Guarantees for the Success Frequency of an Algorithm for Finding Dodgson-Election Winners

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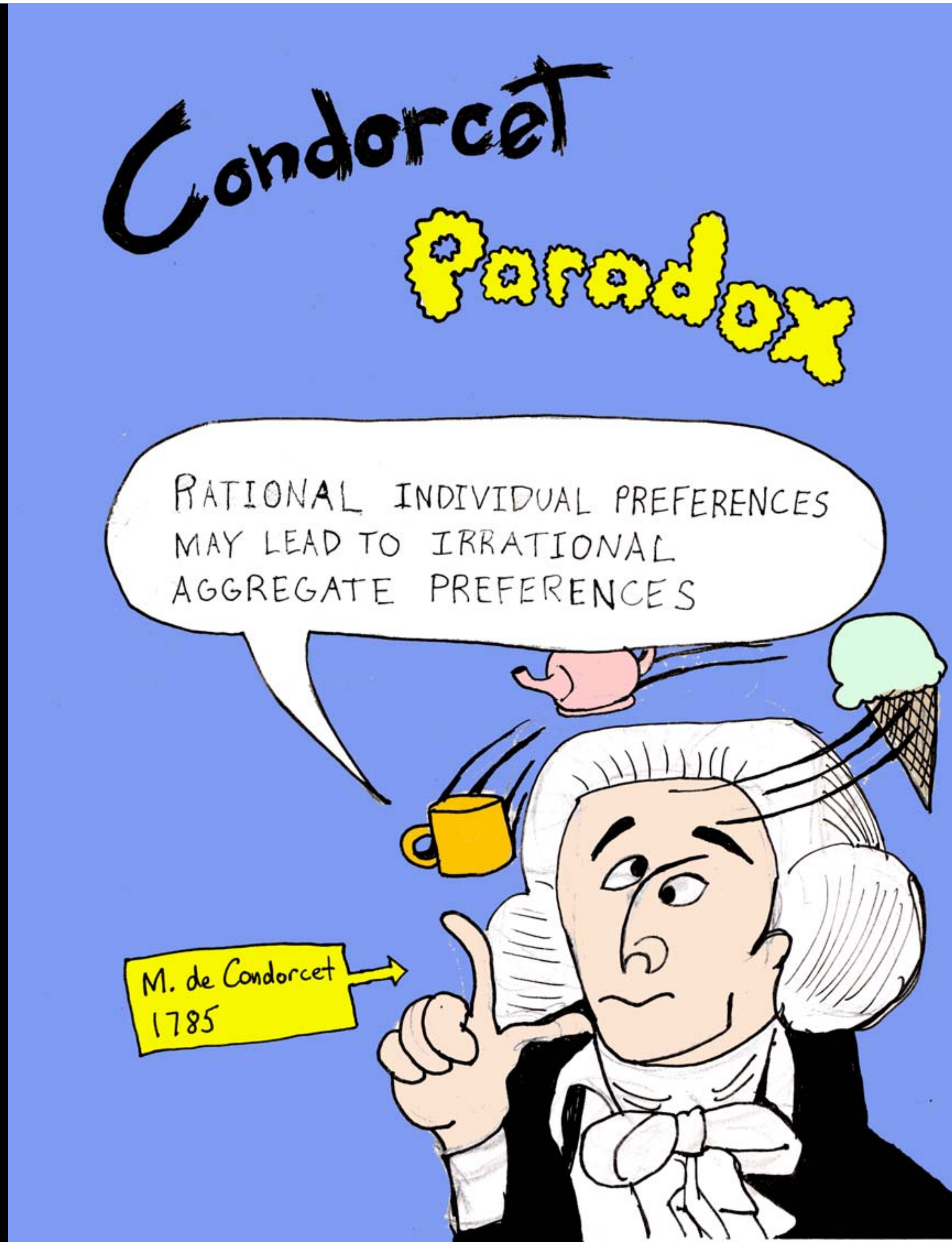
