ranking function over } \mathcal{D}
\end{aligned}
$$

Note that classifying the documents (e.g. accepted/rejected) is a special case of a total preorder $\preceq$

## Example

|  | Referees |  |  |  |  | Total Preorder |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | $\otimes$ | $4 \preceq\{1,3\} \preceq 2$ |
| 1 | $s_{1,1}$ | - | $s_{1,3}$ | $s_{1,4}$ | - | $s_{1}$ | $r(1)=2$ |
| $\stackrel{\text { ® }}{\text { ® }}$ | $s_{2,1}$ | $s_{2,2}$ | - | - | $s_{2,5}$ | $s_{2}$ | $r(2)=1$ |
| $\bigcirc$ | - | $s_{3,2}$ | $s_{3,3}$ | $s_{3,4}$ | - | $s_{3}$ | $r(3)=2$ |
| 4 | $s_{4,1}$ | - | $s_{4,3}$ | - | $s_{4,5}$ | $s_{4}$ | $r(4)=4$ |

## Problem Formulation

- Problem 1: Find an appropriate combination operator $\otimes$
- Problem 2: Find an appropriate total preorder $\preceq$
- Solution: Apply the opinion calculus
(i) Transform scores $s_{i, j}$ into opinions $\varphi_{i, j}$
(ii) Apply the combination operator $\otimes$ defined for independent opinions $\Rightarrow \varphi_{i}$
(iii) Use various probabilistic transformations $f \in\{g, h, p\}$ to turn each $\varphi_{i}$ into a Bayesian opinion $f\left(\varphi_{i}\right)$
(iv) Use the natural total order $\preceq_{0}$ of Bayesian opinions to define $\preceq$


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## Opinions

- The opinion calculus is an algebraic version of the Dempster's theory of lower and upper probabilities (Dempster, 1967) for two-valued hypotheses $H \in\{$ yes, no $\}$
- Terminology and references:
- (Hajek and Valdes, 1991) $\rightarrow$ Dempster pairs, dempsteroids
- (Jøsang, 1997) $\rightarrow$ opinions, subjective logic
- (Daniel, 2002) $\rightarrow$ d-pairs, Dempster's semigroup
- An opinion relative to $H$ is a triple $\varphi=(b, d, i) \in[0,1]^{3}$
- $b+d+i=1$
- $b=$ degree of belief of $H$
- $d=$ degree of disbelief $H$
- $i=$ degree of ignorance relative to $H$
- Dempster's theory provides a probabilistic interpretation for $b$, $d$, and $i$


## Opinion Triangle



## Opinion Classes



## Combining Opinions

- Let $\varphi_{1}=\left(b_{1}, d_{1}, i_{1}\right)$ and $\varphi_{2}=\left(b_{2}, d_{2}, i_{2}\right)$ be independent:

$$
\varphi_{1} \otimes \varphi_{2}=\left(\frac{b_{1} b_{2}+b_{1} i_{2}+i_{1} b_{2}}{1-b_{1} d_{2}-d_{1} b_{2}}, \frac{d_{1} d_{2}+d_{1} i_{2}+i_{1} d_{2}}{1-b_{1} d_{2}-d_{1} b_{2}}, \frac{i_{1} i_{2}}{1-b_{1} d_{2}-d_{1} b_{2}}\right)
$$

- Let $\varphi_{i}=\left(b_{i}, d_{i}, i_{i}\right), 1 \leq i \leq n$, be independent:

$$
\begin{aligned}
& \varphi_{1} \otimes \cdots \otimes \varphi_{n} \\
& \quad=\left(\frac{1}{K}\left[\prod_{i}\left(b_{i}+i_{i}\right)-\prod_{i} i_{i}\right], \frac{1}{K}\left[\prod_{i}\left(d_{i}+i_{i}\right)-\prod_{i} i_{i}\right], \frac{1}{K} \prod_{i} i_{i}\right) \\
& \quad \text { for } K=\prod_{i}\left(b_{i}+i_{i}\right)+\prod_{i}\left(d_{i}+i_{i}\right)-\prod_{i} i_{i}>0
\end{aligned}
$$

- Click here to start demo


## The Opinion Monoid

- $\Phi=\{(b, d, i): b+d+i=1\}$ is not closed under $\otimes$
- $\otimes$ is undefined for $p=(1,0,0)$ and $n=(0,1,0)$
- Add inconsistent opinion $z=(1,1,-1)$
- Define $p \otimes n=n \otimes p=z$
- Define $\varphi \otimes z=z \otimes \varphi=z$, for all $\varphi \in \Phi$
- $\Phi_{z}=\Phi \cup\{z\}$ is closed under $\otimes$
- $\otimes$ is commutative
- $\otimes$ is associative
- Therefore, $\left(\Phi_{z}, \otimes\right)$ is a commutative semigroup
- $e=(0,0,1)$ is the identity element: $e \otimes \varphi=\varphi \otimes e=\varphi$
- $z=(1,1,-1)$ is the zero element: $z \otimes \varphi=\varphi \otimes z=z$
- Therefore, $\left(\Phi_{z}, \otimes, e\right)$ is a commutative monoid with zero element $z$


