QuickRank

A Recursive Ranking Algorithm

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Introduction

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  - Except the individuals and the alternatives are the same.
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  > *Almost all societies have elementary units called families, which may be grouped into villages or tribes, and these into larger groupings, and so on. If we make a chart of social interactions, of who talks to whom, the clusters of dense interaction in the chart will identify a rather well-defined hierarchic[al] structure.* Simon [1962]

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  - Bonacich's Hypothesis

Review of PageRank

C

A

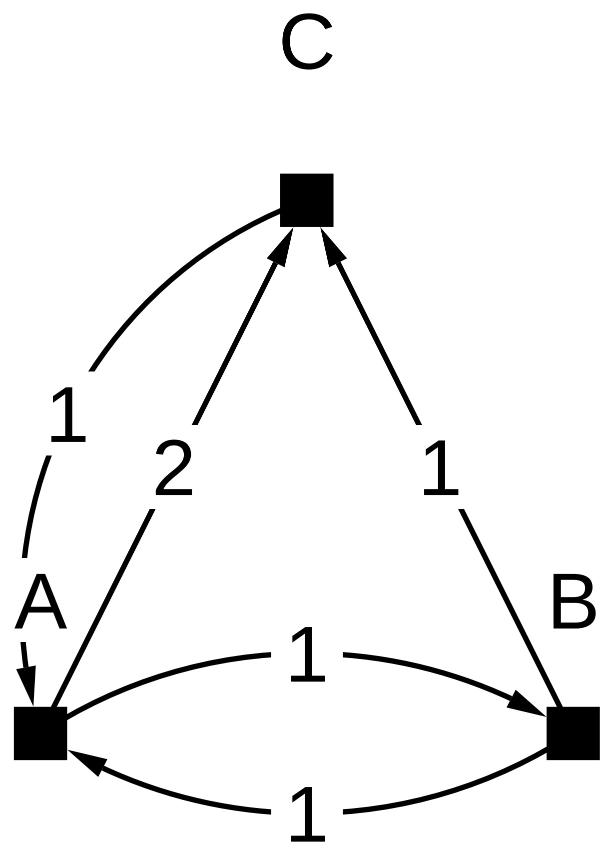
A

C, B, C

B

A, C

Review of PageRank



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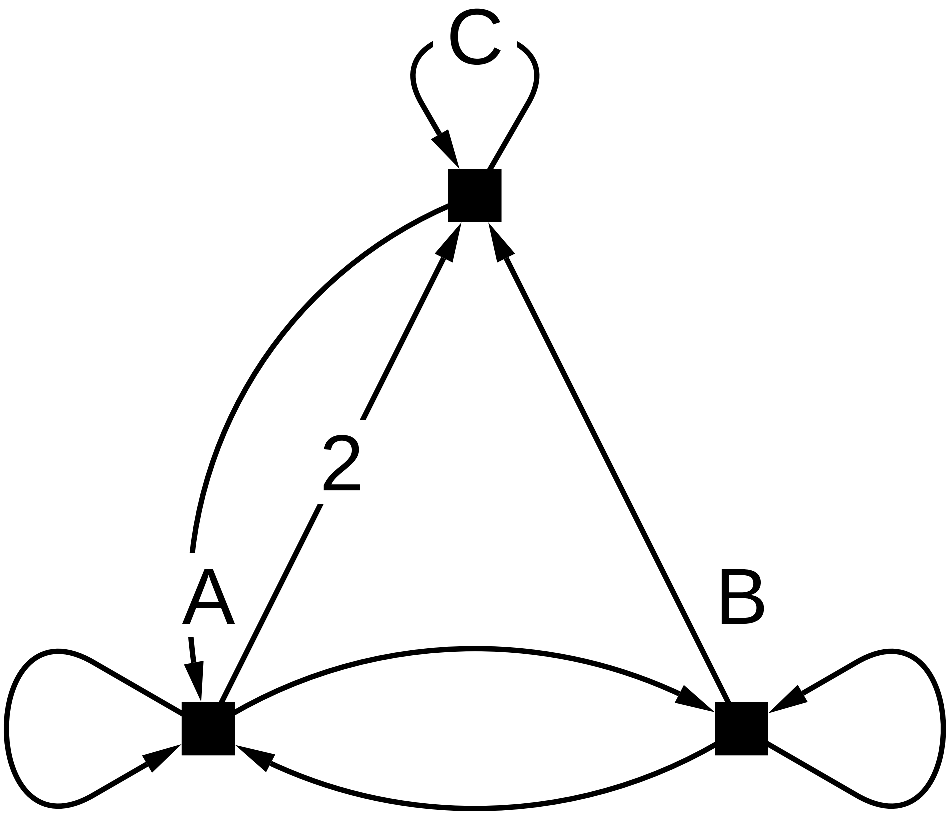

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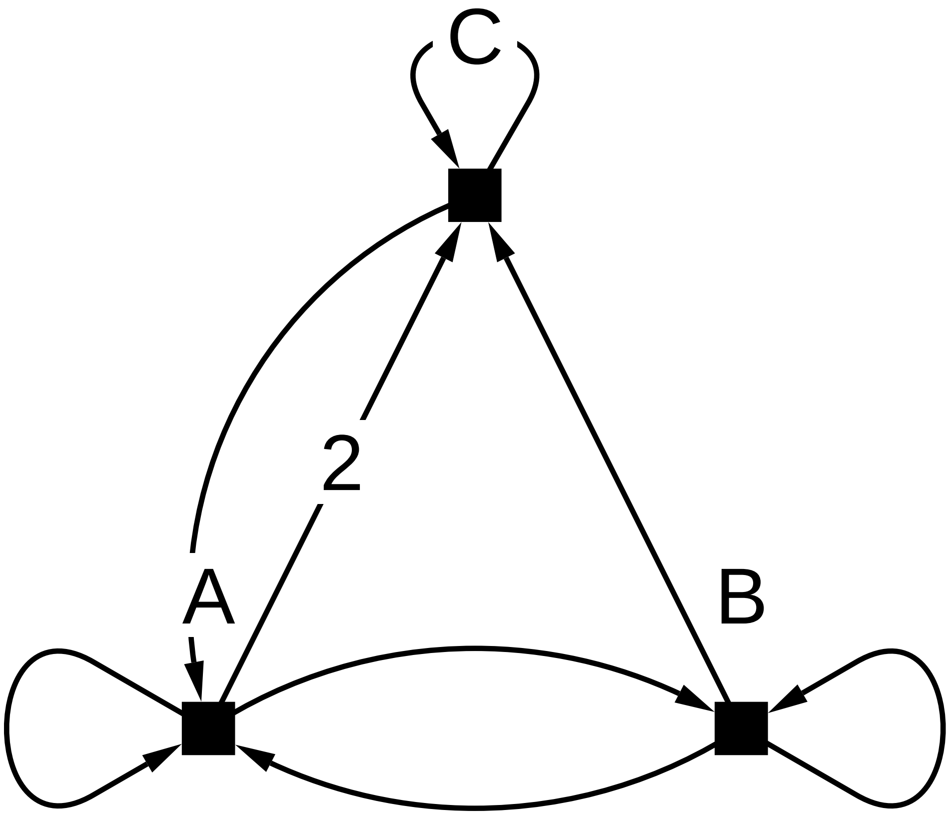

A	B	C
1	2	1

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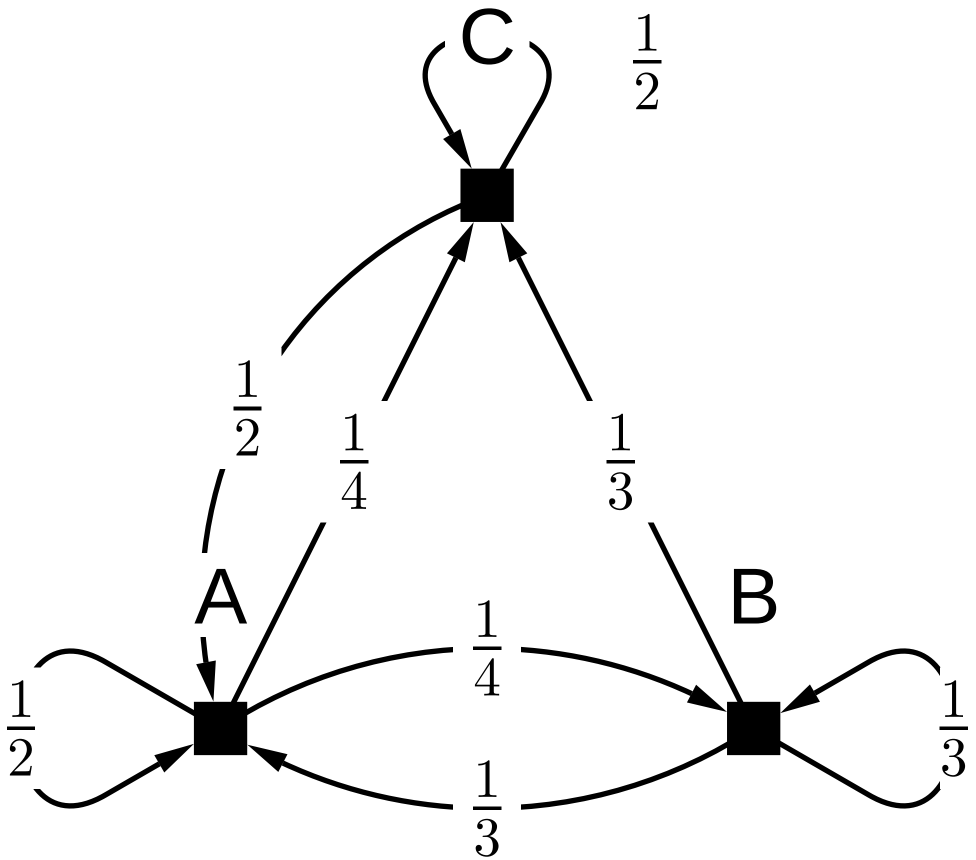

A	B	C
0.25	0.50	0.25

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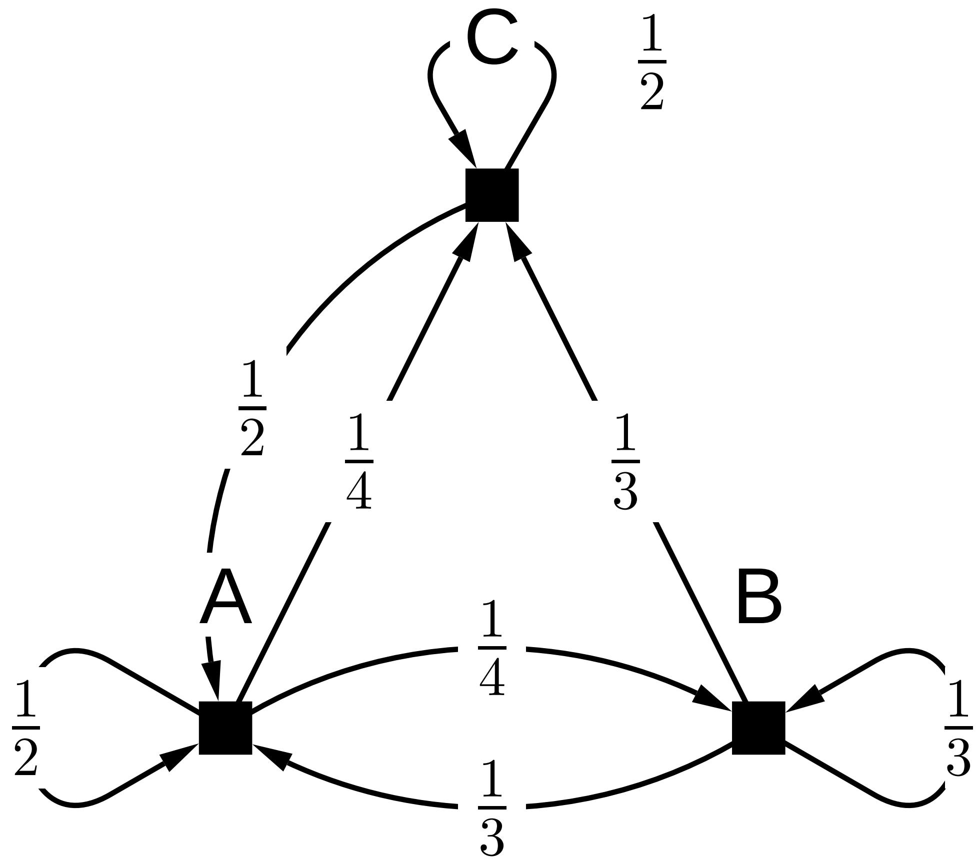