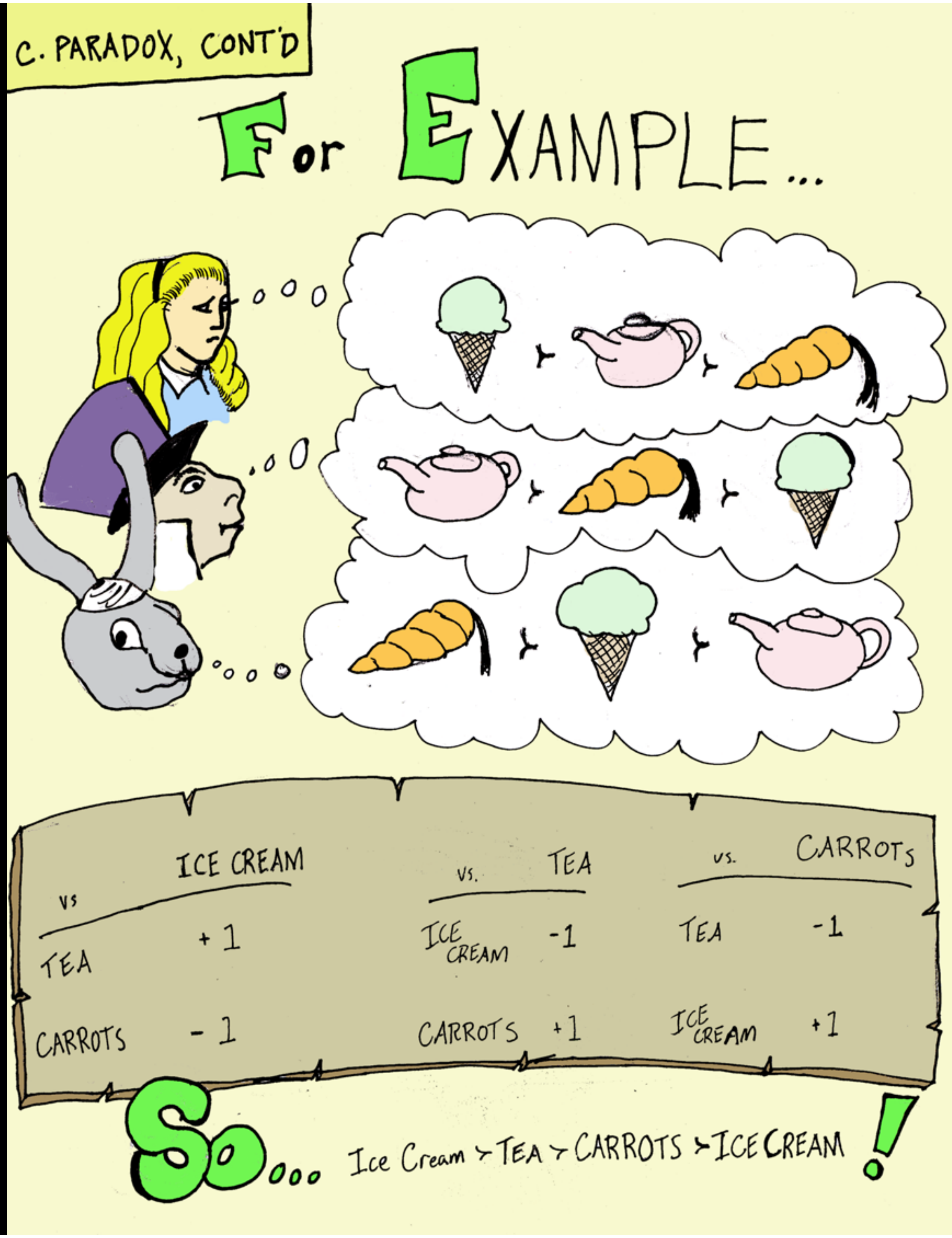
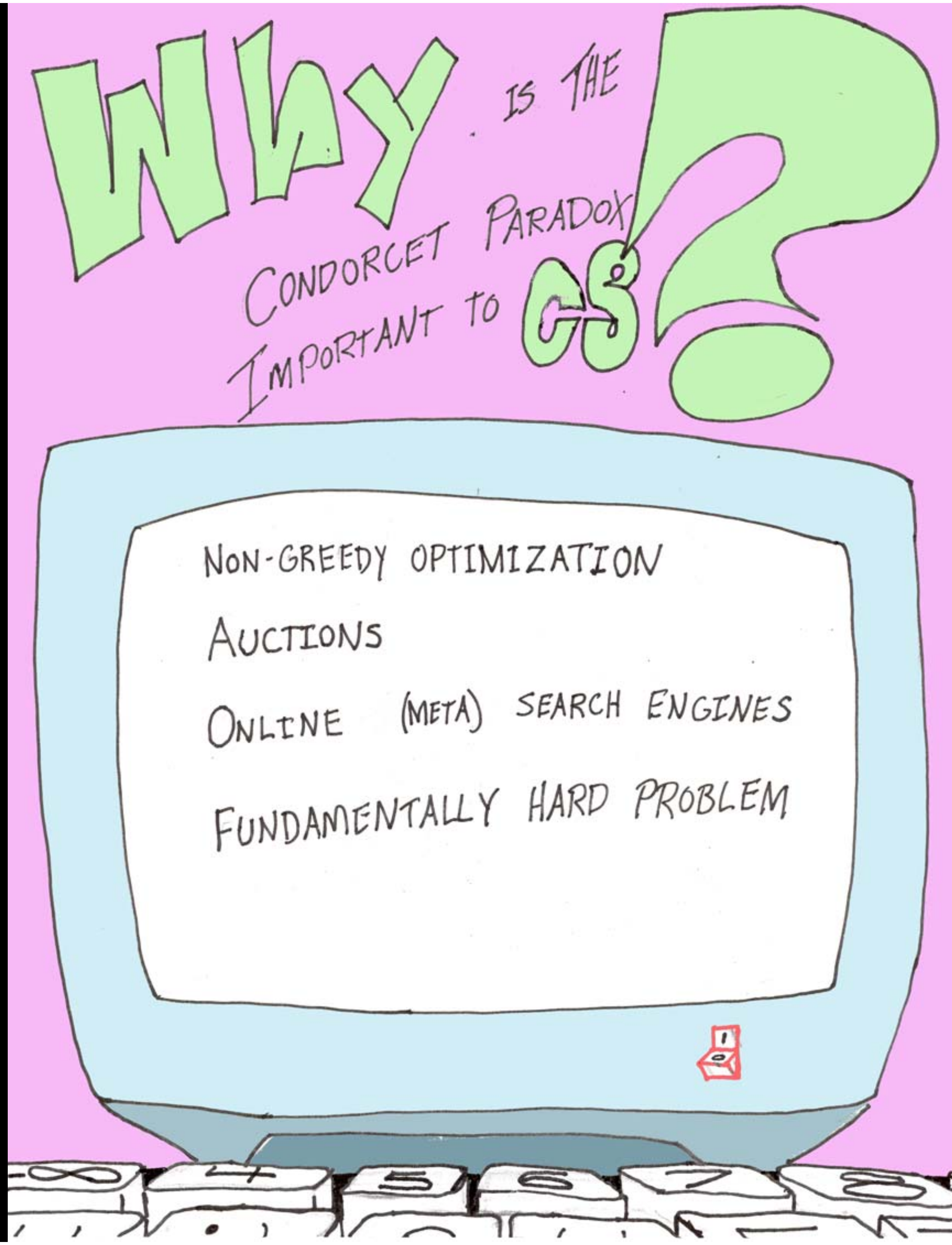
OUTLINE