## Other Opinion Monoids

| Name | Notation | Definition |  | Identity |
| :--- | :---: | :--- | :---: | :---: |
| Zero |  |  |  |  |
| general | $\Phi_{z}$ | $\Phi \cup\{z\}$ | $e$ | $z$ |
| non-negative | $\Phi_{\geq}$ | $\{(b, d, i) \in \Phi: b \geq d\}$ | $e$ | $p$ |
| non-positive | $\Phi_{\leq}$ | $\{(b, d, i) \in \Phi: b \leq d\}$ | $e$ | $n$ |
| simple non-negative | $\Phi_{+}$ | $\{(b, d, i) \in \Phi: d=0\}$ | $e$ | $p$ |
| simple non-positive | $\Phi_{-}$ | $\{(b, d, i) \in \Phi: b=0\}$ | $e$ | $n$ |
| indifferent | $\Phi_{=}$ | $\{(b, d, i) \in \Phi: b=d\}$ | $e$ | $u$ |
| Bayesian | $\Phi_{0}$ | $\{(b, d, i) \in \Phi: i=0\} \cup\{z\}$ | $u$ | $z$ |

## Remarks:

- $\Phi_{+}, \Phi_{-}, \Phi_{=}, \Phi_{0} \backslash\{z\}$ possess a natural total order
- $\Phi_{0} \backslash\{p, n, z\}$ forms a commutative group


## Probabilistic Transformations

- A probabilistic transformation is mapping $f: \Phi_{z} \rightarrow \Phi_{0}$
- Belief transformation:

$$
g(\varphi)=\left(\frac{b}{b+d}, \frac{d}{b+d}, 0\right)
$$

- Plausibility transformation:

$$
h(\varphi)=\left(\frac{1-d}{1+i}, \frac{1-b}{1+i}, 0\right)=\varphi \otimes u
$$

- Pignistic transformation:

$$
p(\varphi)=\left(b+\frac{i}{2}, d+\frac{i}{2}, 0\right)
$$

- For each $f \in\{g, h, p\}$, the total order over $\Phi_{0} \backslash\{z\}$ defines a total preorder over $\Phi$


## Probabilistic Transformations (cont.)



Belief Transformation



Pignistic Transformation

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## Referee Scores as Opinions

- Problem 1: Define a mapping from scores to opinions
- Score: $s=(q, e) \in[0,1] \times[0,1]$
- Opinion: $\varphi=(b, d, i) \in \Phi$
- Mapping: $\Delta:[0,1] \times[0,1] \rightarrow \Phi$
- Solution: Probabilistic interpretation of $q$ and $e$
- $e=P(E)$
$\Rightarrow$ probability of the referee being an expert (event $E$ )
- $q=P(Q \mid E)$
$\Rightarrow$ conditional probability of the document being a high-quality paper (event $Q$ ), given that the referee is an expert (event $E$ )
- If $E$ and $Q$ are probabilistically independent, then

$$
\Delta(s)=(b, d, i)=(e \cdot q, e \cdot(1-q), 1-e)
$$

## Mapping Scores into Opinions



Remarks:

- $\Delta$ is invertible: $\Delta^{-1}(\varphi)=(q, e)=\left(\frac{b}{1-i}, 1-i\right)$, for $\varphi \neq e$
- $s=\Delta^{-1}\left(\Delta\left(s_{1}\right) \otimes \cdots \otimes \Delta\left(s_{k}\right)\right)$


## Combining Scores

$$
\begin{aligned}
s_{1}=(0.80,0.50) \Rightarrow & \varphi_{1} \\
s_{2}=(0.40,0.25) \Rightarrow & \varphi_{2} \\
s_{3}=(0.20,0.75) \Rightarrow & \varphi_{3} \\
& \Downarrow \\
s=(0.37,0.86) & \Leftarrow \varphi
\end{aligned}
$$



## Document Ranking

- Problem 2: Determine document ranking
- Documents: $\mathcal{D}=\{1, \ldots, n\}$
- Scores: $\mathcal{S}=\left\{s_{1}, \ldots, s_{n}\right\}$
- Opinions: $\Delta=\left\{\varphi_{1}, \ldots, \varphi_{n}\right\}$
- Define ranking $r(i)$ for all $i \in \mathcal{D}$
- Solution: Use probabilistic transformation of $\varphi_{i}$
- $f\left(\varphi_{i}\right)=\left(b_{i}, 1-b_{i}, 0\right)$ for $f \in\{g, h, p\}$
- $i \preceq j \Leftrightarrow f\left(\varphi_{i}\right) \preceq_{0} f\left(\varphi_{j}\right) \Leftrightarrow b_{i} \leq b_{j}$
- $i \prec j \Leftrightarrow i \preceq j \wedge i \nsucceq j$
- Ranking: $r(i)=|\{j \in \mathcal{D}: i \prec j\}|+1$


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## Conclusion

- Peer reviewing leads to an important judgement aggregation problem
- It can be solved using the opinion calculus (Dempster-Shafer theory)
- The method can be implemented efficiently
- Future work and open problems:
- Get into a conference management tools (CyberChair, ...)
- Empirical study based on data from real conferences
- Compare/evaluate different probabilistic transformations

