Merging Judgments and the Problem of Truth-Tracking

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The discursive dilemma

Group of 7 people

$(P \wedge Q) \leftrightarrow R$

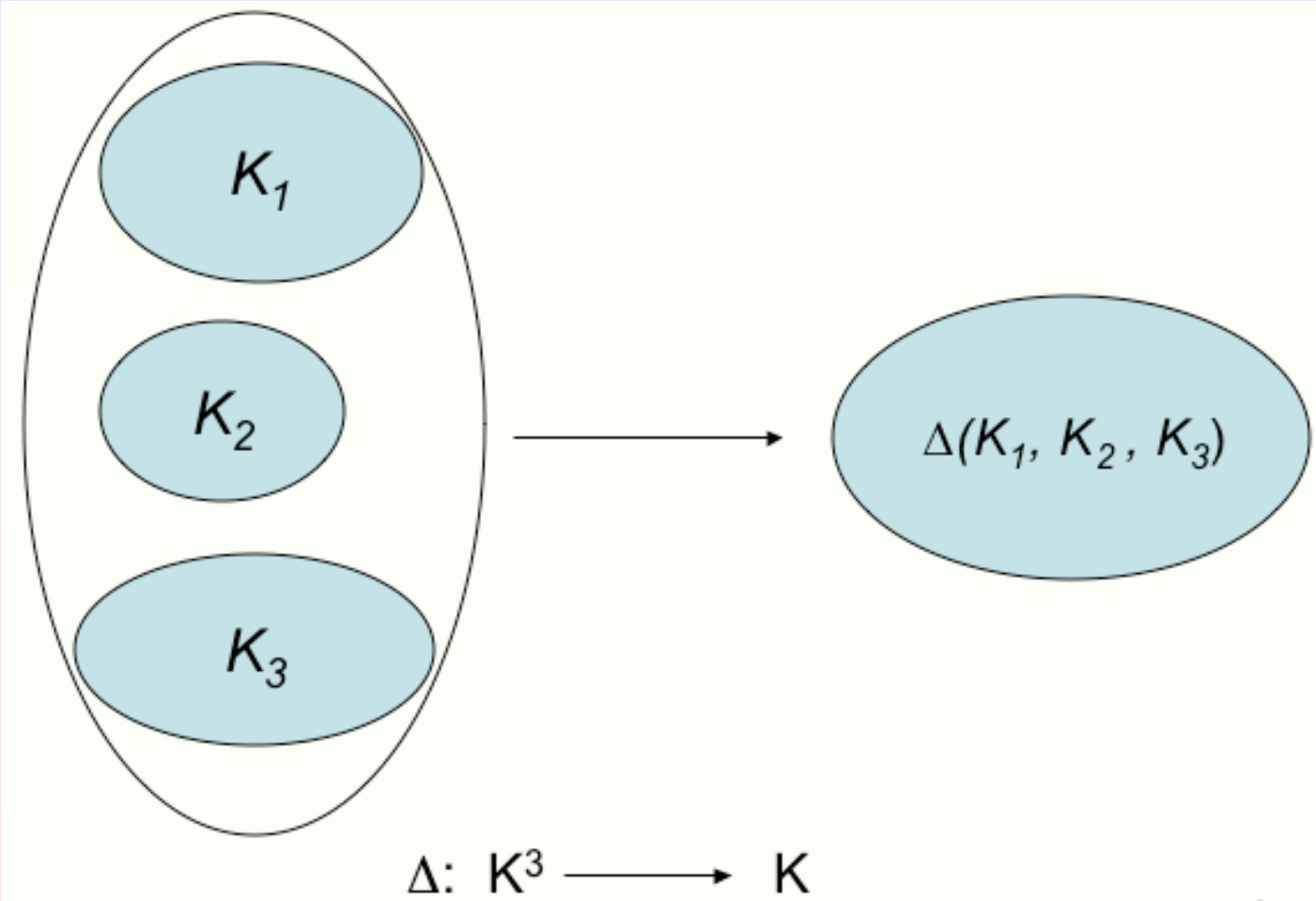
	P	Q	R
Members 1,2,3	Yes	Yes	Yes
Members 4,5	Yes	No	No
Members 6,7	No	Yes	No
Majority	Yes	Yes	No

Two escape routes: premise-based procedure (PBP) or conclusion-based procedure (CBP). PBP and CBP lead to two different results.

Need for an aggregation procedure that assigns a collective judgment set (reasons + conclusion) to the individual judgment sets.

The reasons for a decision are as important as the decision

Belief merging: an aggregation procedure imported from AI



Belief merging: the intuitive idea

- Belief merging (Konieczny & Pino-Pérez) requires the satisfaction of integrity constraints (IC): these are extra conditions imposed on the collective outcome.
- Distance-based approach in belief merging: collective outcomes (satisfying IC) determined via minimization of *distance* with respect to profiles of individual bases.
- What happens when we apply methods from belief merging to collective decision problems?

Belief merging applied to the discursive dilemma

Agenda $X = \{P, Q, R\}$ with $IC = \{(P \wedge Q) \leftrightarrow R\}$

$\text{Mod}(K_1)$=$\text{Mod}(K_2)$=$\text{Mod}(K_3)$=$\{(1,1,1)\}$
$\text{Mod}(K_4)$=$\text{Mod}(K_5)$=$\{(1,0,0)\}$ and $\text{Mod}(K_6)$=$\text{Mod}(K_7)$=$\{(0,1,0)\}$

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	Δ^E_{IC}
(1,1,1)	0	0	0	2	2	2	2	8
(1,1,0)	1	1	1	1	1	1	1	7
(1,0,1)	1	1	1	1	1	3	3	11
(1,0,0)	2	2	2	0	0	2	2	10
(0,1,1)	1	1	1	3	3	1	1	11
(0,1,0)	2	2	2	2	2	0	0	10
(0,0,1)	2	2	2	2	2	2	2	14
(0,0,0)	3	3	3	1	1	1	1	13

The problem of truth-tracking

Assumption: There is a factual truth that can (and should) be tracked by the aggregation procedure.