A	B	C
0.30	0.37	0.33

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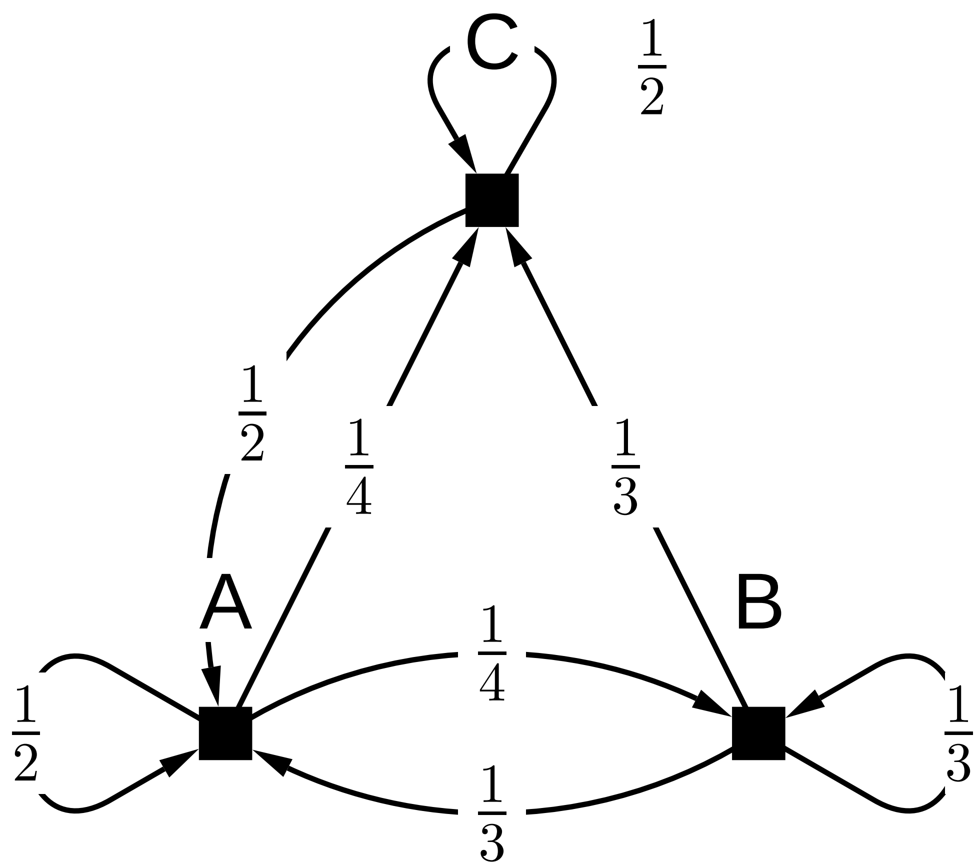
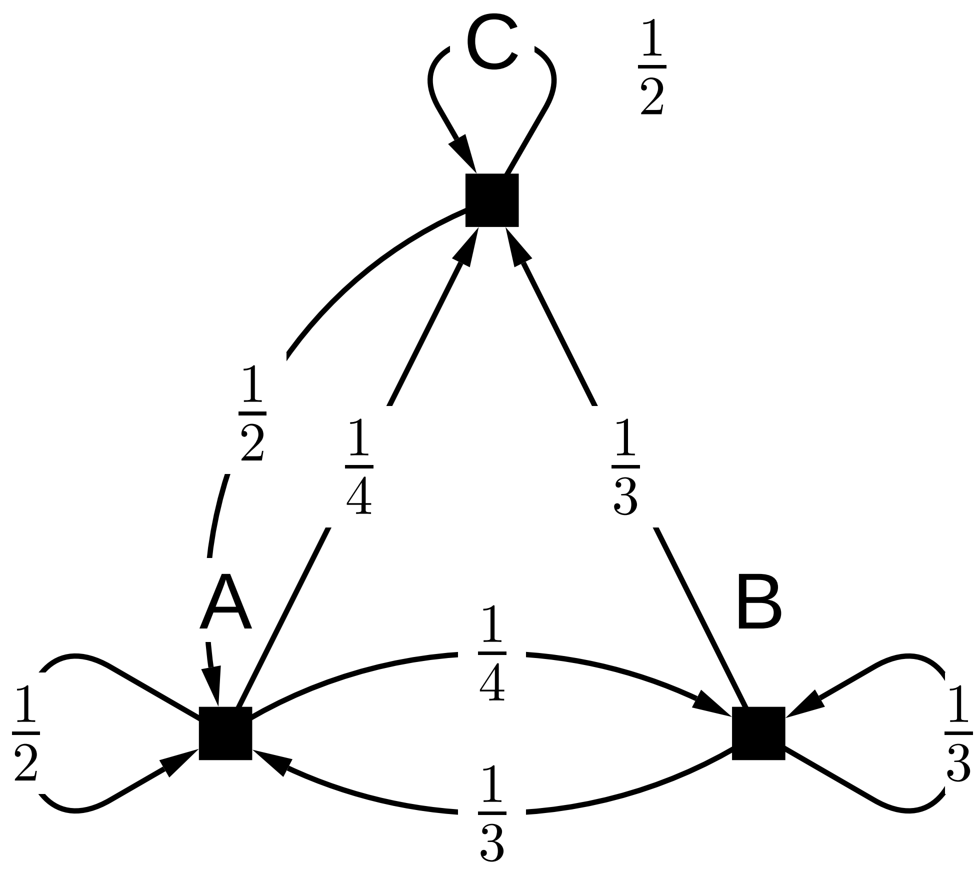


A	B	C
0.32	0.32	0.36

Review of PageRank



A	B	C
0.33	0.28	0.39

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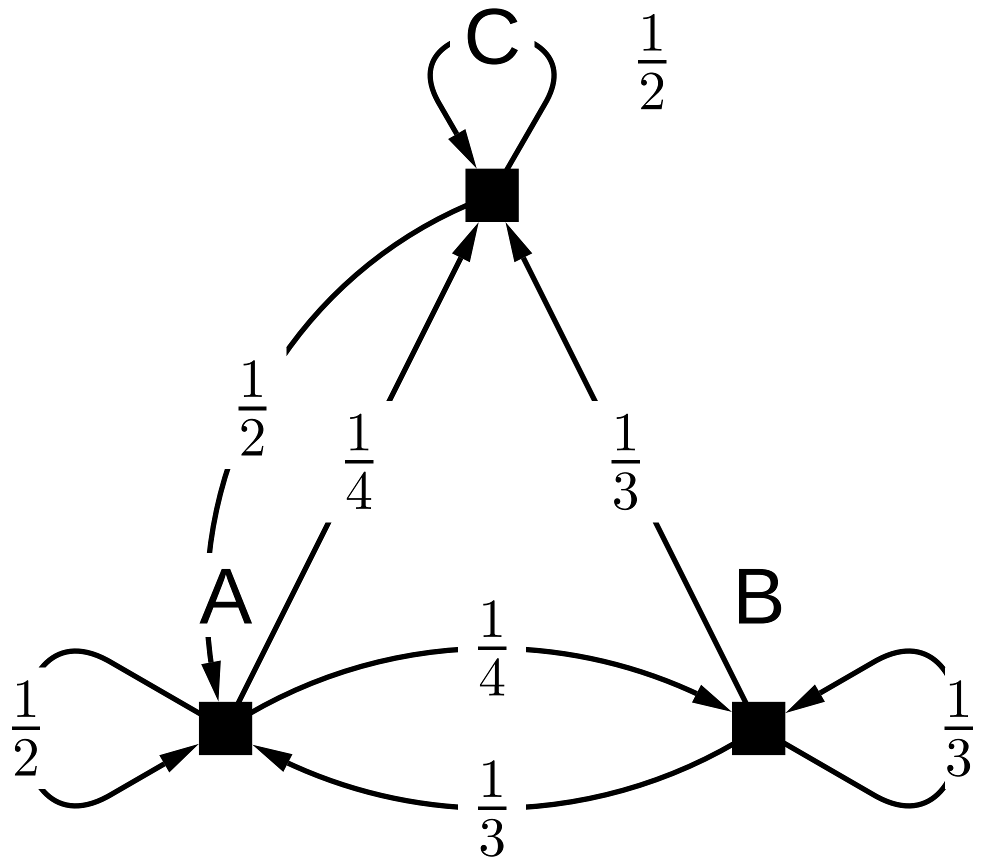

A	B	C
0.36	0.21	0.43

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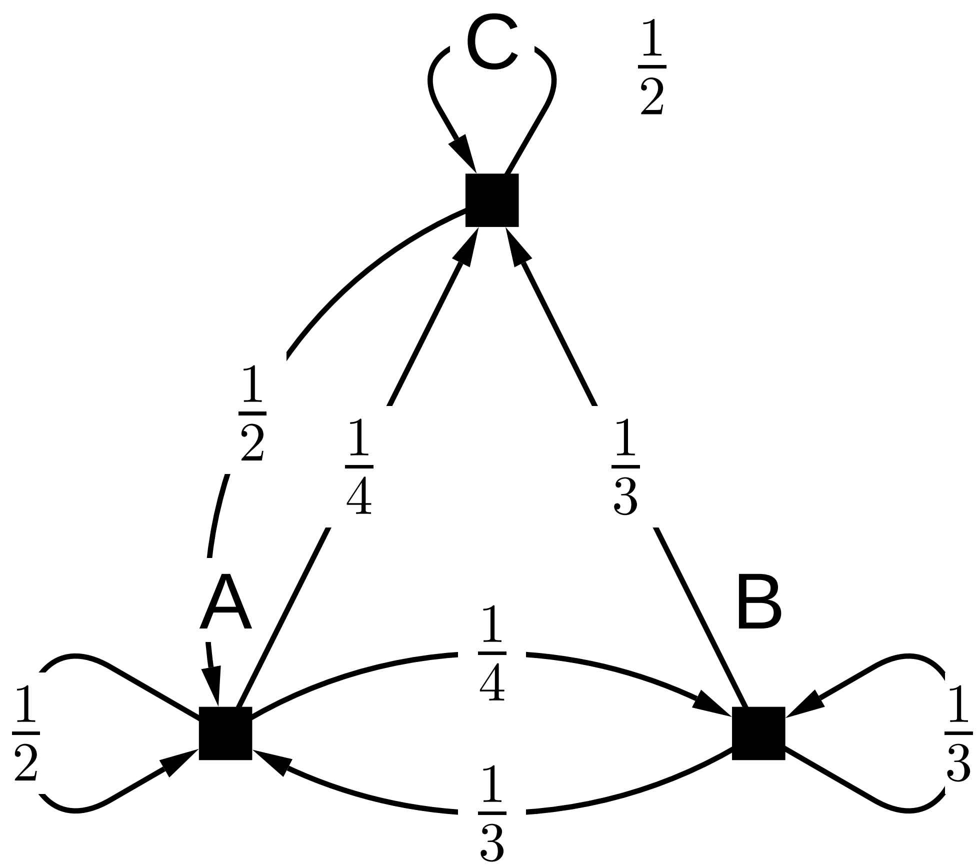

A	B	C
2	3	1

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- Fixing an enumeration of individuals, a ranking algorithm corresponds to a mapping:

$$\begin{array}{ccccc} \text{Pos. Matrix} & \times & \text{Prob. Dist.} & \to & \text{Prob. Dist.} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} & & \begin{bmatrix} 0.25 \\ 0.50 \\ 0.25 \end{bmatrix} & & \begin{bmatrix} 0.36 \\ 0.21 \\ 0.43 \end{bmatrix} \end{array}$$

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- We view this as aggregation of importance judgments to yield a collective ranking.

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$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}/3 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}/3 \rightarrow \begin{bmatrix} \frac{3}{11} \\ \frac{2}{11} \\ \frac{4}{11} \end{bmatrix}$$

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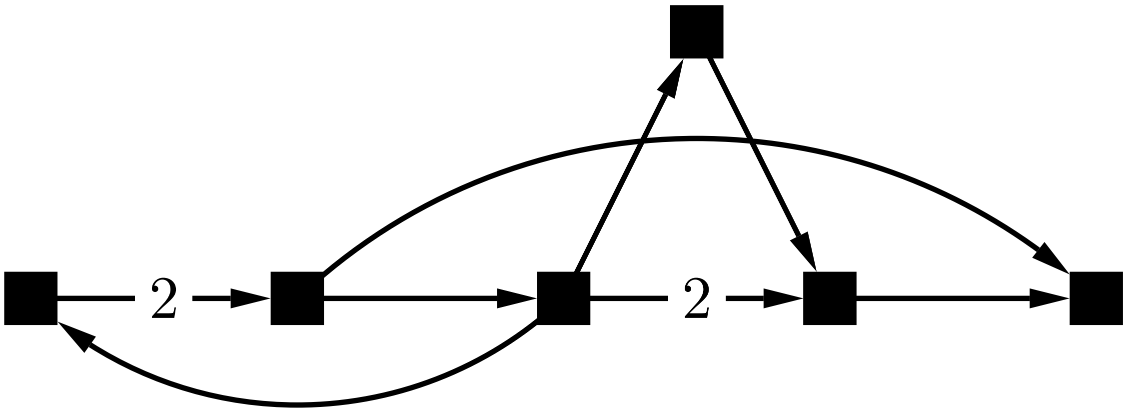
- This generalizes Indegree.

