On the Robustness of Preference Aggregation in Noisy Environments

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Outline

- Motivation
- Definition of Robustness
- Results:
  - About Robustness in general.
  - Sketch of results about specific voting rules.
- Conclusions

Voting in Noisy Environments

Motivation	Definition	Results	Conclusions

- Election: set of **voters** $N=\{1,\ldots,n\}$, **alternatives** / **candidates** $A=\{x_1,\ldots,x_m\}$.
- Voters have linear preferences R^i; winner of the election determined according to a **social choice function** / **voting rule**.
- Preferences may be faulty:
  - – Agents may misunderstand choices.
  - – Robots operating in an unreliable environment.

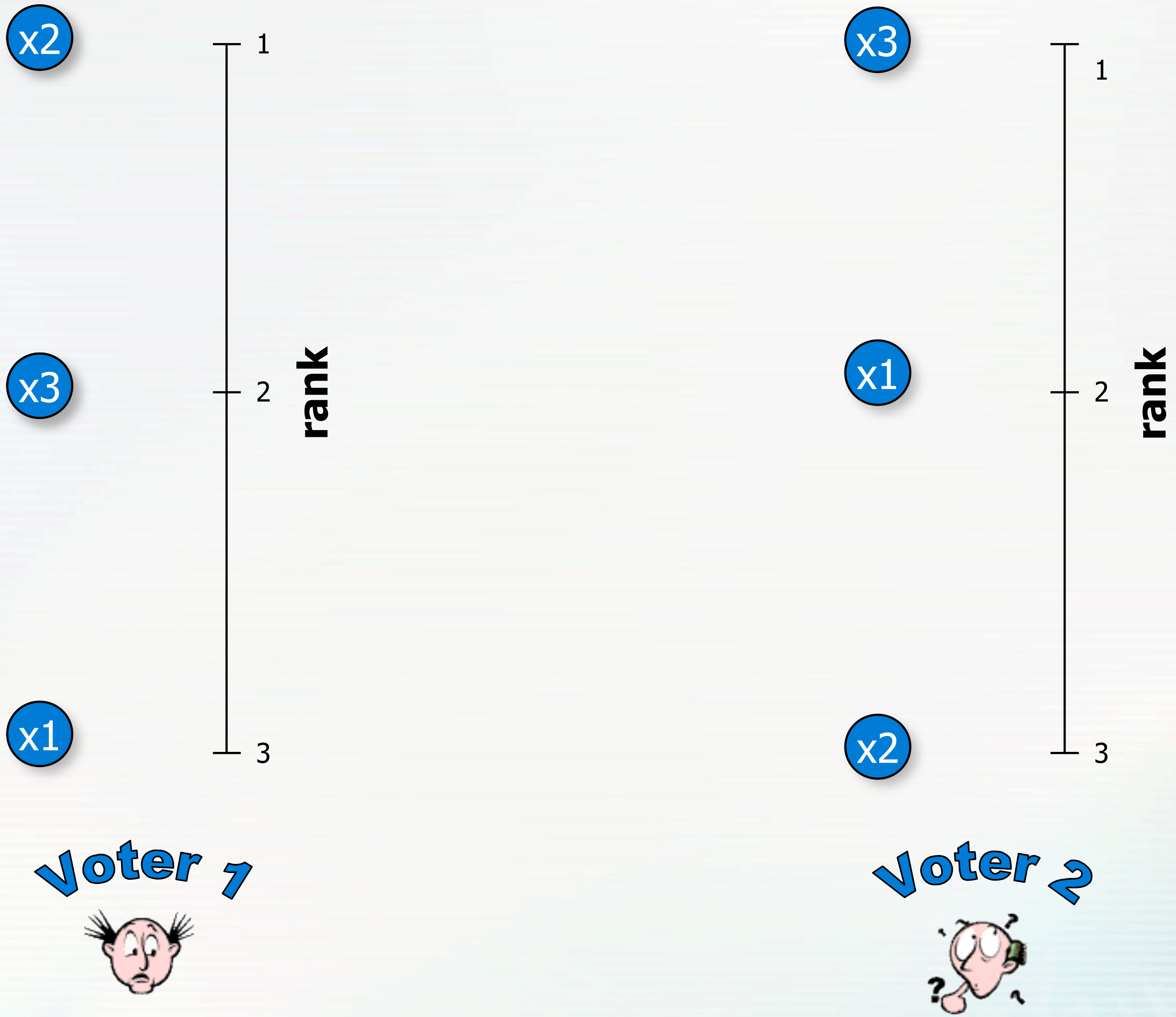
Possible Informal Definitions of Robustness

- **Option 1**: given a uniform distribution over preference profiles, what is the probability of the outcome **not** changing, when the faults are adversarial?
- Reminiscent of manipulation.
- **Option 2** (ours): given the worst preference profile and a uniform distribution over faults, what is the probability of the outcome **not** changing?