- Belief merging avoids paradoxical outcomes. But how good is it in selecting the *right* outcome?
- Bovens & Rabinowicz (2006) have tested PBP and CBP in terms of truth-trackers.

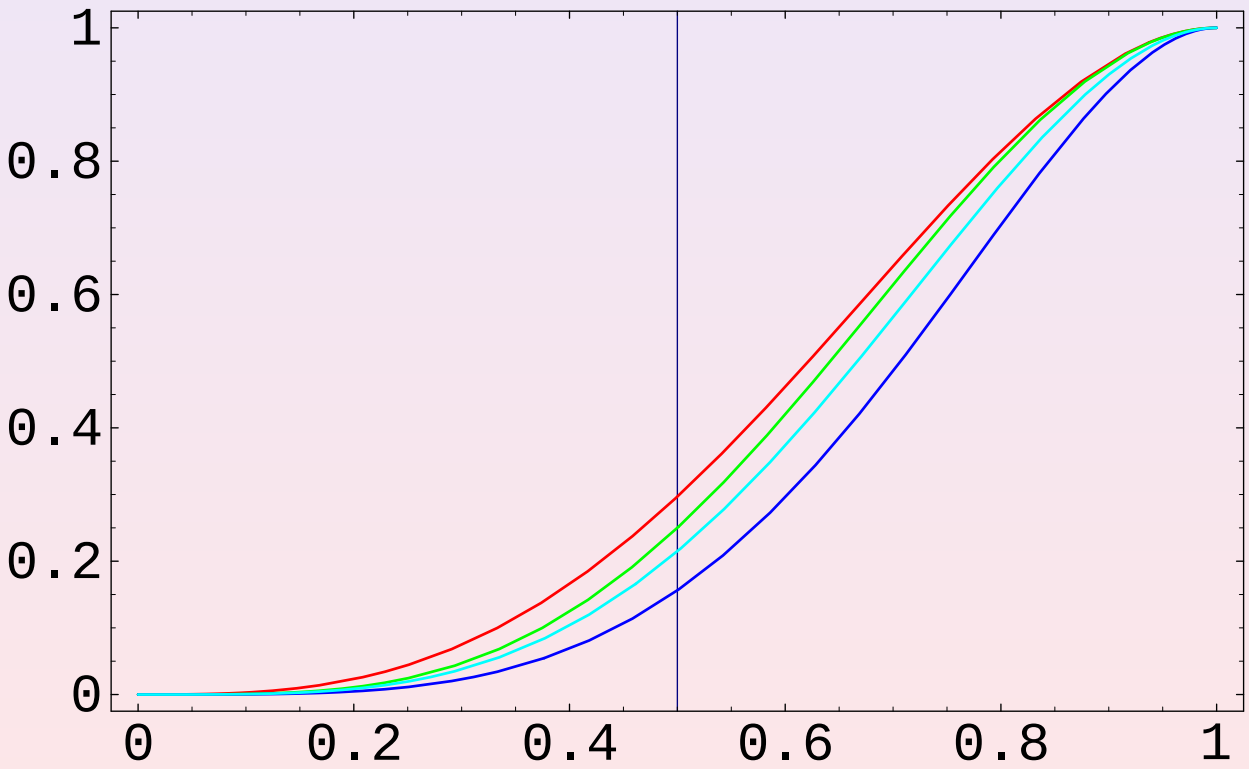
Our framework

- The chance that an individual correctly judges the truth or falsity of the propositions P and Q (her *competence*) is p.
- The voters are equally competent and independent.
- The prior probability that P and Q are true are equal (q).
- P and Q are (logically and probabilistically) independent.
- We consider the case of $P \wedge Q \leftrightarrow R$
- There are 4 possible situations:
  - $S_1 = \{P, Q, R\} = (1, 1, 1)$
  - $S_2 = \{P, \neg Q, \neg R\} = (1, 0, 0)$
  - $S_3 = \{\neg P, Q, \neg R\} = (0, 1, 0)$
  - $S_4 = \{\neg P, \neg Q, \neg R\} = (0, 0, 0)$