Ranking and Hierarchies



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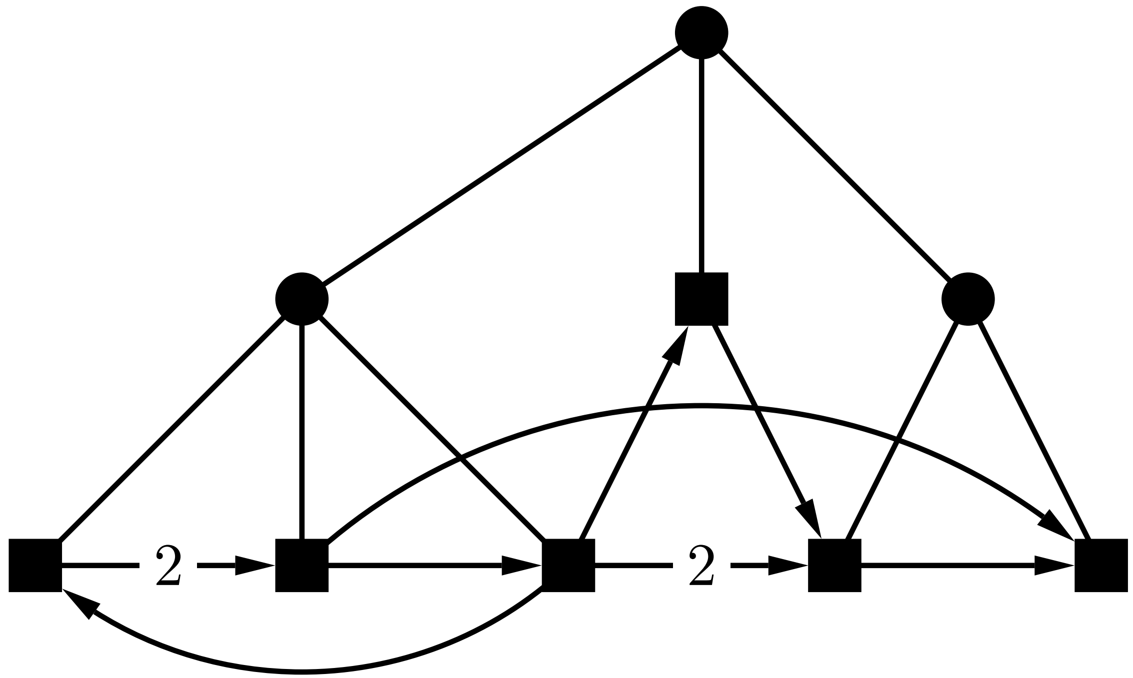

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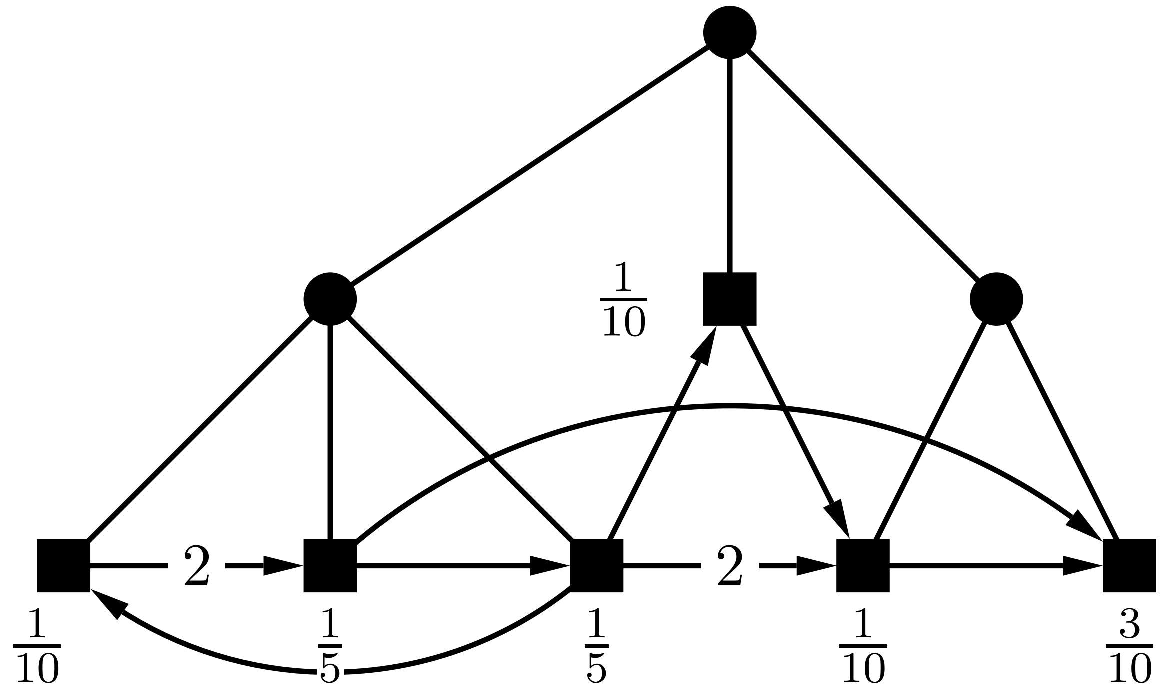

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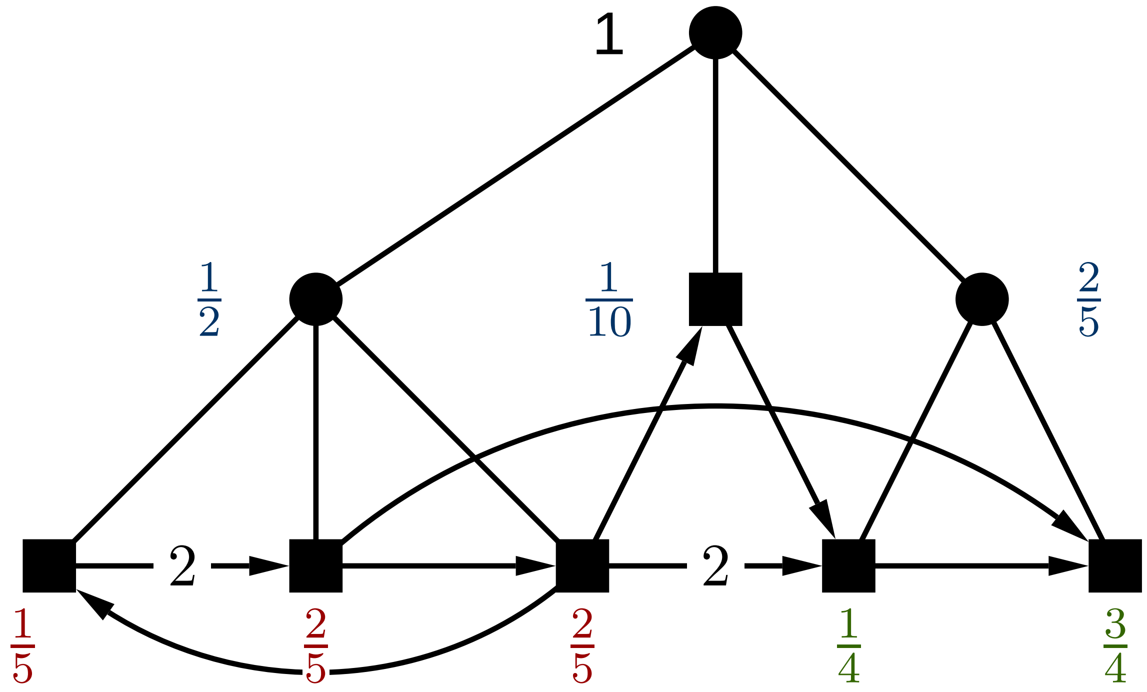

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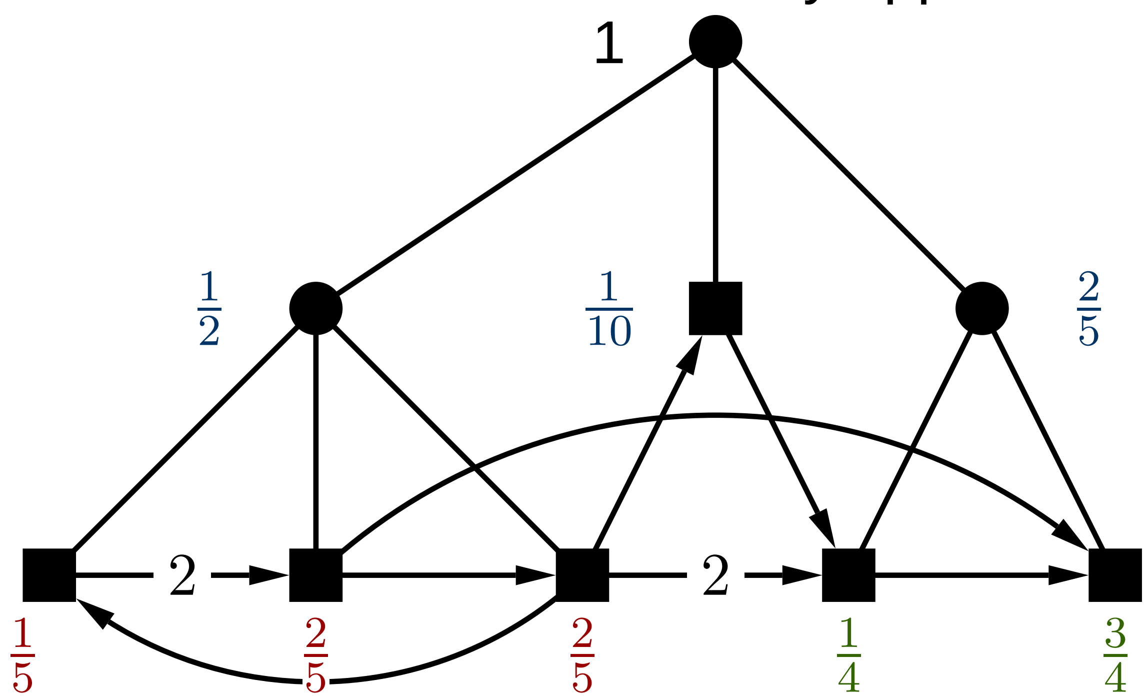

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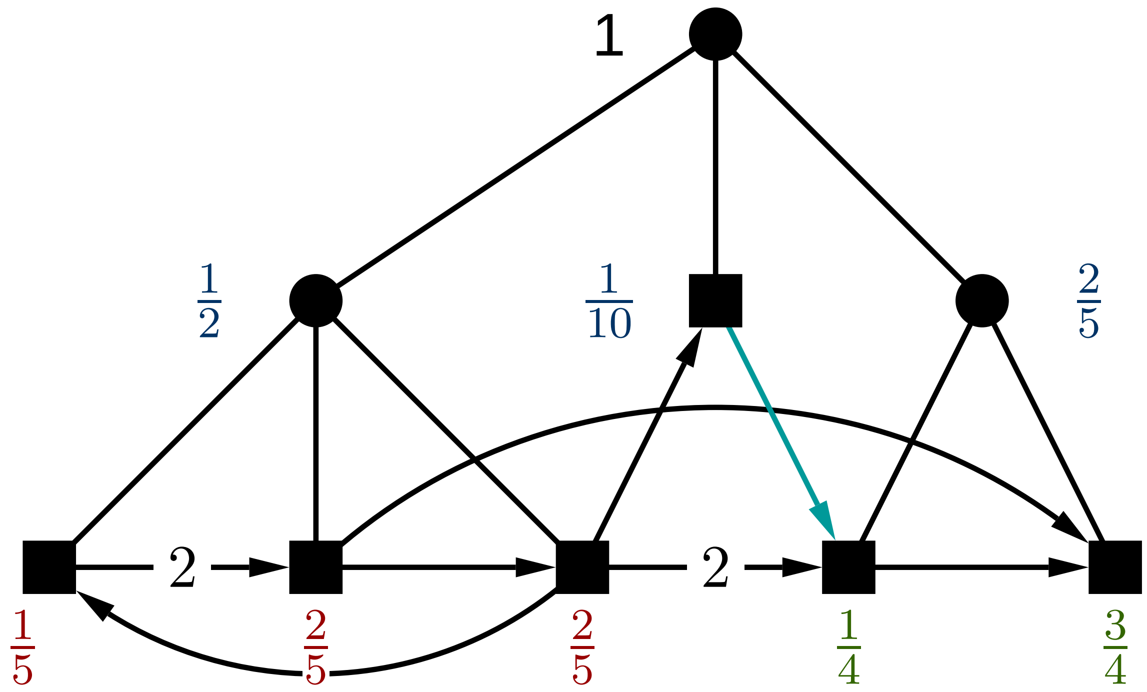