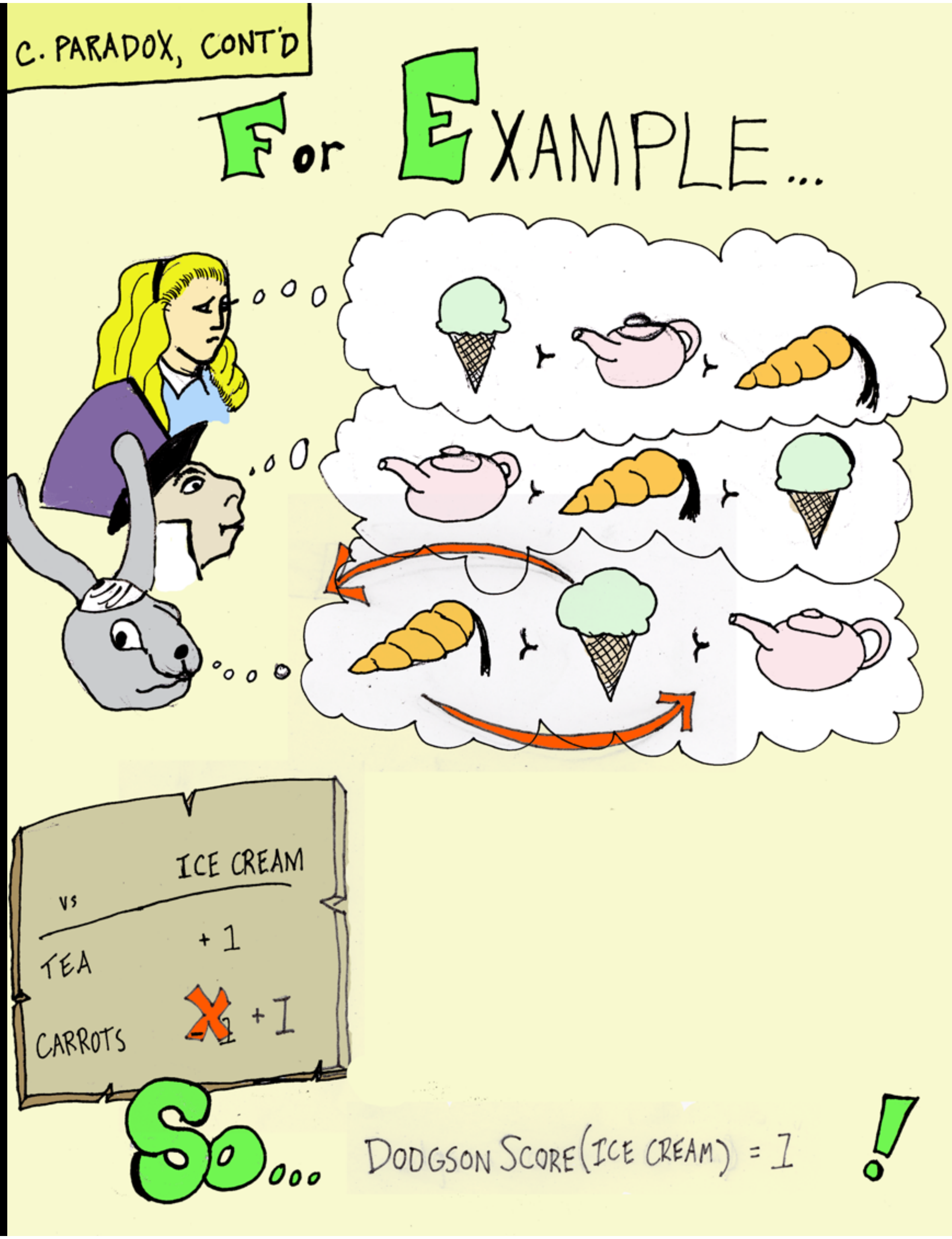
- Condorcet Paradox
- Dodgson Elections
- A "frequently self-knowingly correct" heuristic for computing Dodgson Elections