Formal Definition of Robustness

- Fault: a “switch” between two adjacent candidates in the preferences of one voter.
  - Depends on representation; ∃ consistent, “quite good” representation.

Faults Illustrated



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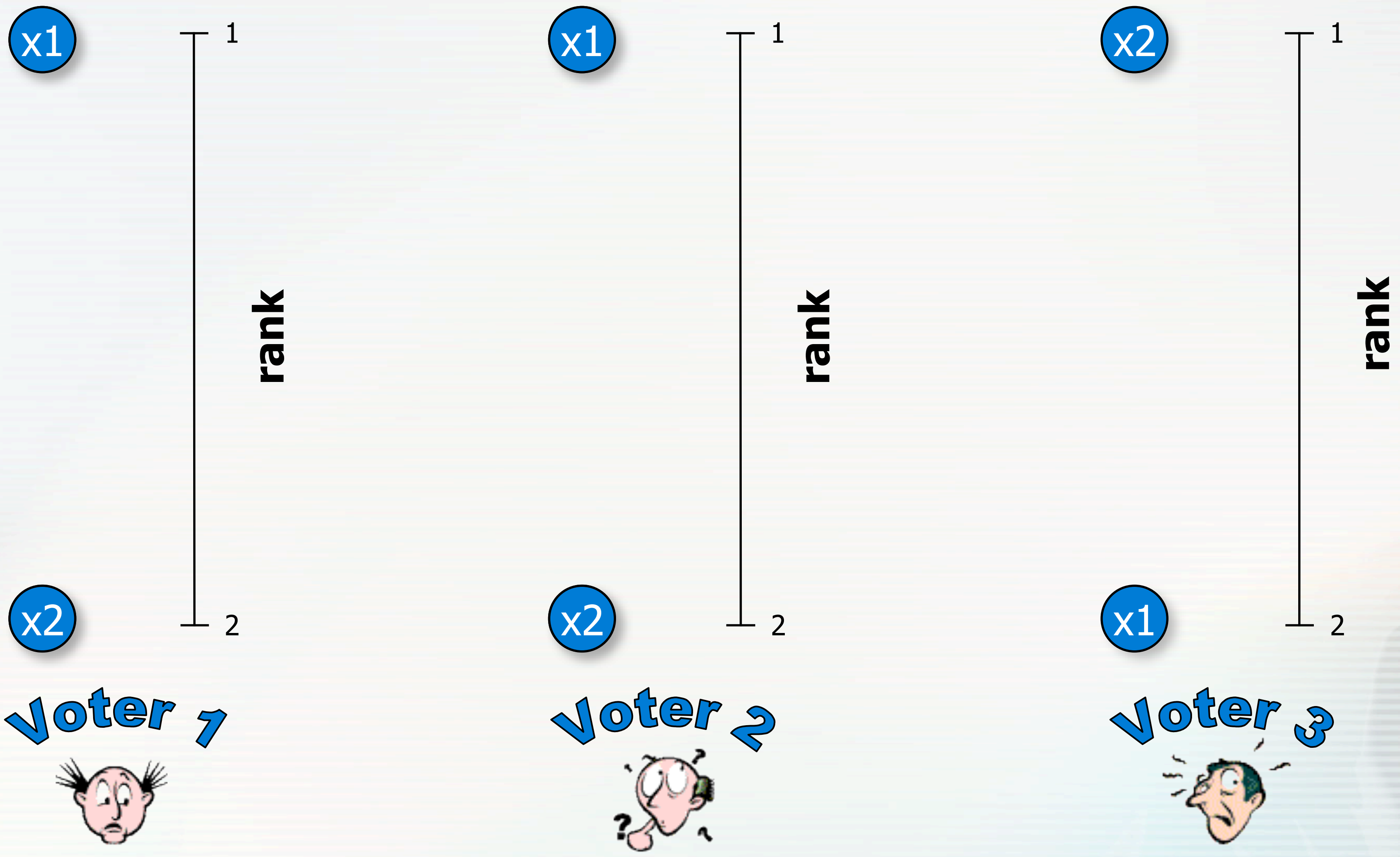
- $D_k(R)$ = prob. dist. over profiles; sample: start with R and perform k independent uniform switches.
- The **k-robustness** of F at R is: $\rho(F,R) = Pr_{R1\sim Dk(R)}[F(R)=F(R_1)]$