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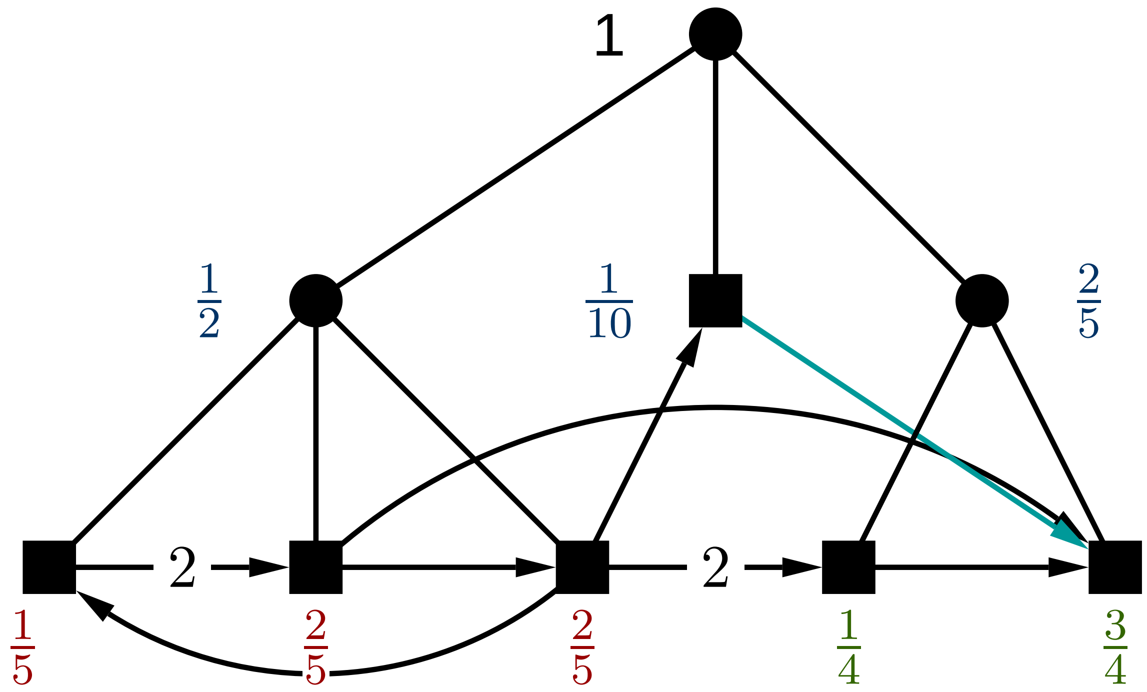

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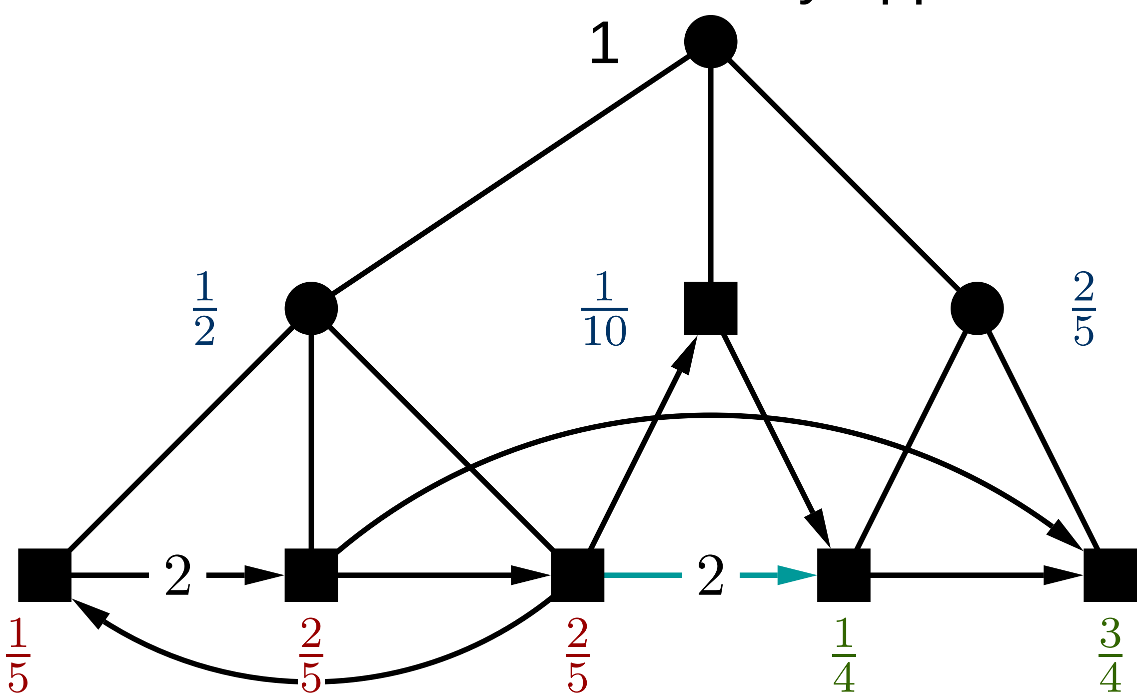

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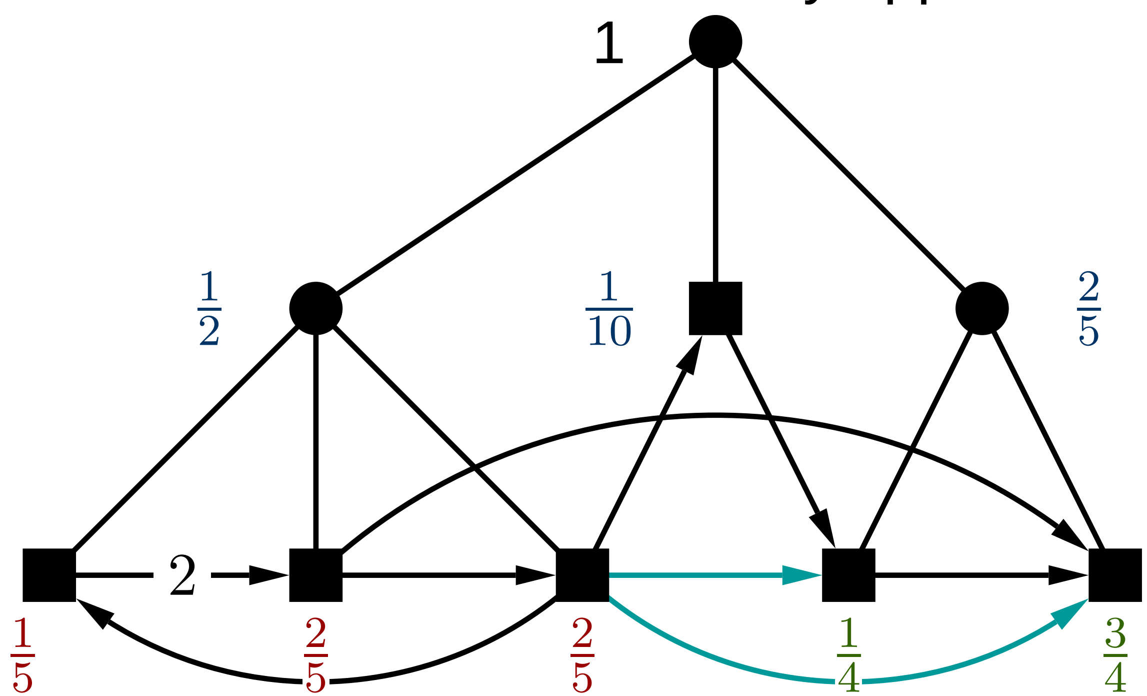

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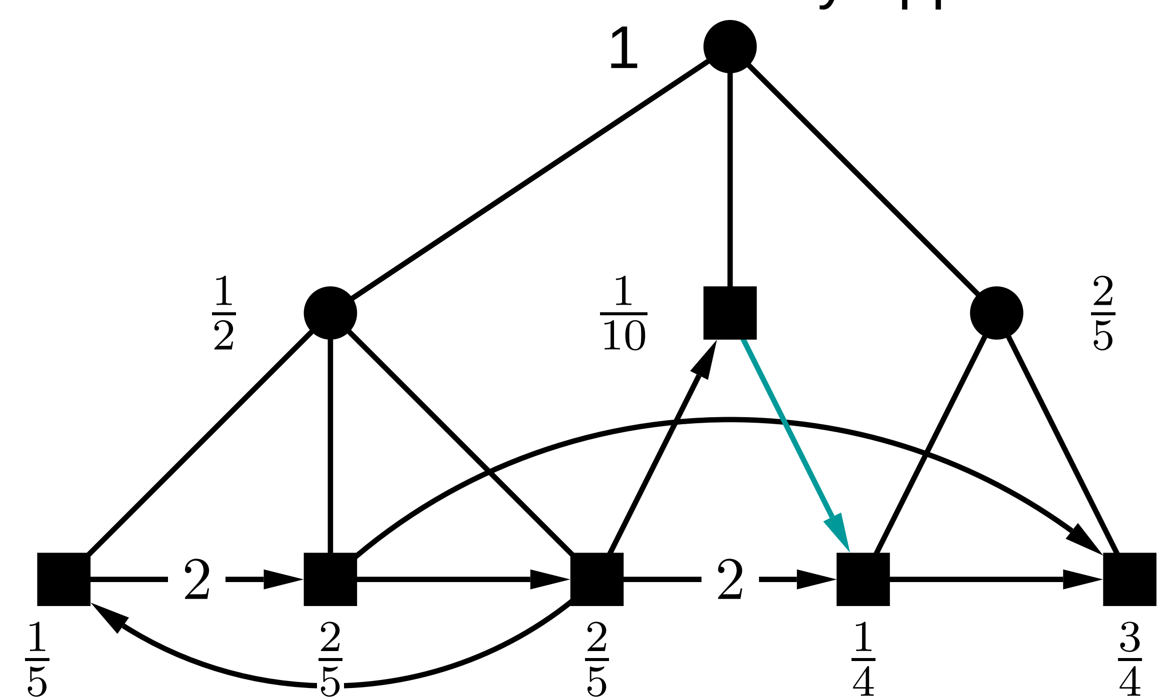

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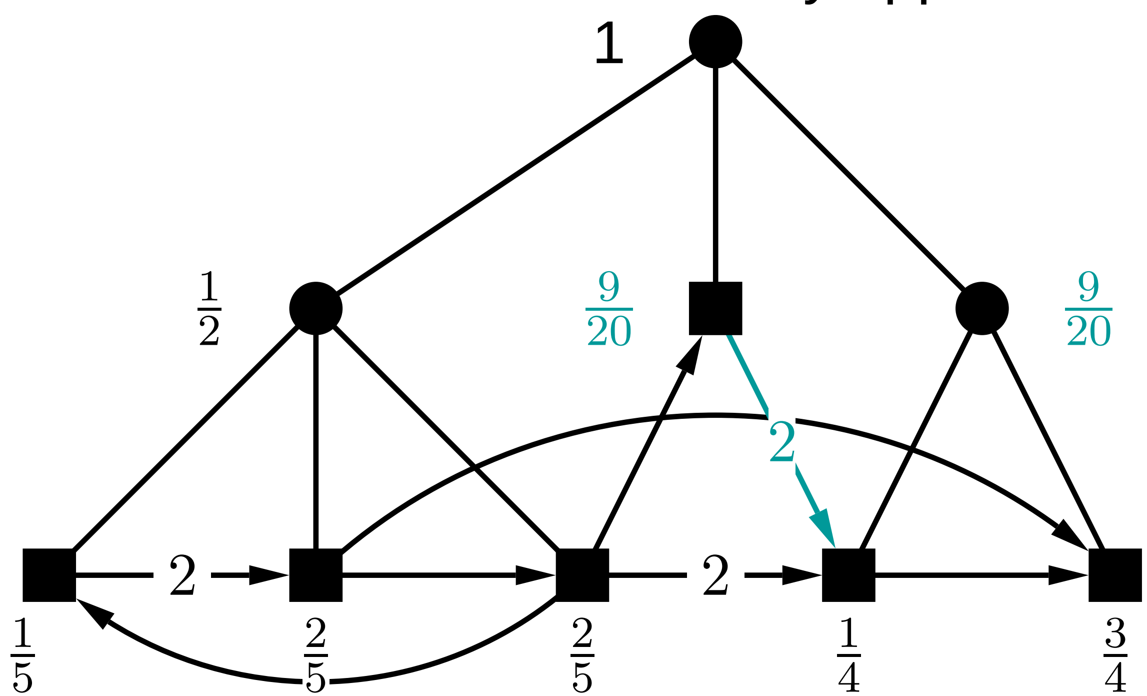

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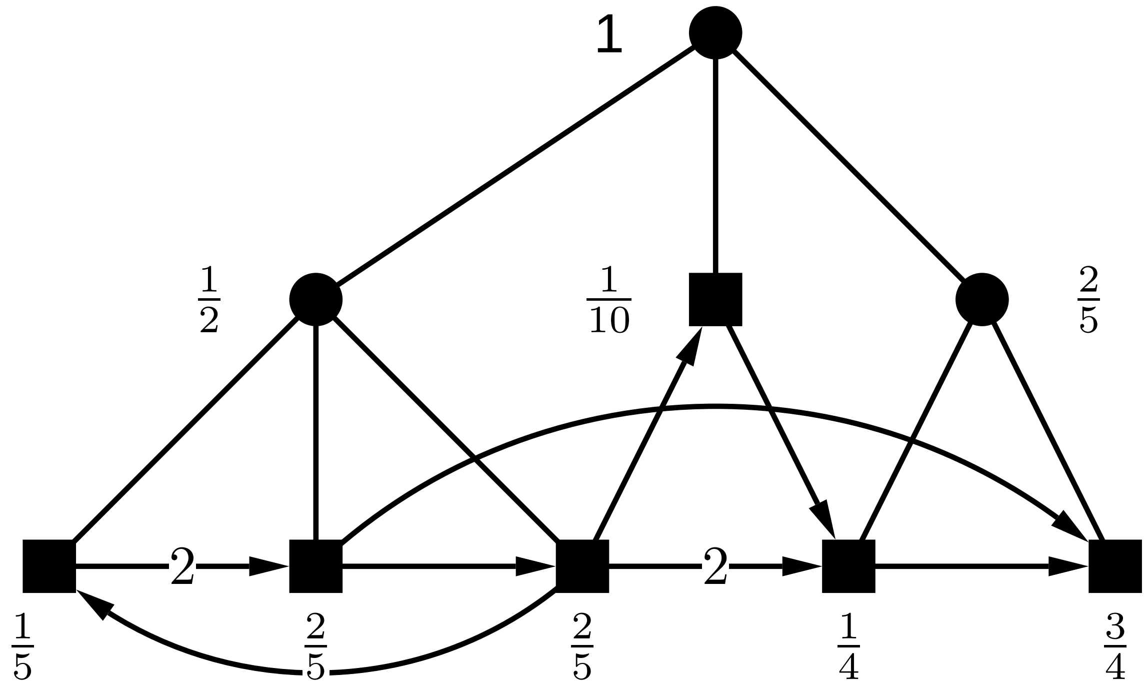
$$\text{Ranking Algorithm} \times \text{Hierarchy} \longrightarrow \text{Ranking Algorithm}$$

satisfying Peer-Review

QuickRank: A Sample Calculation



Apply the Peer-Review Principle to localize the importance judgments.