Condorcet
Paradox
RATIONAL INDIVIDUAL PREFERENCES MAY LEAD TO IRRATIONAL AGGREGATE PREFERENCES
M. de Condorcet 1785


C. PARADOX, CONT'D
For EXAMPLE...
vs ICE CREAM
TEA +1
CARROTS -1
vs. TEA
ICE CREAM -1
CARROTS +1
vs. CARROTS
TEA -1
ICE CREAM +1
Sooo Ice Cream ≻ TEA ≻ CARROTS ≻ ICE CREAM !


WHY IS THE CONDORCET PARADOX IMPORTANT TO CS?
NON-GREEDY OPTIMIZATION
AUCTIONS
ONLINE (META) SEARCH ENGINES
FUNDAMENTALLY HARD PROBLEM


The DODGSON SCORE OF (E.G.) ICE CREAM IS THE FEWEST ADJACENT SWAPS (IN PREFERENCES) NEEDED FOR IT TO (PAIRWISE) BEAT EACH REMAINING CANDIDATE (E.G. TEA AND CARROTS)
C. Dodgson a.k.a. L. Carroll 1876
tip-toez


C. PARADOX, CONT'D
For EXAMPLE...
ICE CREAM
VS
TEA +1
CARROTS ~~-1~~ +1
So... DODGSON SCORE(ICE CREAM) = 1 !

Example 2

$a \succ b \succ c \succ d \succ e$

$a \succ b \succ c \succ d \succ e$

$a \succ d \succ c \succ b \succ e$

$a \succ d \succ c \succ b \succ e$

$a \succ e \succ c \succ d \succ b$

	c
a	-5
b	+1
d	+1
e	+3

Example 2

$$a \succ b \succ c \succ d \succ e$$

$$a \succ b \succ c \succ d \succ e$$

$$a \succ d \succ c \succ b \succ e$$

$$a \succ d \succ c \succ b \succ e$$

$$a \succ e \succ c \succ d \succ b$$

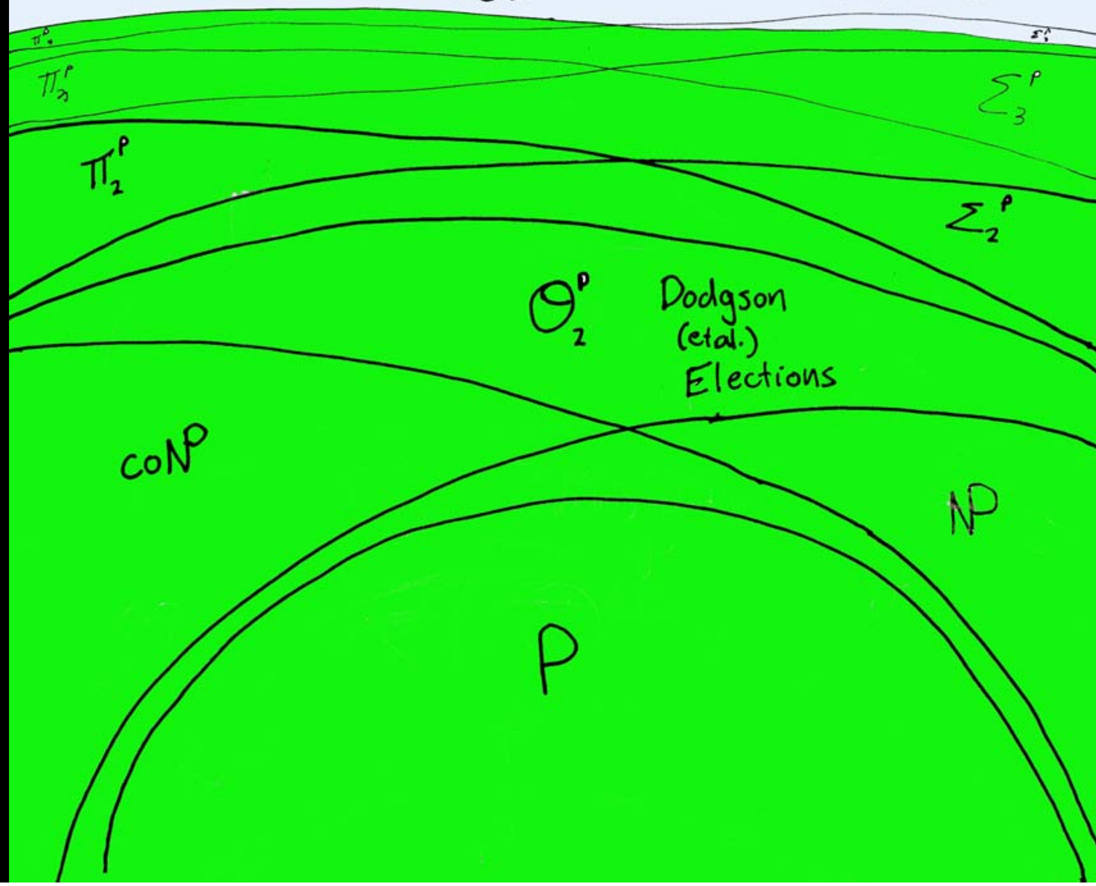
	c
a	+1
b	+1
d	+1
e	+3

Dodgson Score (c) = **6**

DODGSON ELECTIONS are HARD!

Instance: $(\mathbb{C}, V, x)$
Question: Is $x \in \mathbb{C}$ a Dodgson Winner, given votes V

- NP-hard Bartholdi, Tovey, Trick '89
- Θ_2^p-complete Hemaspaandra, Hemaspaandra, and Rothe '97



GREEDYSCORE($\mathbb{C}, V, x$)

```
Score ← 0
For each v ∈ V : (∃ y ∈ ℂ)[y ≻ x in v]
    If y is beating x
        SWAP(y, x)
        Score++
If x now ties-or-defeats all other candidates
    return (Score, "Definitely")
Else
    return (Score, "Maybe")
```

THEOREM 1.1.