Robustness Illustrated

F = Plurality. 1-Robustness at R is 1/3.



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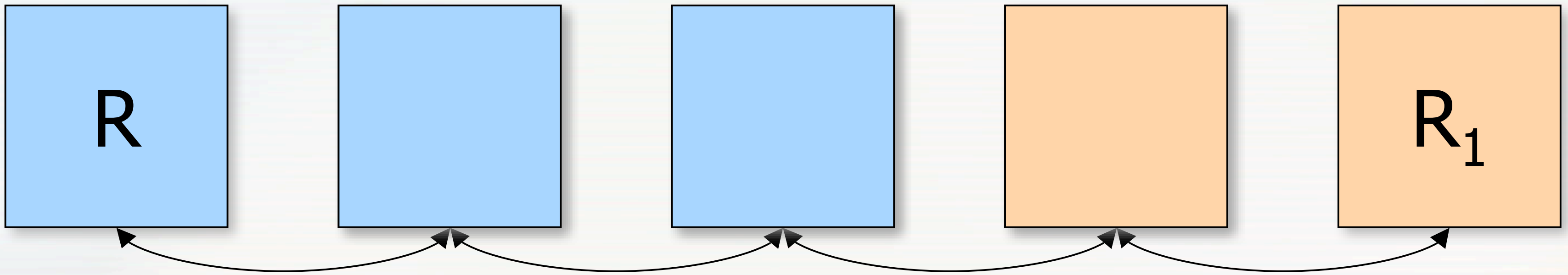
- The **k-robustness** of F is: $\rho(F) = \min_R \rho(F,R)$

Simple Facts about Robustness

- **Theorem:** $\rho_k(F) \geq (\rho_1(F))^k$
- **Theorem:** If Ran(F)>1, then $\rho_1(F) < 1$.
- Proof:



1-robustness of Scoring rules

Motivation	Definition	Results	Conclusions

- Scoring rules: defined by a vector $\alpha=\langle\alpha_1,\ldots,\alpha_m\rangle$, all $\alpha_i \geq \alpha_{i+1}$. Each candidate receives α_i points from every voter which ranks it in the i'th place.
  - Plurality: $\alpha=\langle 1,0,\ldots,0\rangle$
  - Borda: $\alpha=\langle m-1,m-2,\ldots,0\rangle$
- $A_F = \{1 \leq i \leq m-1: \alpha_i > \alpha_{i+1}\}$; $a_F = |A_F|$
- **Proposition:** $\rho_1(F) \geq (m-1-a_F)/(m-1)$
- Proof:
  - A fault only affects the outcome if $\alpha_i > \alpha_{i+1}$.
  - There are a_F such positions per voter, out of m-1.
- **Proposition:** $\rho_1(F) \leq (m-a_F)/m$

Results about 1-robustness

Motivation | Definition | **Results** | Conclusions

Rule	Lower Bound	Upper Bound
Scoring	$(m-1-a_F)/(m-1)$	$(m-a_F)/m$
Copeland	0	$1/(m-1)$
Maximin	0	$1/(m-1)$
Bucklin	$(m-2)/(m-1)$	1
Plurality w. Runoff	$(m-5/2)/(m-1)$	$(m-5/2)/(m-1)+5m/(2m(m-1))$

Conclusions

- k-robustness: worst-case probability that k switches change outcome.
- Connection to 1-robustness:
  - High 1-robustness $\Rightarrow$ high k-robustness.
  - Low 1-robustness $\Rightarrow$ can expect low k-robustness.
- Tool for designers:
  - Robust rules: Plurality, Plurality w. Runoff, Veto, Bucklin.
  - Susceptible: Borda, Copeland, Maximin.
- Future work:
  - Different error models.
  - Average-case analysis.