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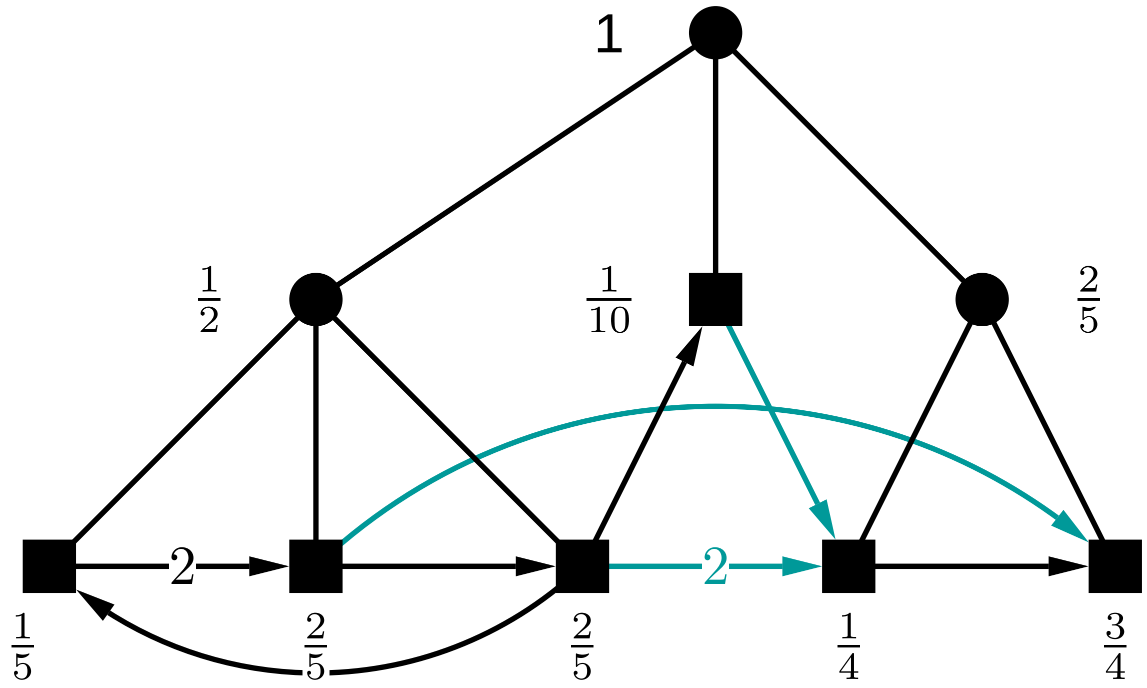

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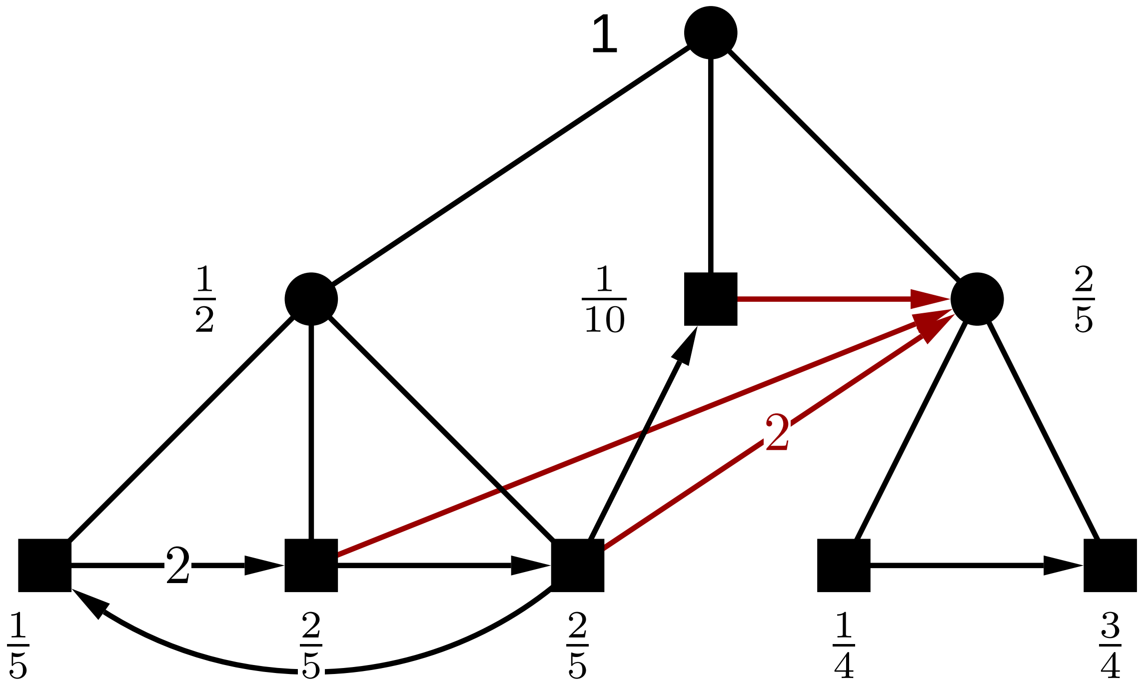

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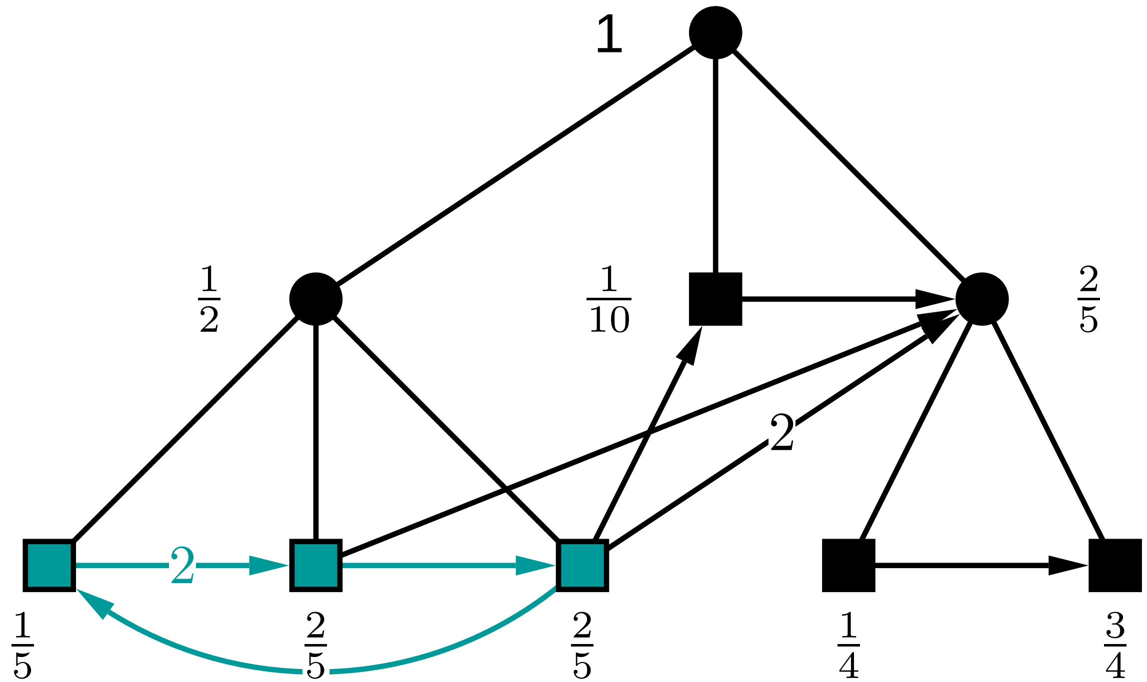

Apply the Peer-Review Principle to focus on a particular subcommunity.

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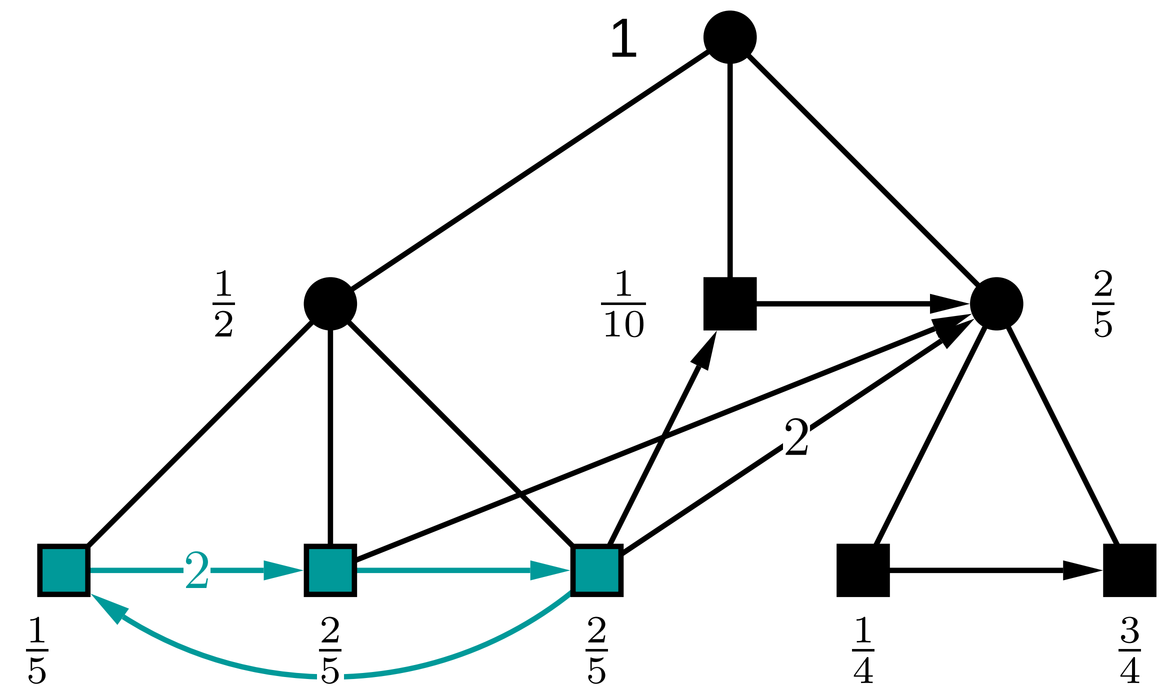

Apply Indegree to rank the subcommunity: $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{5} \\ \frac{2}{5} \\ \frac{2}{5} \end{bmatrix}$