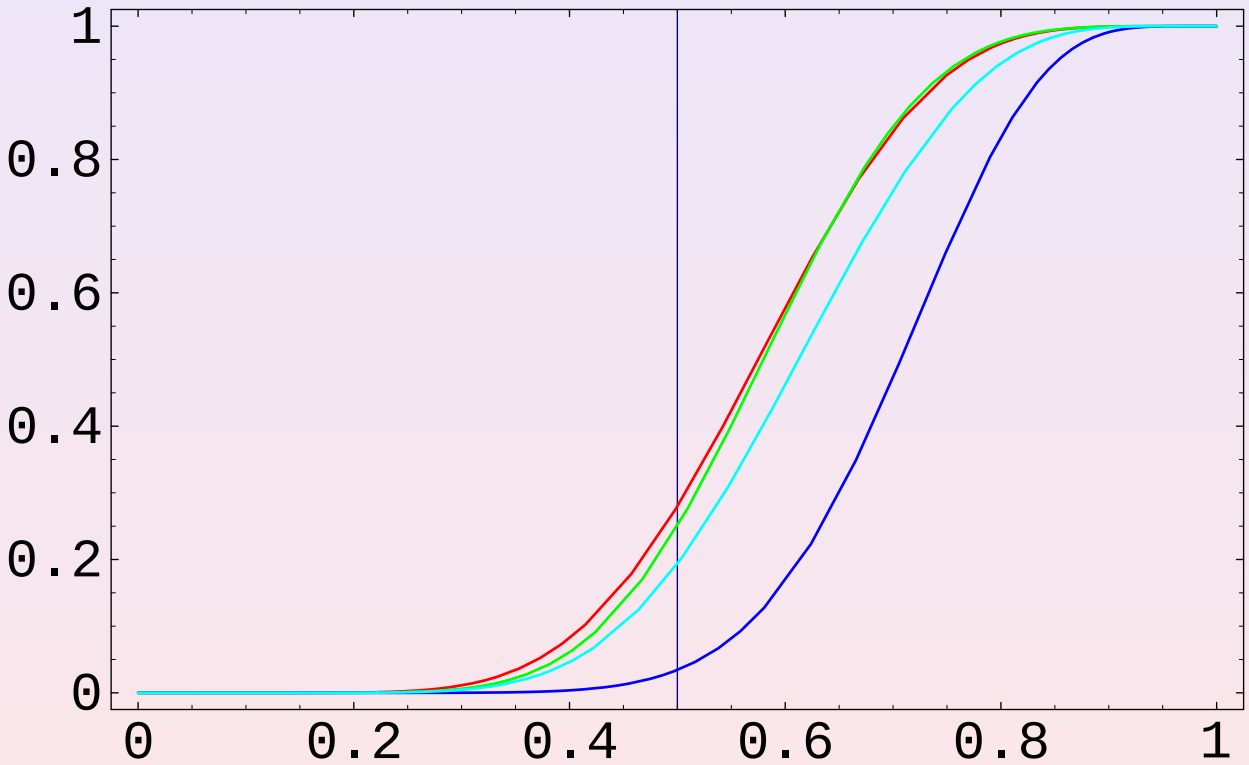
Our framework

- We want to calculate the probability of the proposition F: Fusion ranks the right judgment set first.
- Note that $\mathcal{P}(F) = \sum_{i=1}^{4} \mathcal{P}(F|S_i) \cdot \mathcal{P}(S_i)$, so that we have to calculate the conditional probabilities $\mathcal{P}(F|S_i)$ for $i = 1, \ldots, 4$.
- Let's assume that S_1 is the right judgment set.
- Idea: Fusion gets it right if $d_1 \leq \min(d_1, \ldots, d_4)$.

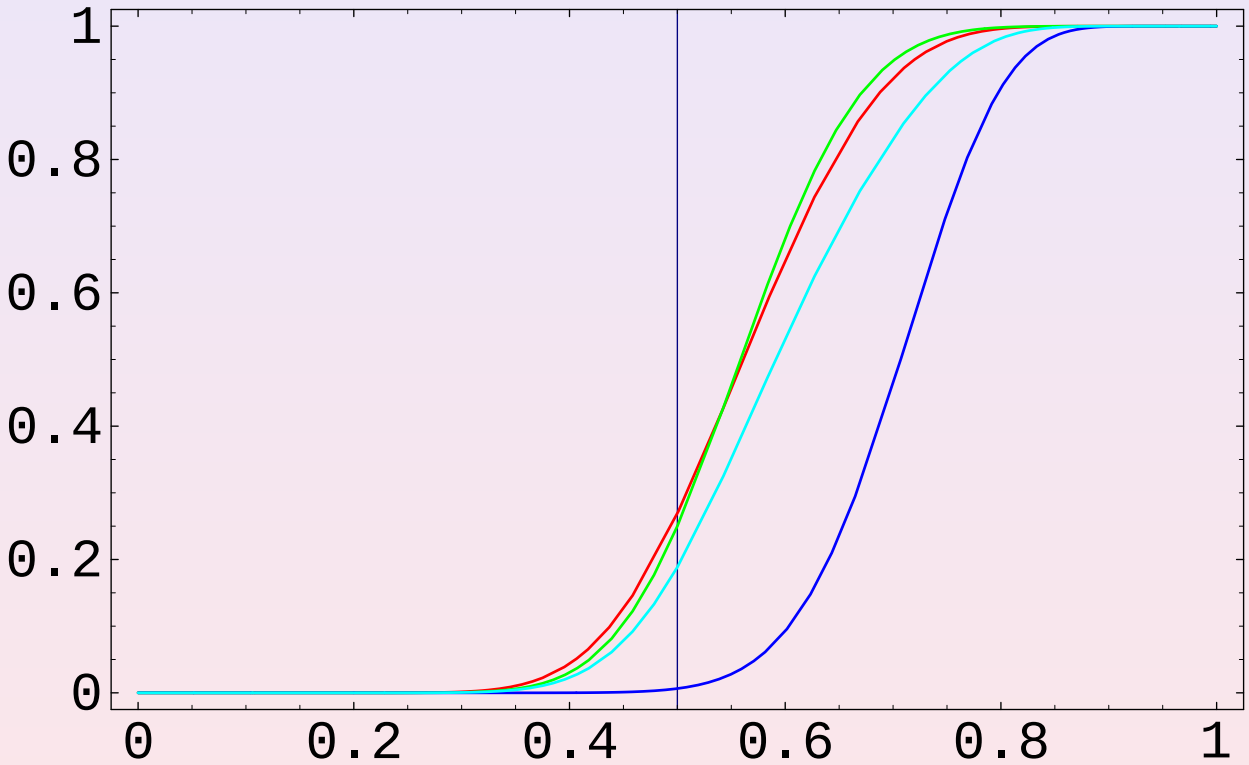
Fusion ranks the right judgment set first (R) compared with PBP (G), CBP (B) and CBP-RR(T) for $N = 3$ and $q = .5$



