Hybrid Elections Broaden Complexity-Theoretic Resistance to Control

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Overview

Goal:

Broaden complexity-theoretic resistance to control!

1. Definitions: Elections, Types of Control, etc.
2. Our Hybridization Scheme
3. Inheritance and Hybrid Elections: Results

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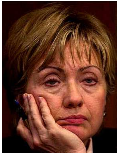
Elections

- Candidates: a finite set C.

George Hillary Barack Ralph John

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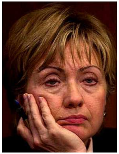

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Election Systems

- Election System: a mapping from sets V of votes to (possibly empty, possibly nonstrict) subsets of C, i.e., the election system outputs the winner(s) of the election.

Voter 1:	G	>	B	>	H	>	R	>	J
Voter 2:	G	>	B	>	H	>	R	>	J
Voter 3:	H	>	B	>	G	>	J	>	R
Voter 4:	H	>	B	>	G	>	J	>	R
Voter 5:	B	>	H	>	G	>	J	>	R

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Plurality Voting: The winners are the candidates who are ranked first the most.

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Election Systems: Condorcet Voting

Condorcet Voting: The winners are all candidates who strictly beat each other candidate in head-on-head majority-rule elections, i.e., get *strictly* more than half the votes in each such election. (There can be at most one and there might be zero.)

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Constructive and Destructive Control

Electoral Control refers to. . .

attempts by an election's organizer ("the chair") to influence the outcome by adding/deleting/partitioning voters or candidates.

- Control issues were first studied by Bartholdi, Tovey, and Trick (1992) in seven different control scenarios, e.g., (constructive) control by adding candidates.
  Results for Plurality Voting and Condorcet Voting.
- Destructive control was studied by Hemaspaandra, Hemaspaandra, and Rothe (AAAI '05).
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Types of Constructive and Destructive Control

The 20 standard types of constructive and destructive control:

- adding candidates,
- deleting candidates,
- partition of candidates in models
  - Ties-Promote (TP) and
  - Ties-Eliminate (TE),
- run-off partition of candidates (TP and TE),
- adding voters,
- deleting voters,
- partition of voters (TP and TE).

Control by Adding Candidates

Constructive (Destructive) Control by Adding Candidates:

Given: A set C of qualified candidates and a distinguished candidate $c \in C$, a set D of possible spoiler candidates, and a set V of voters with preferences over $C \cup D$.

Question: Is there a choice of candidates from D whose entry into the election would assure that c is (not) the unique winner?

Our Hybridization Scheme

Definition: *hybrid*

Let $\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1}$ be election rules that take as input voters' preference orders. Define $\mathit{hybrid}(\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1})$ to be the election rule that does the following:

- If there is at least one candidate and all candidate names (viewed as natural numbers via the standard bijection between Σ^* and $\mathbb{N}$) are congruent, modulo k, to i (for some i, $0 \leq i \leq k-1$) then use election rule $\mathcal{E}_i$.
- Otherwise use $\mathcal{E}_{k-1}$ as the default election rule.

Our Hybridization Scheme: Discussion

Why *hybrid*? And what are its properties?

- The join of sets

$$A \oplus B = \{0x \mid x \in A\} \cup \{1y \mid y \in B\}$$

  preserves both simplicity ($A \in \mathbf{P} \wedge B \in \mathbf{P} \implies A \oplus B \in \mathbf{P}$) and hardness ($C \leq_m^p A \vee C \leq_m^p B \implies C \leq_m^p A \oplus B$).

- Similarly, *hybrid* maintains desirable simplicity properties (e.g., it inherits "winner problem membership in **P**") and hardness properties (it inherits any "resistance-to-control").

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What other approaches did we choose not to use, and why?

1. To hybridize $\mathcal{E}_0$ and $\mathcal{E}_1$, use $\mathcal{E}_0$ exactly if the first voter's most disliked candidate's name is lexicographically less than his/her second-most-disliked candidate's name.
   This choice is bad, as it is sensitive to voter deletion!
2. Or use the modulo k value of the smallest candidate's name to control switching between the k systems.
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3. Or use . . . Bad choice again!

Bottom Line: *hybrid* **keeps deletions/partitions of voters or candidates from jumping uncontrollably between systems.**

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What aspects of the input does *hybrid* use, what aspects is it choosing not to exploit, and for what price?

- *hybrid* uses the candidates' names and only the candidates' names.
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Immunity, Susceptibility, Vulnerability, and Resistance

Definition [Bartholdi, Tovey, and Trick (1992)]:

Let $\mathcal{E}$ be an election system and Φ be a given control type.