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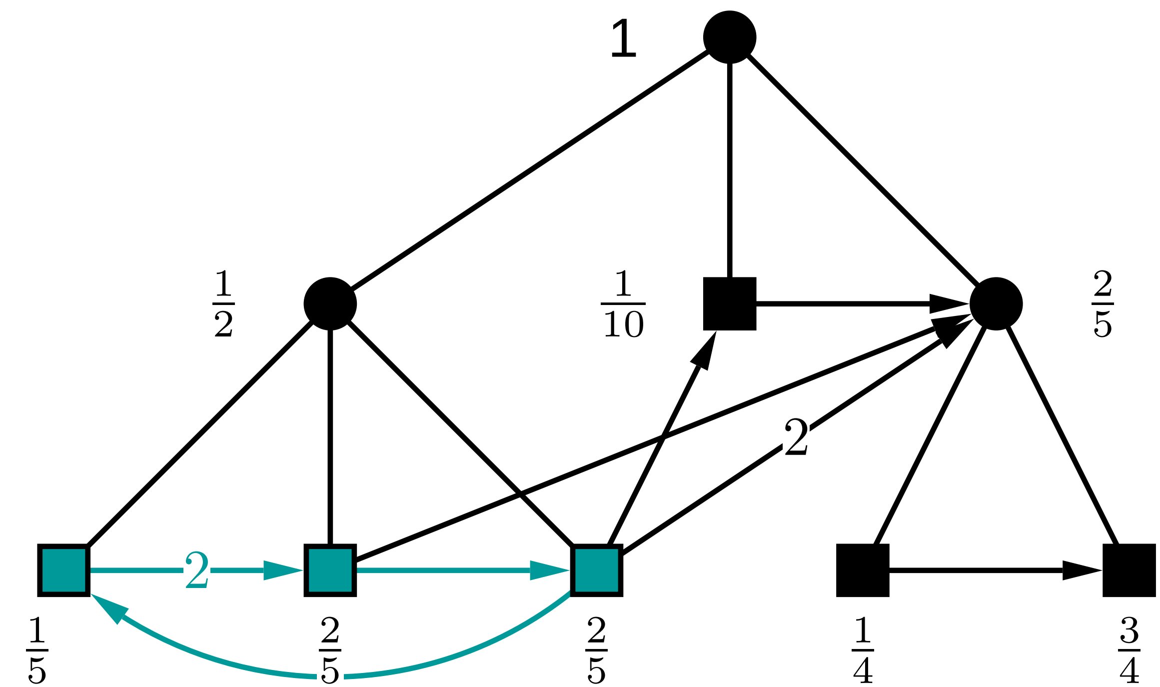

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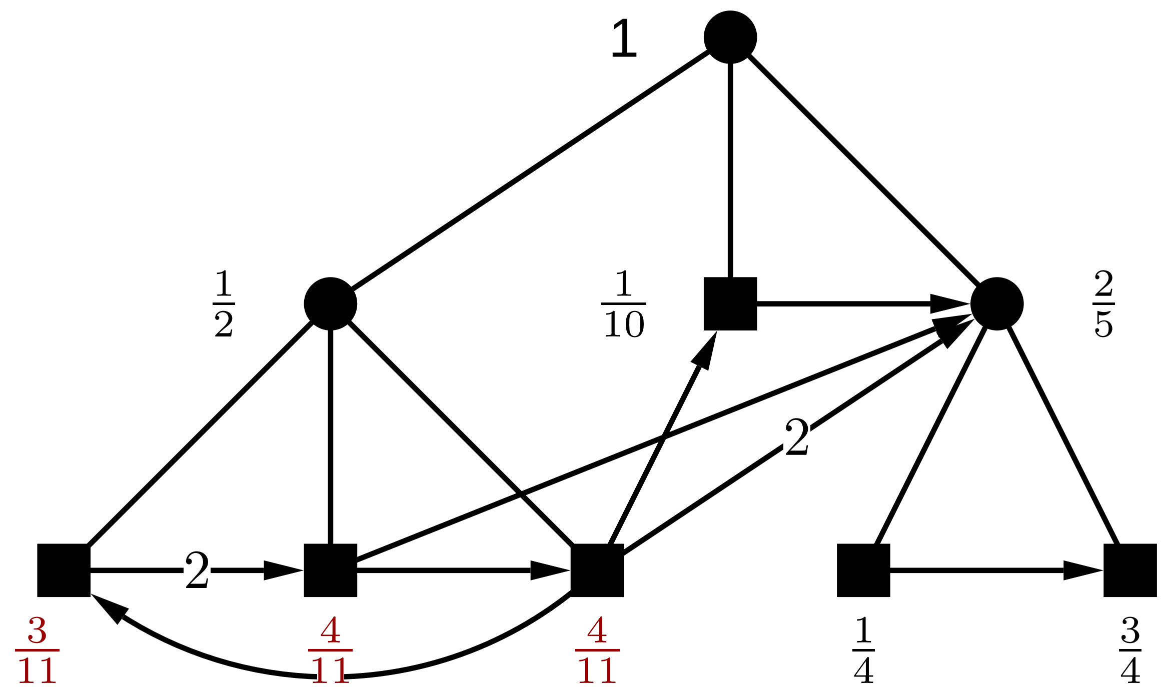

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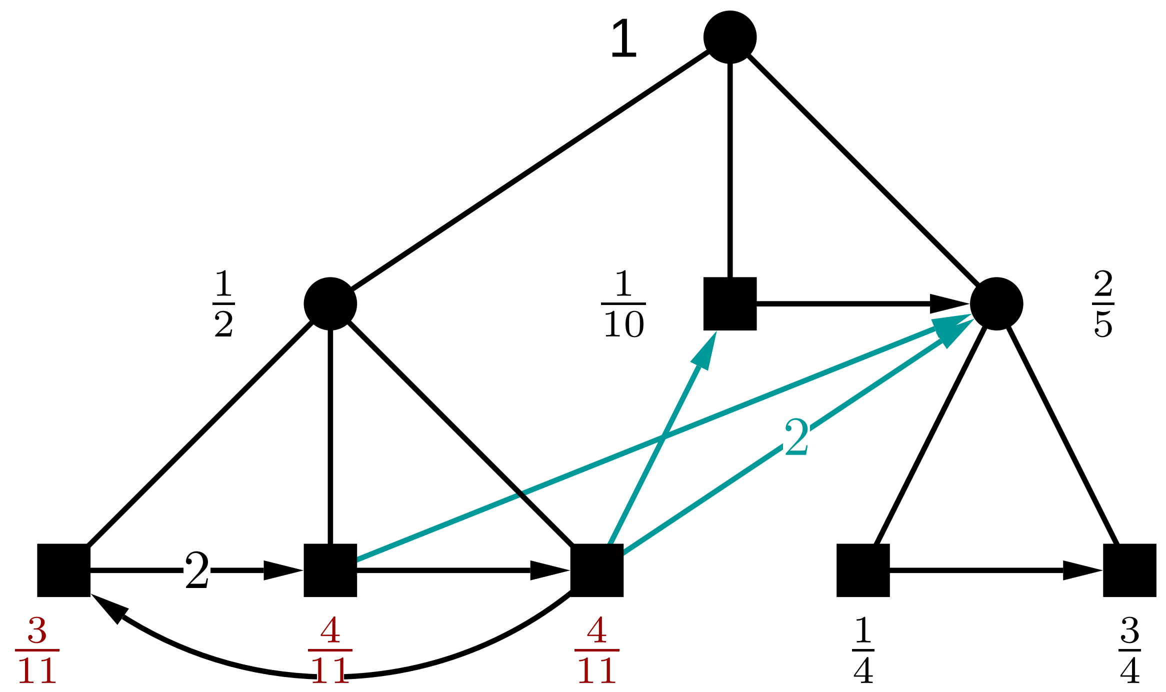

Apply Bonacich's Hypothesis to aggregate non-local judgments.

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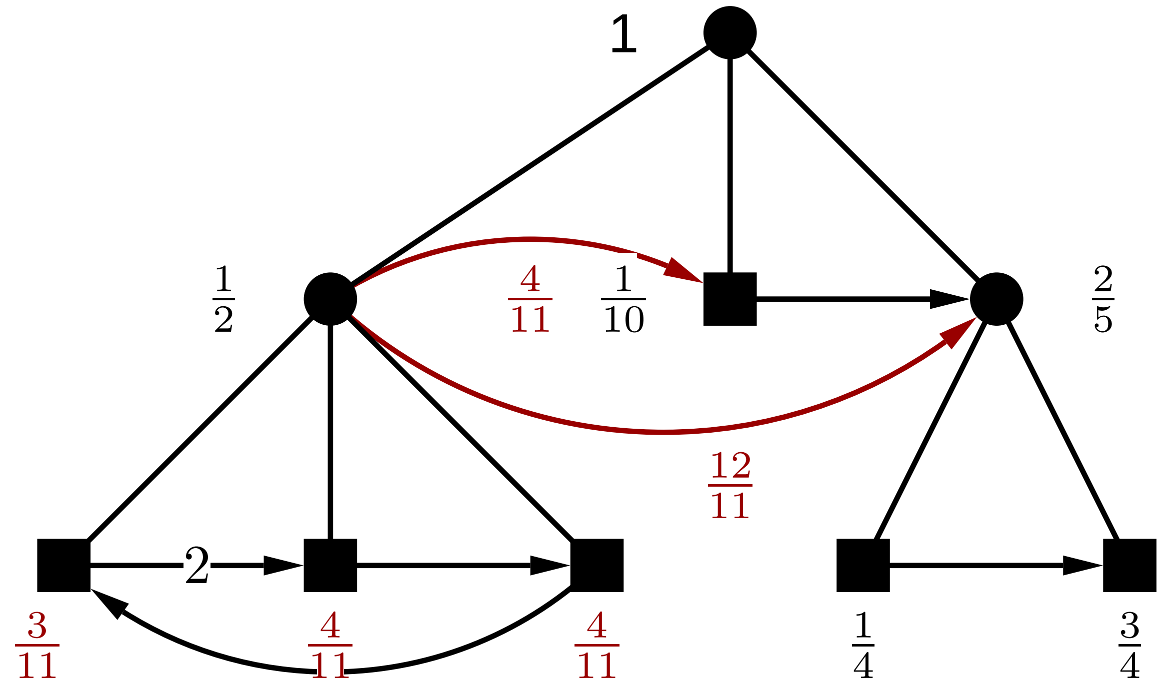

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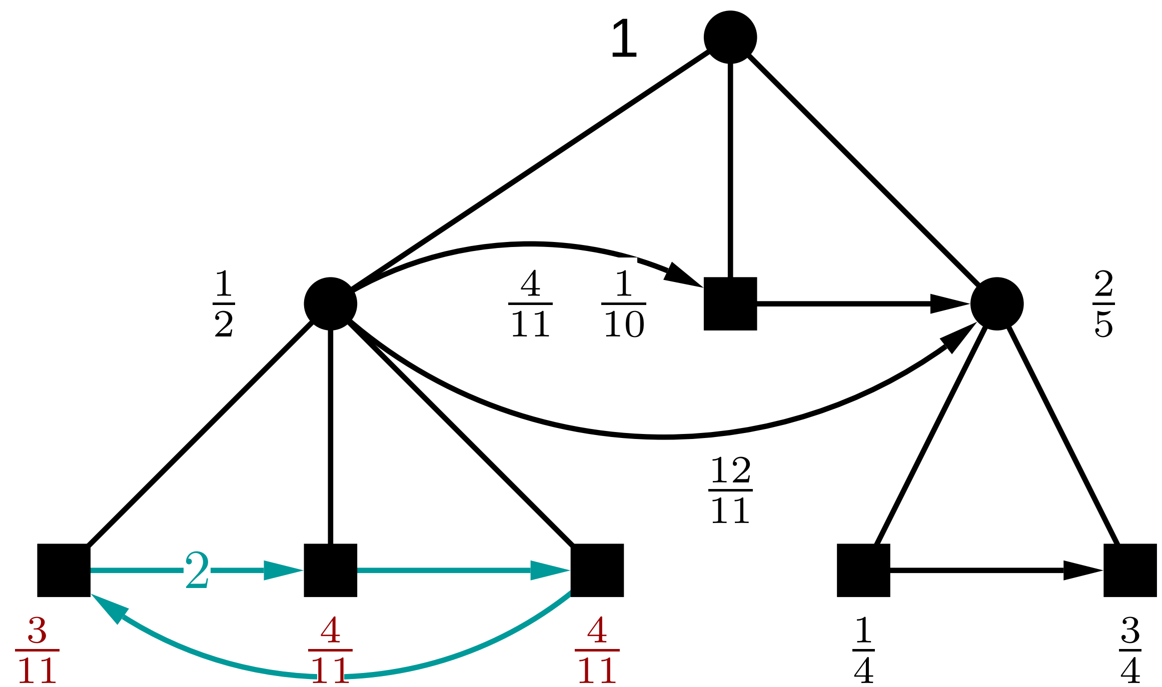

Apply Bonacich's Hypothesis and the Peer-Review Principle to aggregate local judgments: $2 \cdot \frac{3}{11} + \frac{4}{11} + \frac{4}{11}$