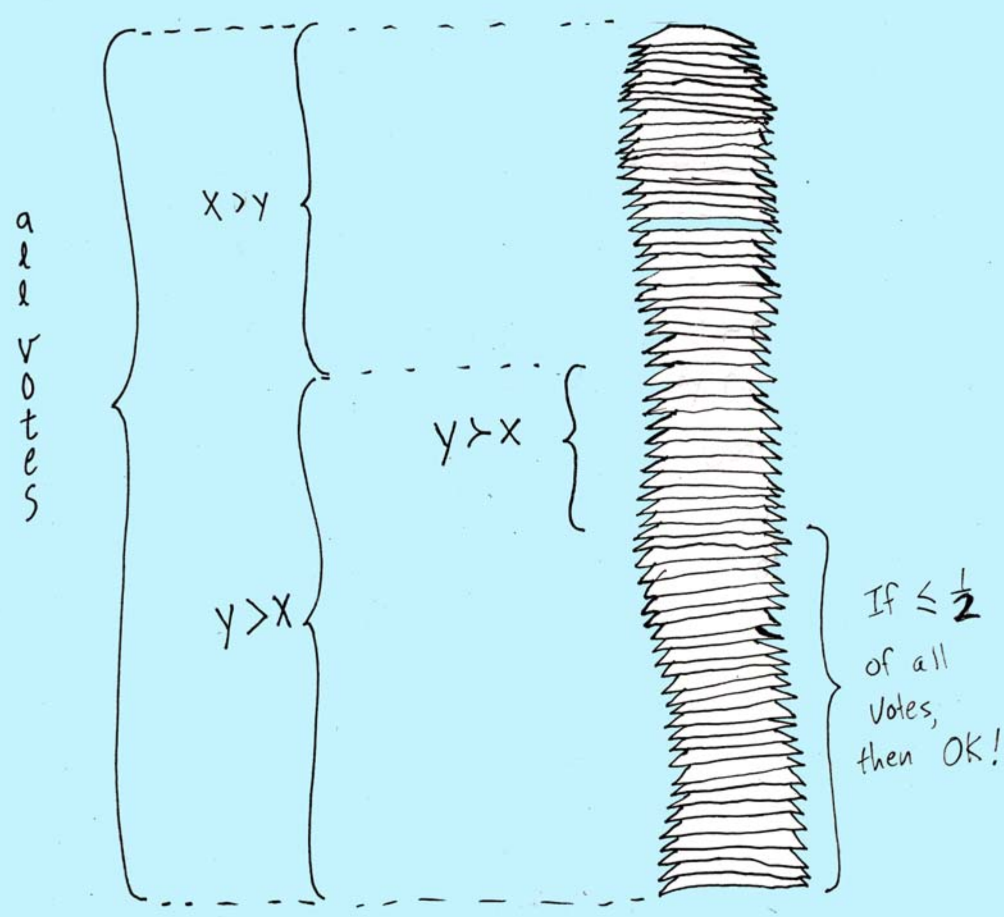
GreedyScore is self-knowingly correct. I.e., if its second output argument is "**Definitely**" then its first argument really is the Dodgson Score

THEOREM 1.2.

For an election having m candidates and n votes, if the votes are chosen independenty and uniformly a random, then prob. GreedyScore outputs "maybe" is at most $2(m-1)e^{-n/8m^2}$

THEOREM 1.2 PROOF SKETCH

$x, y \in \mathbb{C}$



THEOREM 1.2, CONT'D

(REMEMBER: m candidates)

For a vote: $\Pr(y > x) = \frac{1}{2}$

$\Pr(y > x) = \frac{1}{m}$

For n votes:

$$E[\#\text{votes s.t. } y > x] = \tfrac{1}{2} n$$

$$E[\#\text{votes s.t. } y > x] = \tfrac{n}{m}$$

$$\left.\begin{array}{l} \Pr\left[(\#\text{votes s.t. } y > x) > \frac{1}{2} n + \frac{1}{4}\frac{n}{m}\right] \\ \Pr\left[(\#\text{votes s.t. } y > x) < \frac{3}{4}\frac{n}{m}\right] \end{array}\right\} < e^{-n/(8m^2)}$$

FUTURE Work

1. Our analysis assumes votes chosen independently; what if consider dependencies?

2. Experimental validation