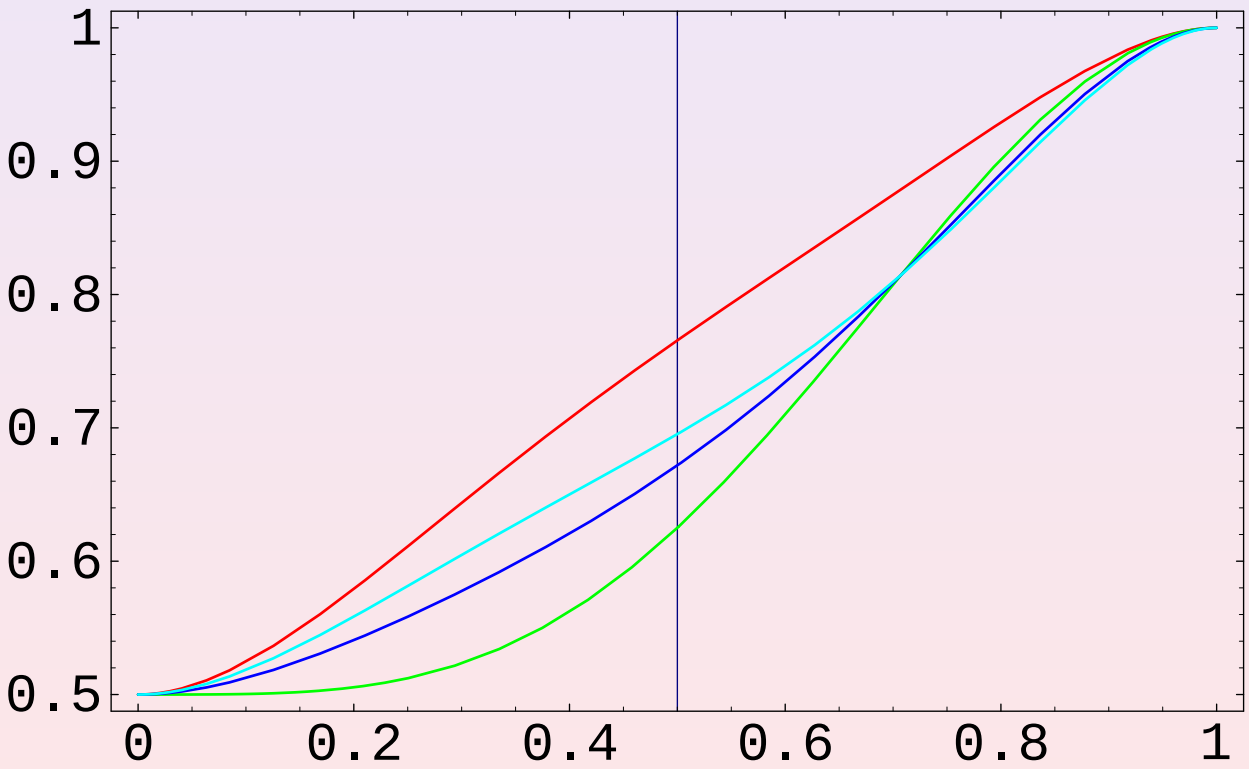
Same for $N = 11$



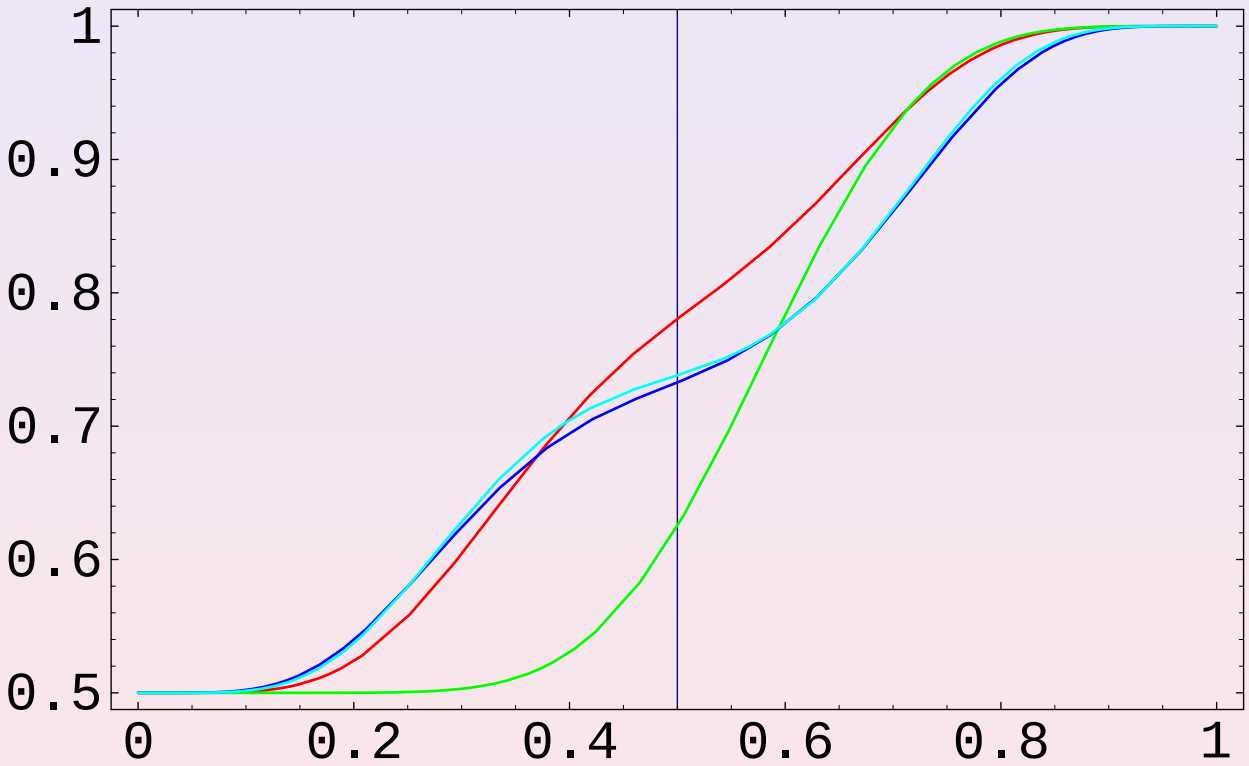
Same for $N = 21$



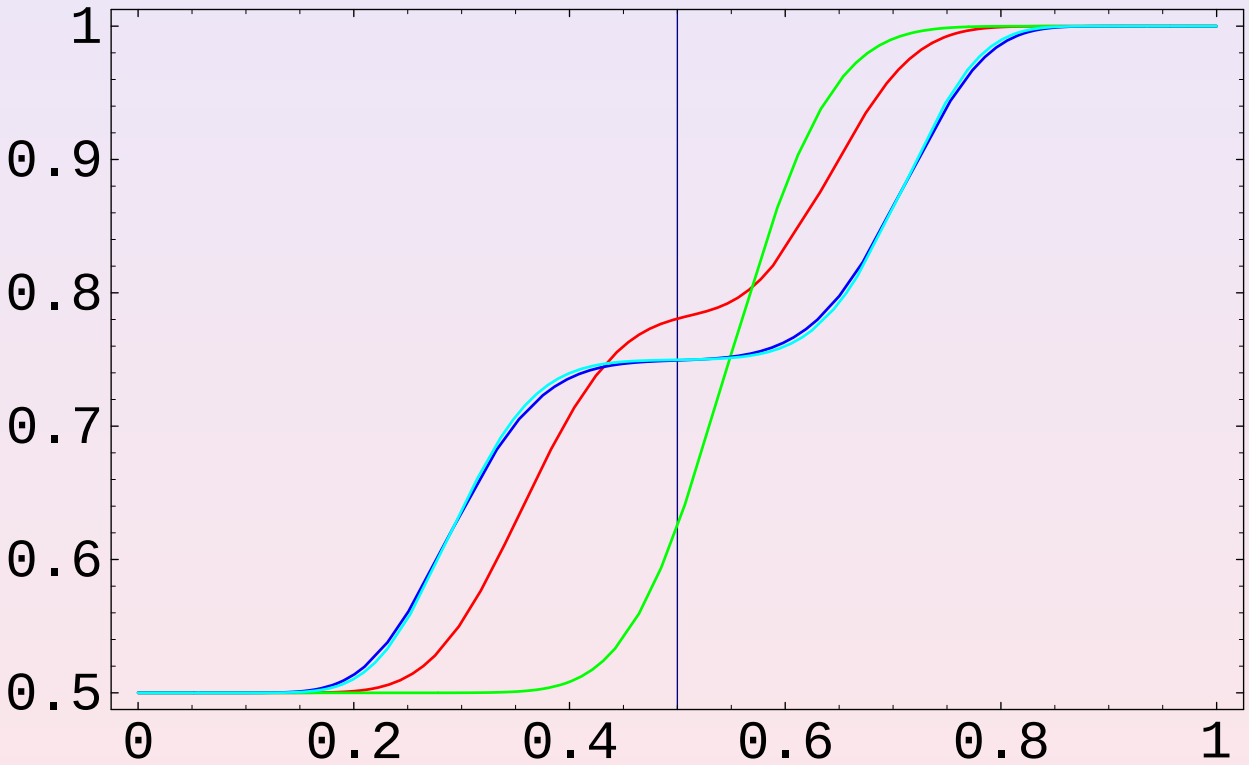
Fusion ranks a judgment set with the right result (not necessarily for the right reasons) first (R) compared with PBP (G), CBP (B) and CBP-RR (T) for $N = 3$ and $q = .5$