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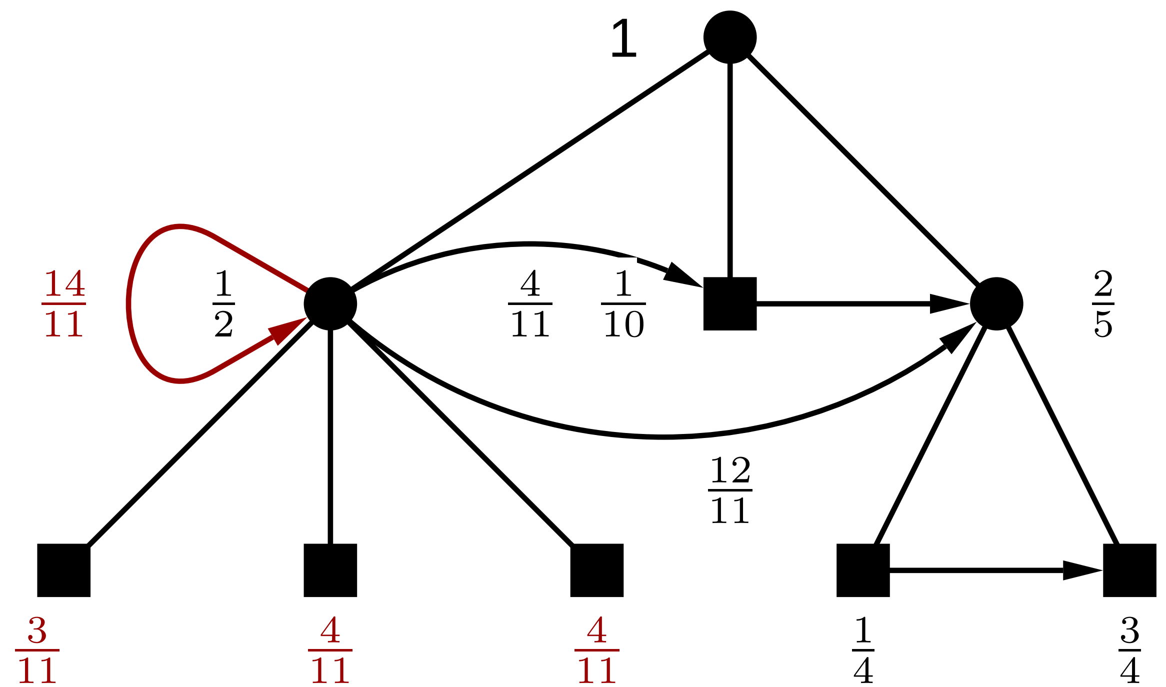

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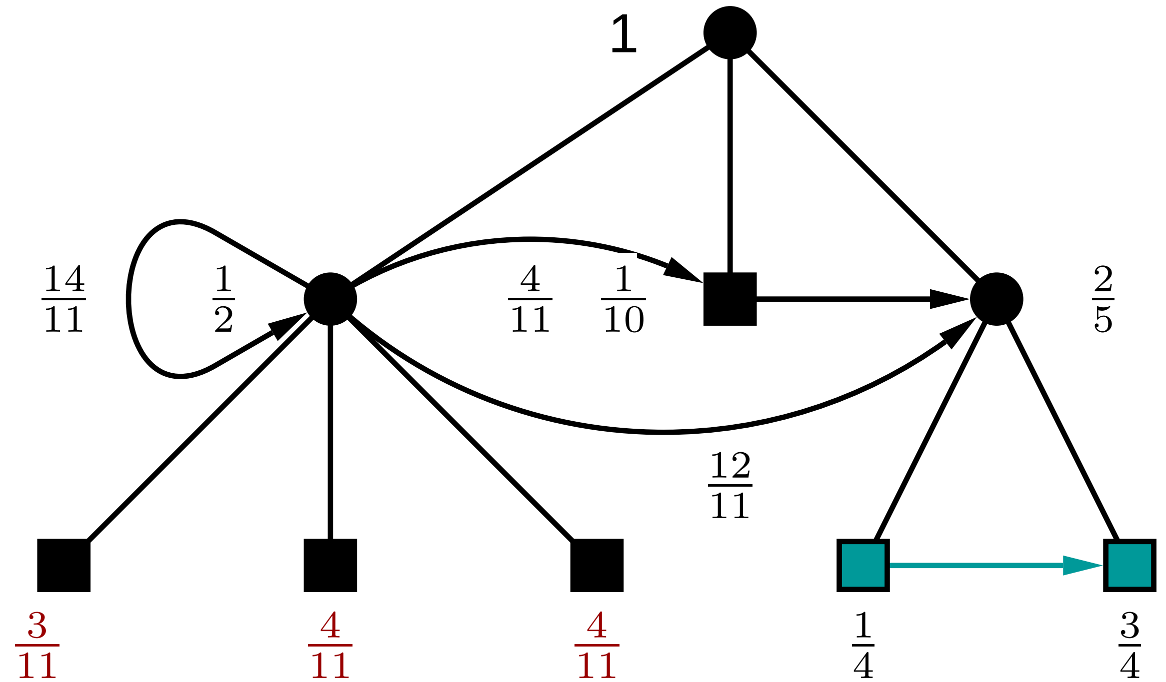

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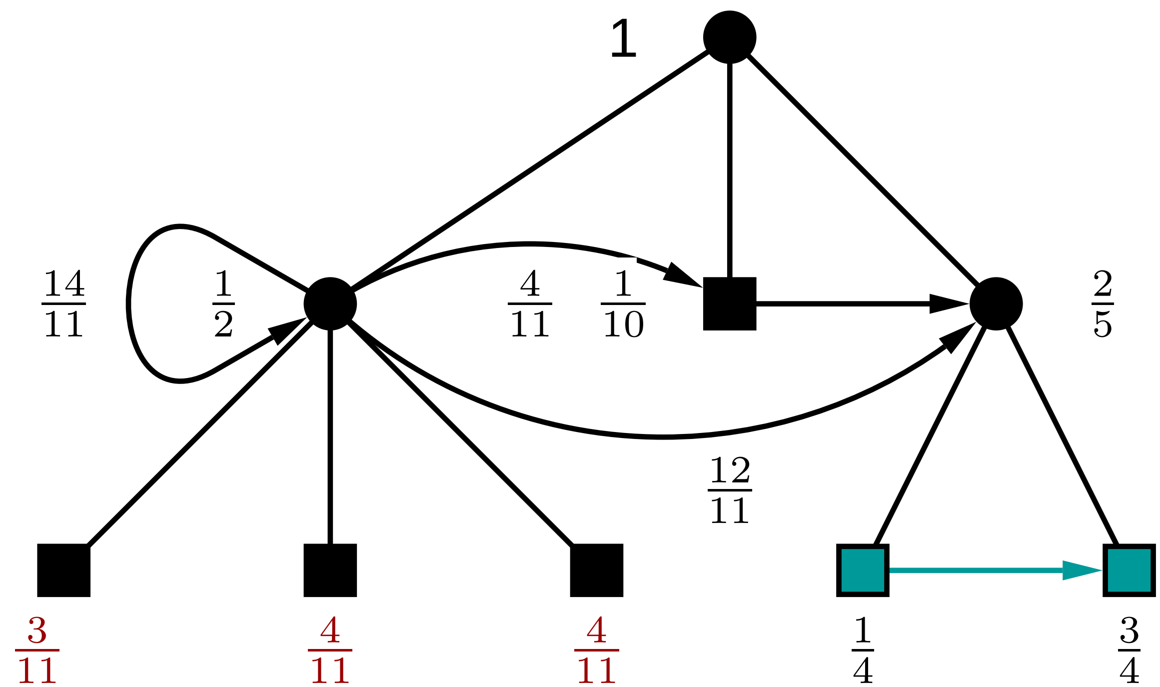

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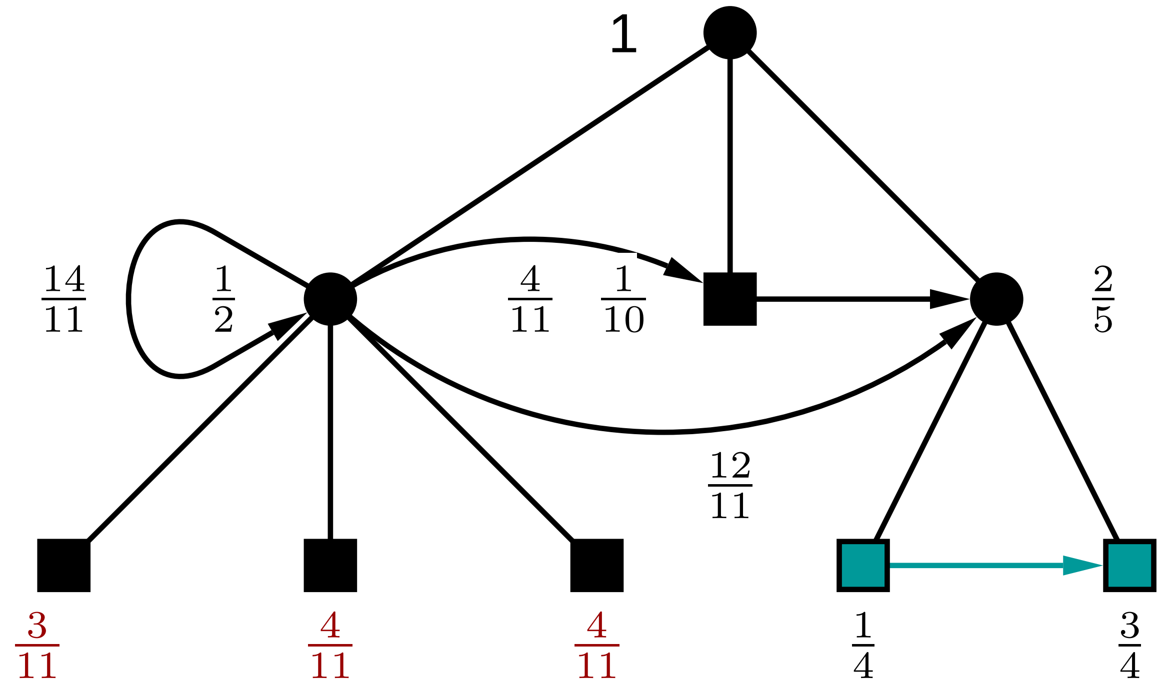

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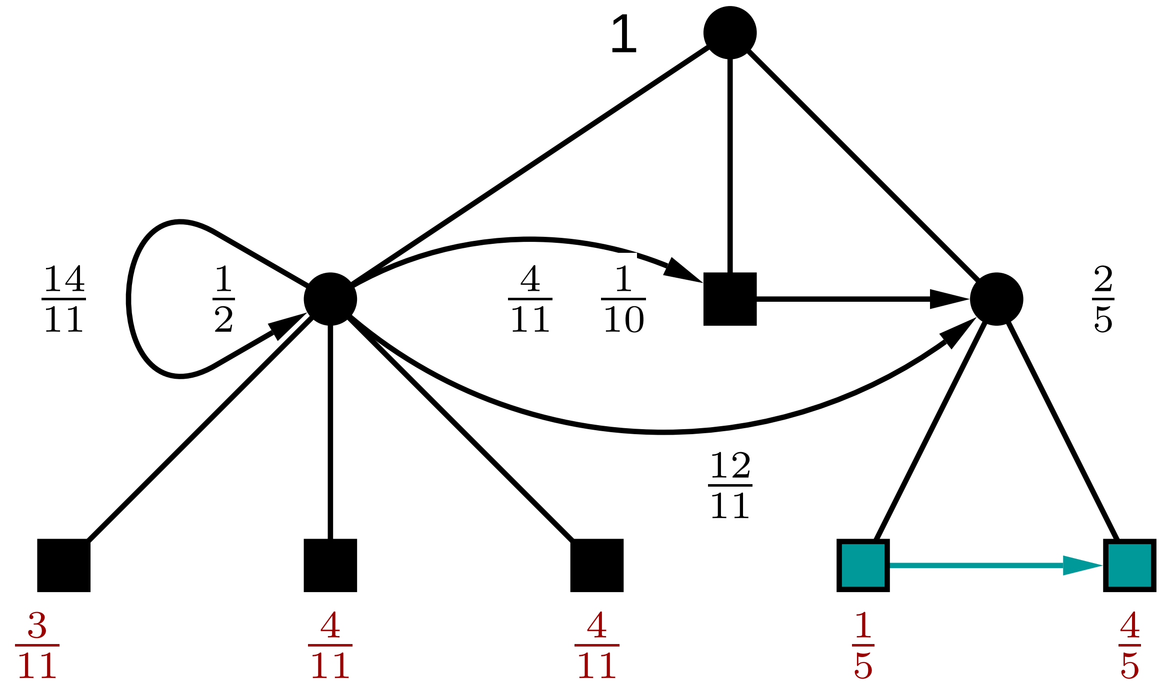