- $\mathcal{E}$ is immune to Φ-control if it is never possible for the chair to reach his/her goal by asserting Φ-control.
- Otherwise, $\mathcal{E}$ is susceptible to Φ-control.
- $\mathcal{E}$ is (computationally) vulnerable to Φ-control if it is susceptible to Φ-control and the corresponding language problem is computationally easy (i.e., solvable in **P**).
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Inheritance

Definition:

A property Γ is strongly inherited (respectively, inherited) by *hybrid* if the following holds for all $k \in \mathbb{N}^+$ and for all candidate-anonymous election systems $\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1}$: *hybrid*$(\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1})$ has property Γ if at least one $\mathcal{E}_i$ has (respectively, all of $\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1}$ have) property Γ.

Proposition:

"Winner / unique winner problem membership in **P**" and "winner / unique winner problem membership in **NP**" are inherited by *hybrid*.

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Susceptibility and Resistance

Control by	Susceptibility	Resistance
Adding Candidates	SI	SI
Deleting Candidates	SI	SI
Partition of Candidates (TE)	SI	SI
Partition of Candidates (TP)	SI	SI
Run-off Partition of Candidates (TE)	SI	SI
Run-off Partition of Candidates (TP)	SI	SI
Adding Voters	SI	SI
Deleting Voters	SI	SI
Partition of Voters (TE)	SI	SI
Partition of Voters (TP)	SI	SI

Resistance to All 20 Standard Types of Control

Theorem:

Let $k \in \mathbb{N}^+$ and let $\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1}$ be election systems. Let Φ be one of the standard twenty types of control. If for at least one i, $0 \leq i \leq k-1$, $\mathcal{E}_i$ is candidate-anonymous and resistant to Φ, then $hybrid(\mathcal{E}_0, \mathcal{E}_1, \ldots, \mathcal{E}_{k-1})$ is resistant to Φ.

Corollary:

hybrid strongly inherits resistance to each of the standard twenty types of control.

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There exist election systems (e.g., $hybrid(\text{plurality}, \text{Condorcet})$) that are resistant to the ten types of constructive control.

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Resistance to All 20 Standard Types of Control

Lemma:

There exists a candidate-anonymous election system, $\mathcal{E}_{\text{not-all-one}}$, that is resistant to (a) destructive control by deleting voters, (b) destructive control by adding voters, and (c) destructive control by partition of voters in the TE model.

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Immunity and Vulnerability

Control by	Immunity	Vulnerability
Adding Candidates	Not I / I*	I
Deleting Candidates	I / Not I*	I iff **P** $=$ **NP**
Partition of Candidates (TE)	Not I	I iff SI iff **P** $=$ **NP**
Partition of Candidates (TP)	Not I	I iff SI iff **P** $=$ **NP**
Run-off Partition of Candidates (TE)	Not I	I iff SI iff **P** $=$ **NP**
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Adding Voters	I	I
Deleting Voters	I	I
Partition of Voters (TE)	I	I
Partition of Voters (TP)	I	I

Questions?

Questions? . . . Answers!

asks:

“Doesn’t this paper take extreme advantage of the worst-case nature of NP-hardness?”

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asks:
"Doesn't this paper take extreme advantage of the worst-case nature of NP-hardness?"

replies: "The 'extreme advantage' comment is deceptive. Natural NP-complete problems routinely have large swaths of input on which they are easy. This is no different."

Questions? ... Answers!

claims: "Your *hybrid* chooses the default system $\mathcal{E}_{k-1}$ with high probability, since it is unlikely that all candidate names are congruent modulo k. And so it really in practical effect is just the default system, and so is not interesting."

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replies: "No, George, *hybrid* simply provides a flexible framework to route the problem to various systems. Disjoint union is the right analog. By your argument, SAT is 'not interesting,' since SAT can be solved with an overwhelmingly high probability of success, as only a small proportion of inputs are syntactically valid formulas."

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asks: "In usual elections, candidate names are known, so the chair can preprocess the data. Doesn't this limit the practical significance of your result?"

Questions? . . . Answers!

asks: “In usual elections, candidate names are known, so the chair can preprocess the data. Doesn’t this limit the practical significance of your result?”

replies: “No, Ralph, you’re assuming that the names fed to the system are the *actual* names. Our hardness-of-control result is *just about problems*; the relation to the real world is up to whoever uses it.”

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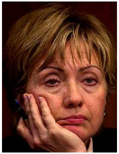 & 

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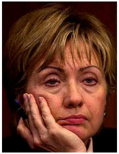 & 

ask: "What about the Conitzer–Sandholm hybridization scheme? And what about its generalization by Elkind and Lipmaa?"

replies: "Those are interesting, but quite different approaches to hybridization.

Briefly put, they go serial (by sticking in a sequential preround), whereas we directly put into parallel a collection of systems.

Mathematically, ours is the cleaner approach."

Questions? . . . Answers!

asks:

"What do you mean by 'mathematically cleaner'?"

Questions? . . . Answers!

asks:

"What do you mean by 'mathematically cleaner'?"

replies: "Due to the disjoint-union-like nature of *hybrid*, we relatively directly get the two most critical inheritances, namely regarding susceptibility and resistance, though admittedly some of the other, and less important, inheritance issues are not tremendously clean for us."