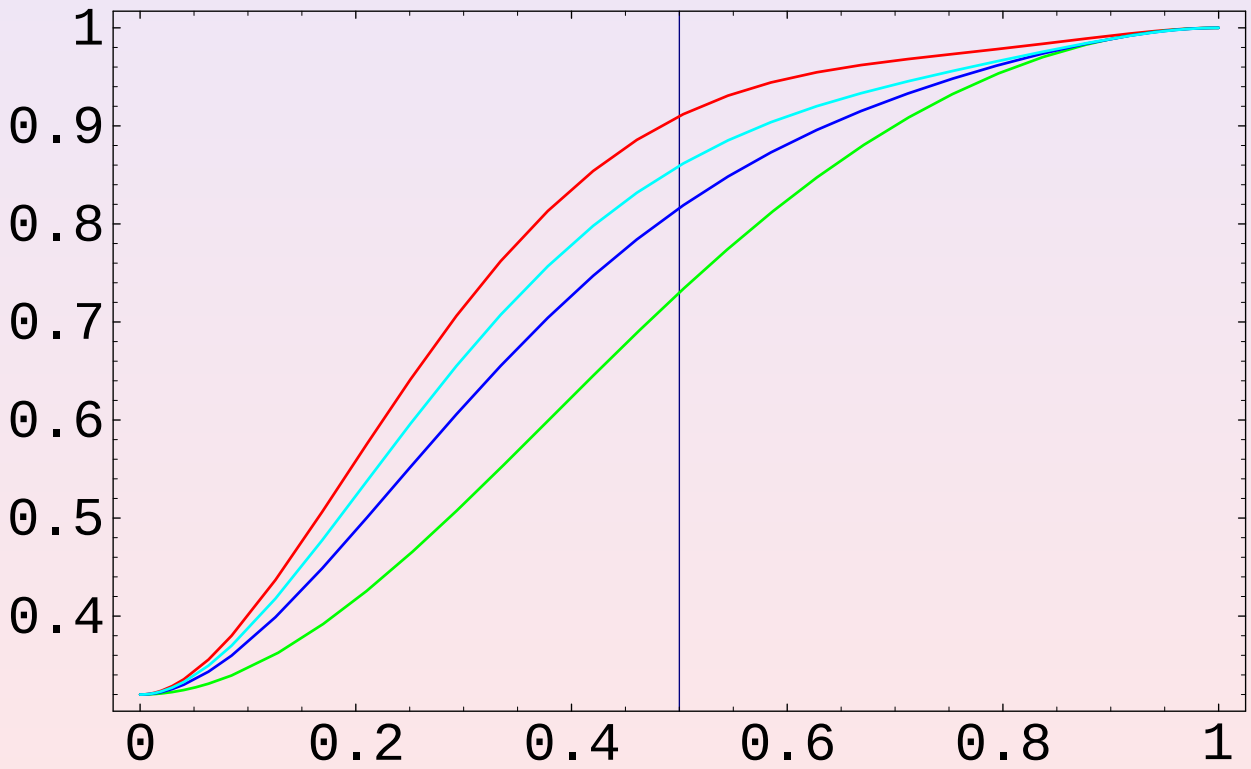
Same for $N = 11$



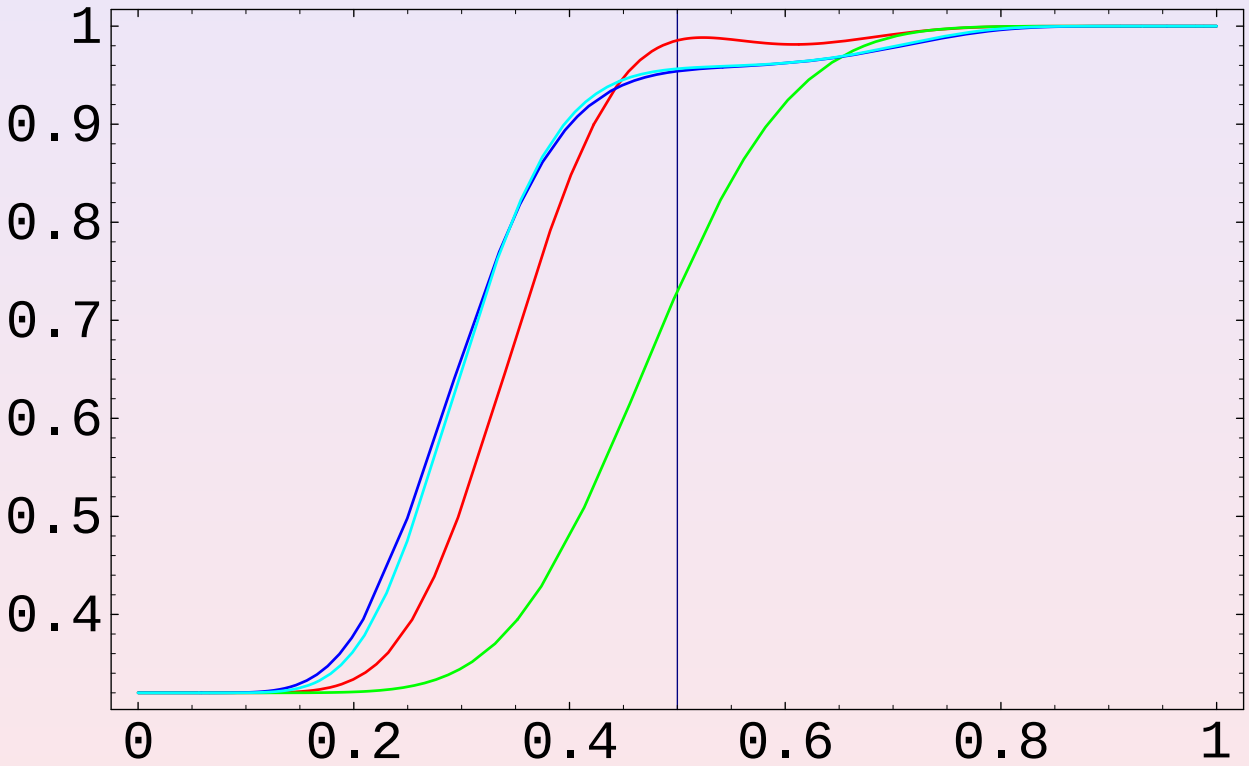
Same for $N = 31$



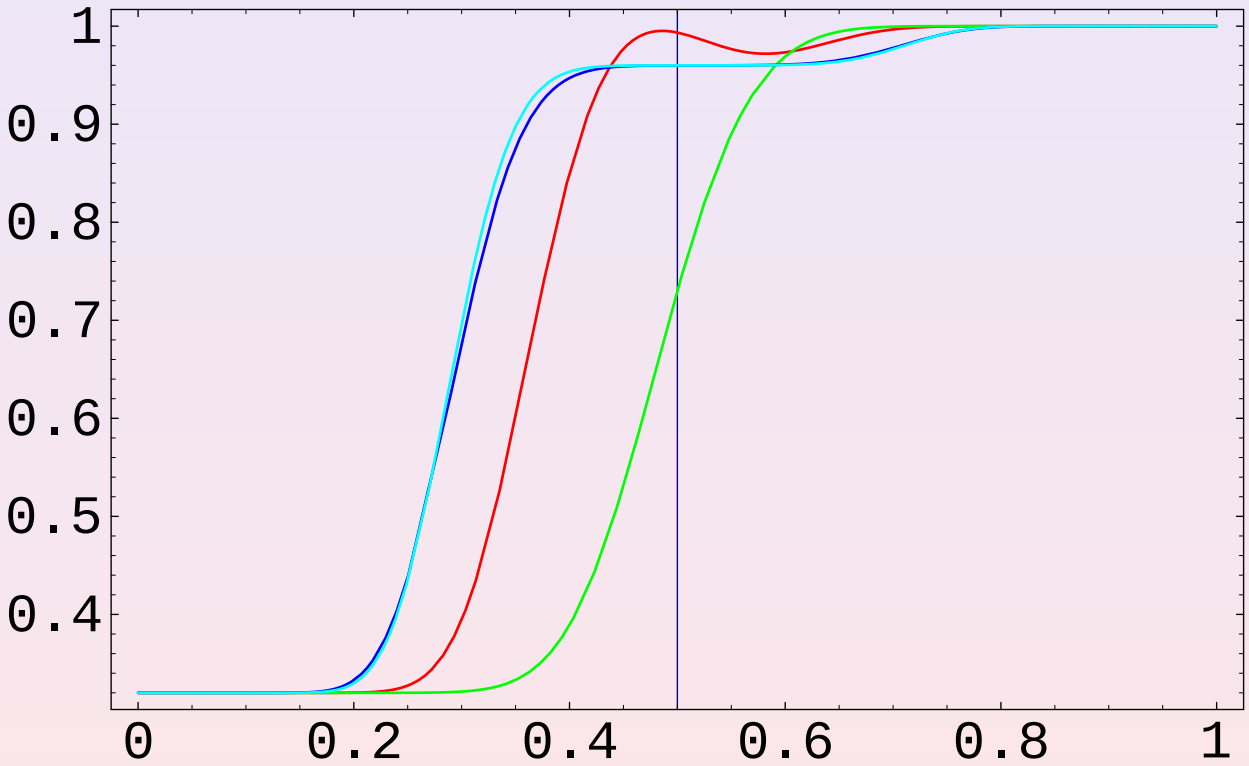
Fusion ranks a judgment set with the right result (not necessarily for the right reasons) first (R) compared with PBP (G), CBP (B) and CBP-RR (T) for $N = 3$ and $q = .2$



