The Distributed Negotiation of Egalitarian Resource Allocations

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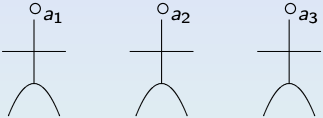
What is an allocation of (indivisible) resources ?

`4 resources:` $\spadesuit_{r_1}$ $\heartsuit_{r_2}$ $\diamondsuit_{r_3}$ $\clubsuit_{r_4}$

`3 agents:`



`allocation:`

$$A = \begin{pmatrix} A/R: & r_1 & r_2 & r_3 & r_4 \\ a_1: & yes & no & no & no \\ a_2: & no & yes & yes & no \\ a_3: & no & no & no & no \end{pmatrix}$$

Figure: Allocation of indivisible resources.

What is a socially optimal allocation of resources ?

You have to make a social choice ! (Endriss et al., JAIR 2006)

- maximise the sum of happiness (cf. Bentham) $\longrightarrow$ utilitarian model
- maximise the happiness of the unhappiest $\longrightarrow$ egalitarian model
- other possibilities: minimise jealousy (envy), maximise performance, etc...

Why study egalitarian allocations of resources ?

Fundamental reasons:

- ensure fairness (when people equally deserve resources)
- little is known about them (computationally)
- learn something about cooperation / negotiation
- assess the degree of fairness of other allocation mechanisms

What is our framework ?

Mathematical description and assumptions:

- cooperative agents $a_1, \ldots, a_n$ and indivisible resources $r_1, \ldots, r_m$
- atomic utilities $u_{i,j} \in \mathbb{R}^+$ = utility for a_i of r_j
- allocation is $A_{i,j} \in \{0, 1\}$, $A_{i,j} = 1$ means a_i is allocated r_j, A is subject to the constraints

$$\forall j \in \{1, \ldots, m\}, \sum_{i=1}^{n} A_{i,j} \leq 1$$

- agent welfare: $w_i(A) = c_i + \sum_{j=1}^{m} A_{i,j} u_{i,j}$ (c_i = "social rank")
- objective: find A maximising

$$sw_e(A) = \min_{i=1}^{n}(w_i(A))$$

How do we solve that problem ?

- The negotiations look like a "ping-pong" game i) $\leftrightarrow$ ii):
  - i) bound the value of optimal social welfare $max(sw_e) \in [L, U]$
  - ii) try to find a solution A such that $sw_e(A) \geq (L + U)/2$
- The length of the match is logarithmic ! However, ii) = "social consensus search" is quite complex...

Illustration

Match steps are numbered 1,2,...,8

Bounds $[L, U]$

- 0. $[0, 10]$
- 2. $[5, 10]$
- 4. $[5, 7.5]$
- 6. $[5, 6.25]$
- 8. $[5.625, 6.25]$

$max(sw_e) = 6$

Exist A / $sw_e(A) \geq (L + U)/2$?

- 1. yes
- 3. no
- 5. no
- 7. yes

How to find a social consensus ?

This involves **individual** and **collective** reasoning:

- agents find allocation **good for themselves**, i.e with satisfying new social rank $(c_i + \sum_{j=1}^{n} A_{i,j} u_{i,j} \geq (L+U)/2)$
- they check their **solutions are compatible**, i.e. no two agents take the same resource $(\forall j, \sum_{i=1..n}(A_{i,j}) \leq 1)$
- simpler for an agent to reason only over solutions accepted by other agents, so
- social consensus can/should grow iteratively from individual consensus !

Computational aspects of consensus search

We use annotated binary search trees of fuzzy allocations for both individual and collective reasoning...

$$(?, ?, ?, ?), no^{possible}$$

$$(yes, ?, ?, ?), yes^{useless} \qquad (no, ?, ?, ?), no^{impossible}$$

$$\begin{pmatrix} yes & no & no & no \\ no & ? & ? & ? \end{pmatrix}, no^{possible}$$

$$\begin{pmatrix} yes & no & no & no \\ no & yes & ? & ? \end{pmatrix}, no^{possible} \qquad \begin{pmatrix} yes & no & no & no \\ no & no & ? & ? \end{pmatrix}, no^{impossible}$$

$$\ldots \qquad \ldots$$

Figure: Reasoning with binary search trees.

How to find a social consensus efficiently ?

Many tricks / heuristics can be combined:

- "exploit social rank": agents pass on the possible consensus to each other and revise them, from unhappiest to happiest
- "exploit preference order": the depth of search trees used by agents are kept small when they think in priority about most useful resources
- "take the bare minimum": ignore opportunistically any consensus that over-consumes resources (developed an algebraic operator for doing that safely)
- agent or resource clustering...

How can distributed negotiations be organised ?

The organisation is fully / rigorously described by:

- a communication language
  $\mathtt{Msg} = \mathtt{tell}(\mathtt{sender}, \mathtt{recipient}, \mathtt{content}, \mathtt{bounds})$
- a protocol corresponding to deterministic finite state automaton (Endriss et al. 2004) describing how messages received affect the mental state of agents

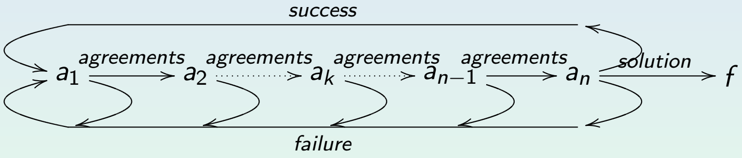


Figure: Communication protocol.

- a policy = set of dialogue constraints (Sadri et al., 2002) that rules the behaviour of agents depending on their mental states

Conclusions

- mechanism guarantees finite convergence to optimal solution (all theoretical results proved in Matt and Toni, CIA 2006)
- mechanism useful for assessing the degree of fairness of other mechanisms (Matt and Toni, TR 2006)
- social negotiations can be distributed (JADE platform)
- complexity still too high (NPC cf. Bouveret et al., 2005), but inherent to preference model
- future work: extend mechanism to other preference models leading to lower complexity
- research opportunities: mathematical and combinatorial properties of optimal consensus spaces and associated operators ? mechanism strategy-proof ?

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Discussion

- Thanks for your attention !
- Any questions ?