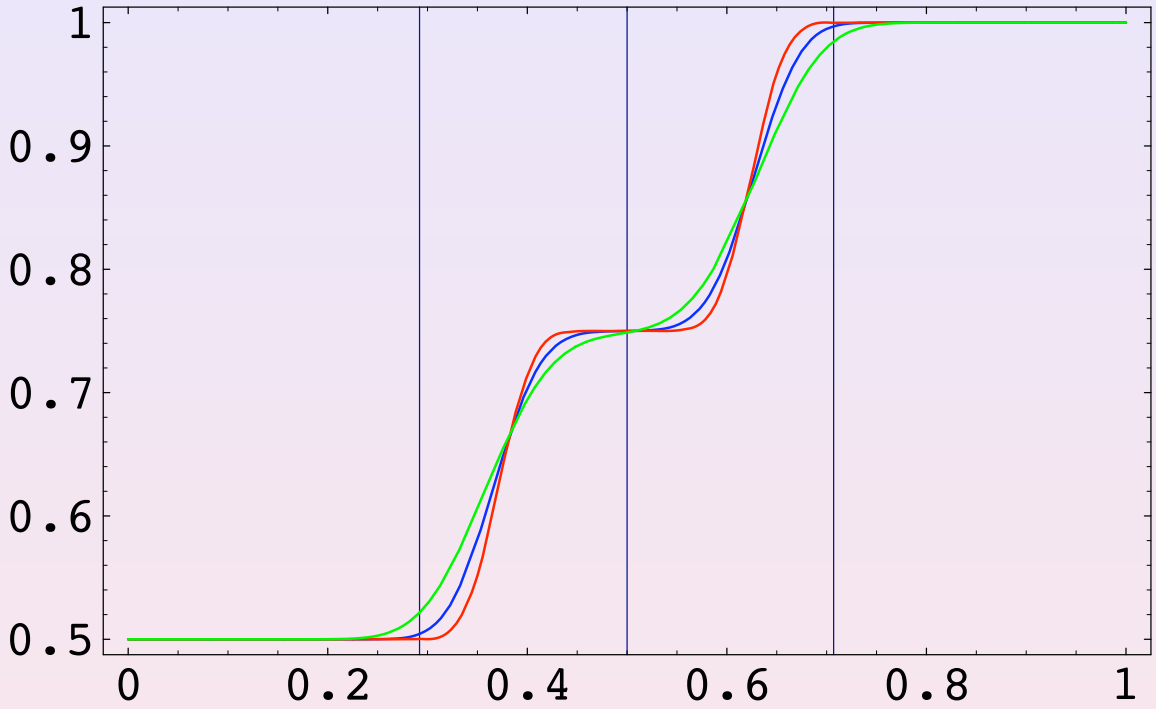
Same for $N = 21$



Same for $N = 51$



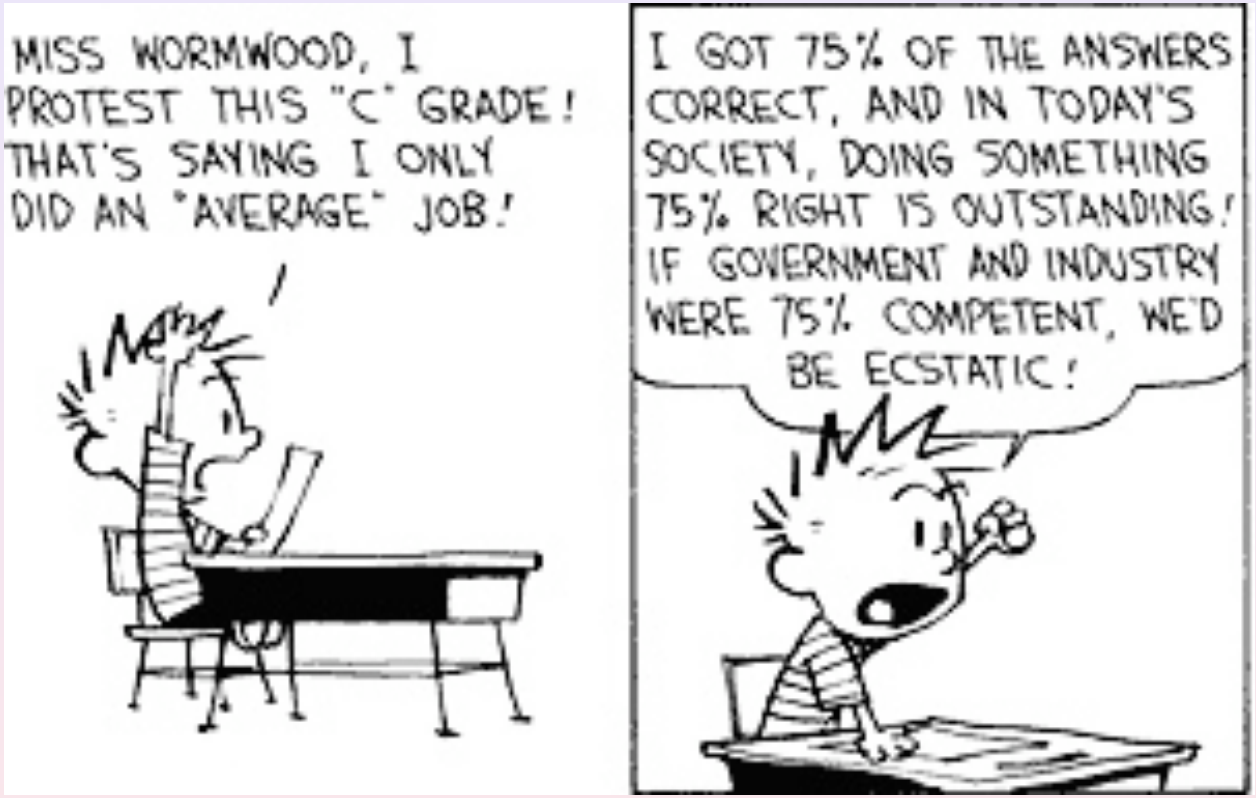
Fusion ranks first right conclusion for $N = 51$ (G), 101 (B), 201 (R) with q=.5



As N converges to infinity, the function for the fusion procedure converges to a step function. In B&R: two crucial values of p are $1 - \sqrt{.5}$ and $\sqrt{.5}$. The CBP tends (i) to .5 for all $p \in (0, 1 - \sqrt{.5})$, (ii) to .75 for all $p \in (1 - \sqrt{.5}, \sqrt{.5})$ and, finally (iii) to 1 for $p \in (\sqrt{.5}, 1)$. The fusion operator strongly outperforms the CBP.

Interpretation

- The fusion approach does especially well for middling values of the competence p.
- For other values of p, the fusion approach is often in between PBP and CBP (whichever is better in the case at hand).
- Hypothesis: Fusion works best for realistic cases ($p \approx .5$) and takes the best of both worlds, i.e. PBP and CBP.


MISS WORMWOOD, I PROTEST THIS "C" GRADE! THAT'S SAYING I ONLY DID AN "AVERAGE" JOB!
I GOT 75% OF THE ANSWERS CORRECT, AND IN TODAY'S SOCIETY, DOING SOMETHING 75% RIGHT IS OUTSTANDING! IF GOVERNMENT AND INDUSTRY WERE 75% COMPETENT, WE'D BE ECSTATIC!

Conclusions and future work

- Belief merging as a valuable tool to aggregate individual judgment sets:
  - no paradox
  - ranking on all possible social outcomes
- We examined how good a truth-tracker the fusion approach is.
- In future work, we will:
  - work with a larger number of voters,
  - a larger number of premises, and
  - use other distance measures.