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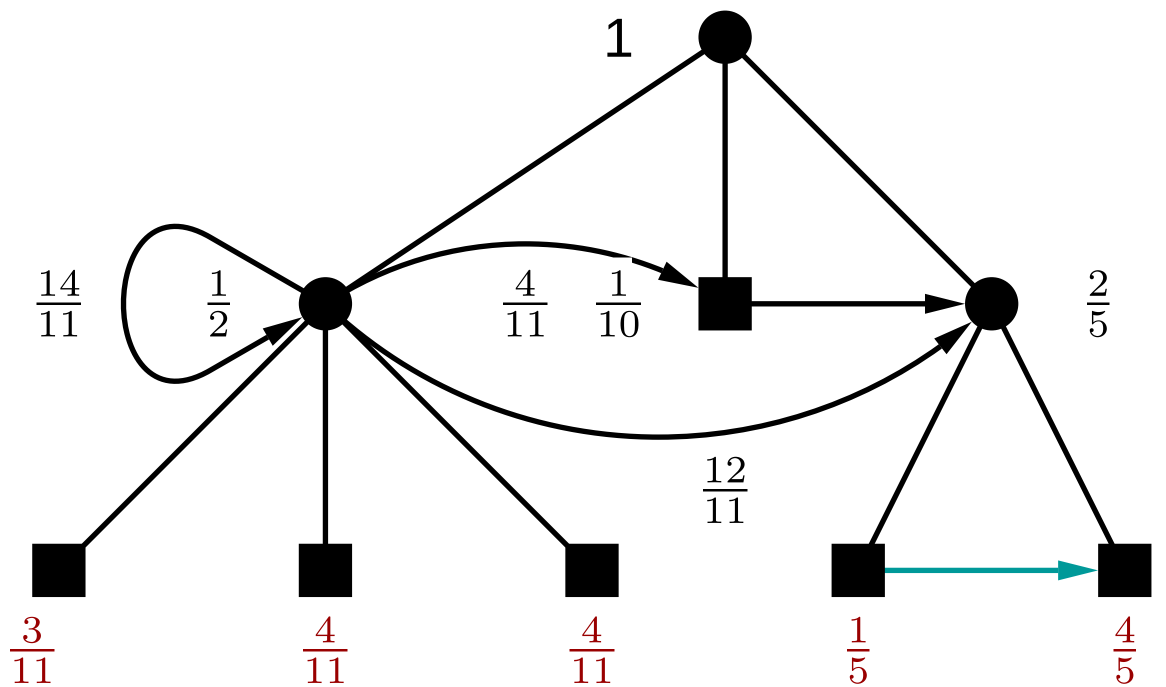

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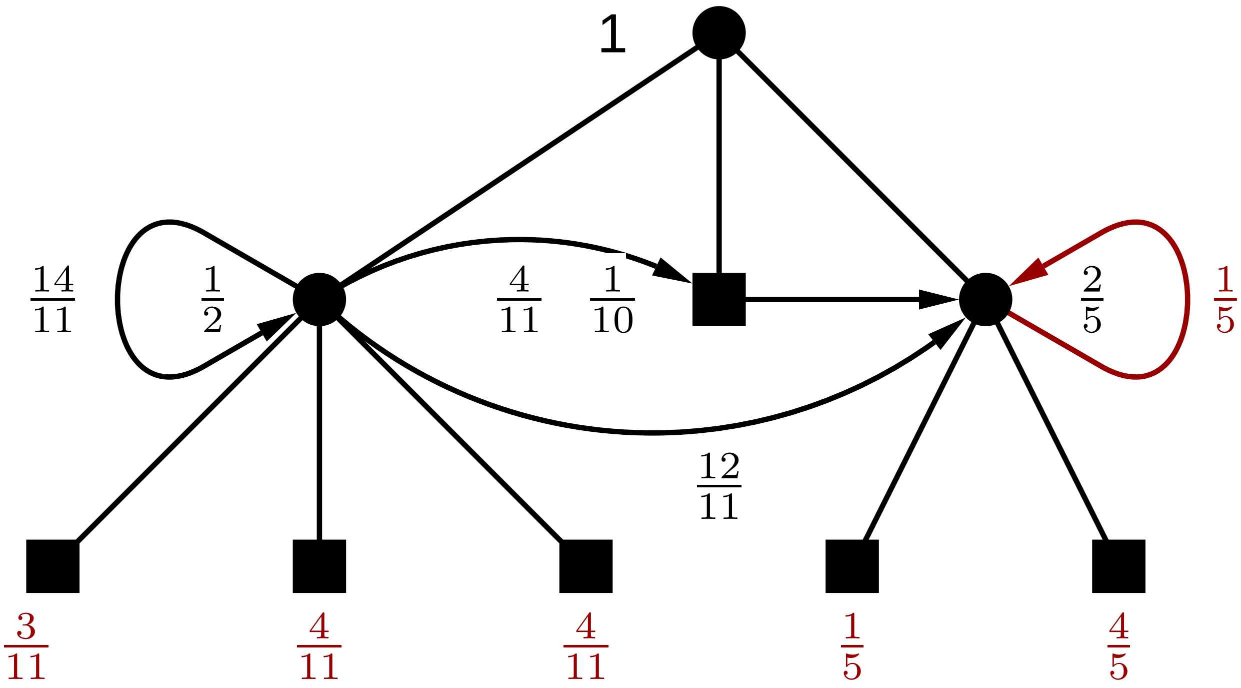

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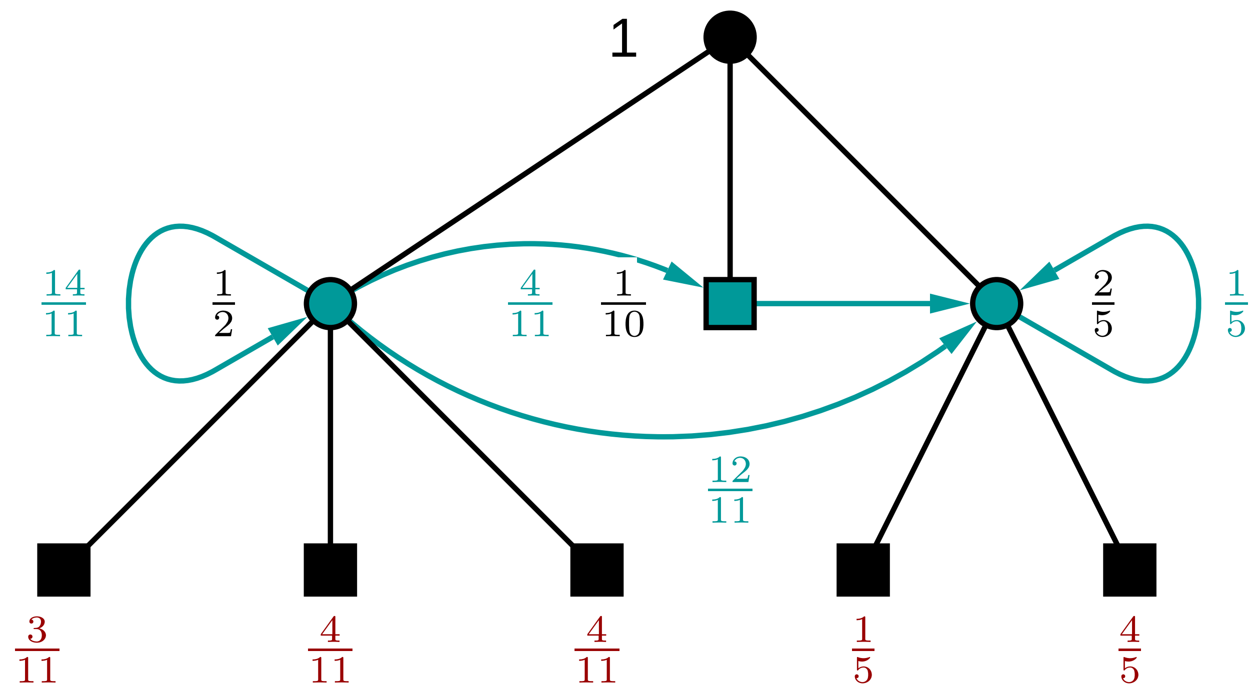

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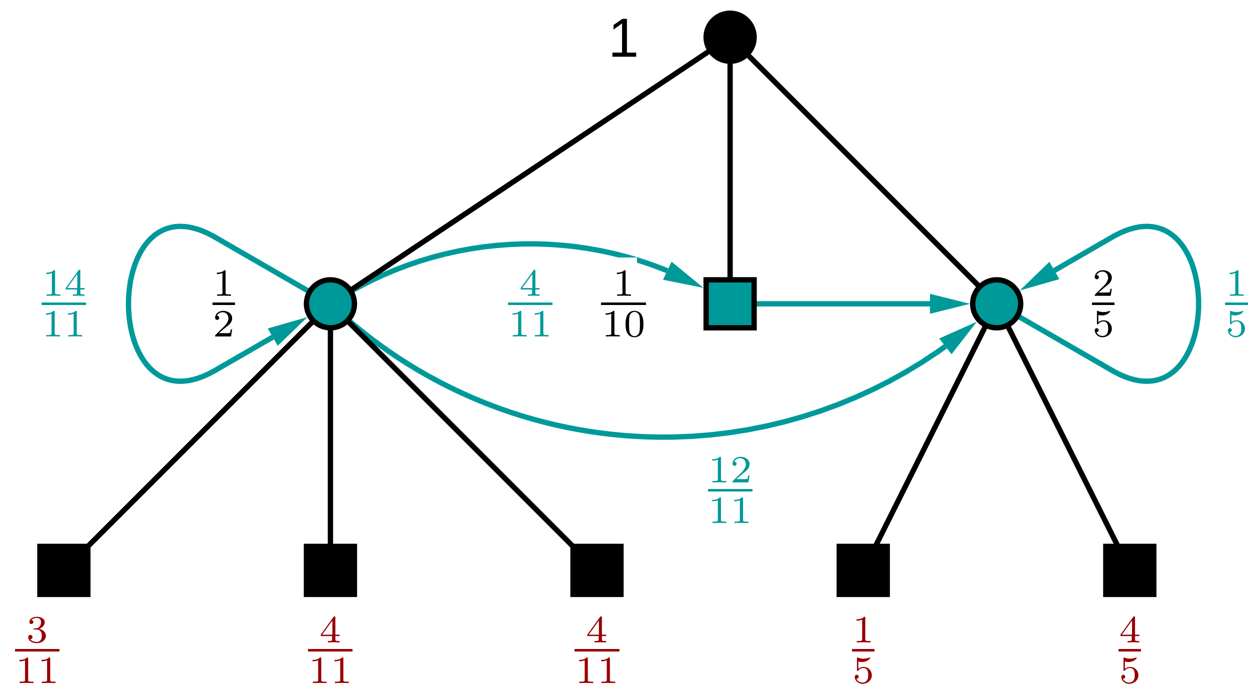

Apply Indegree to rank the subcommunity: $\begin{bmatrix} \frac{14}{11} & 0 & 0 \\ \frac{4}{11} & 1 & 0 \\ \frac{12}{11} & 1 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.1 \\ 0.4 \end{bmatrix}$

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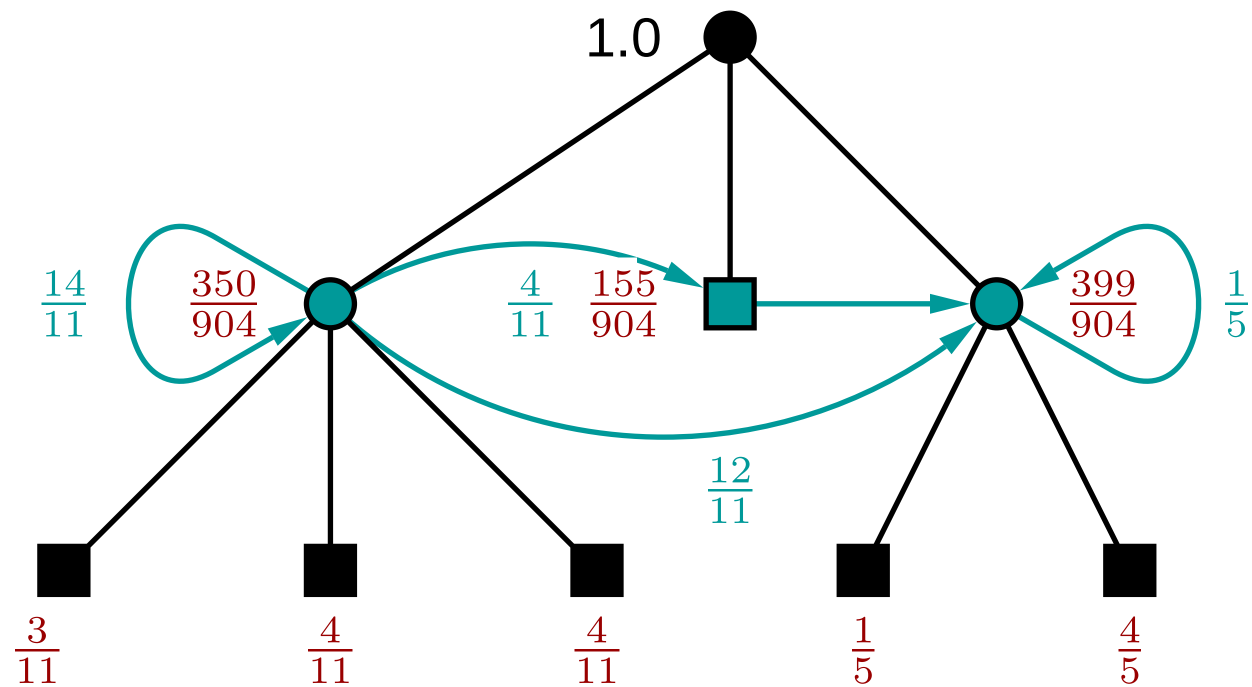
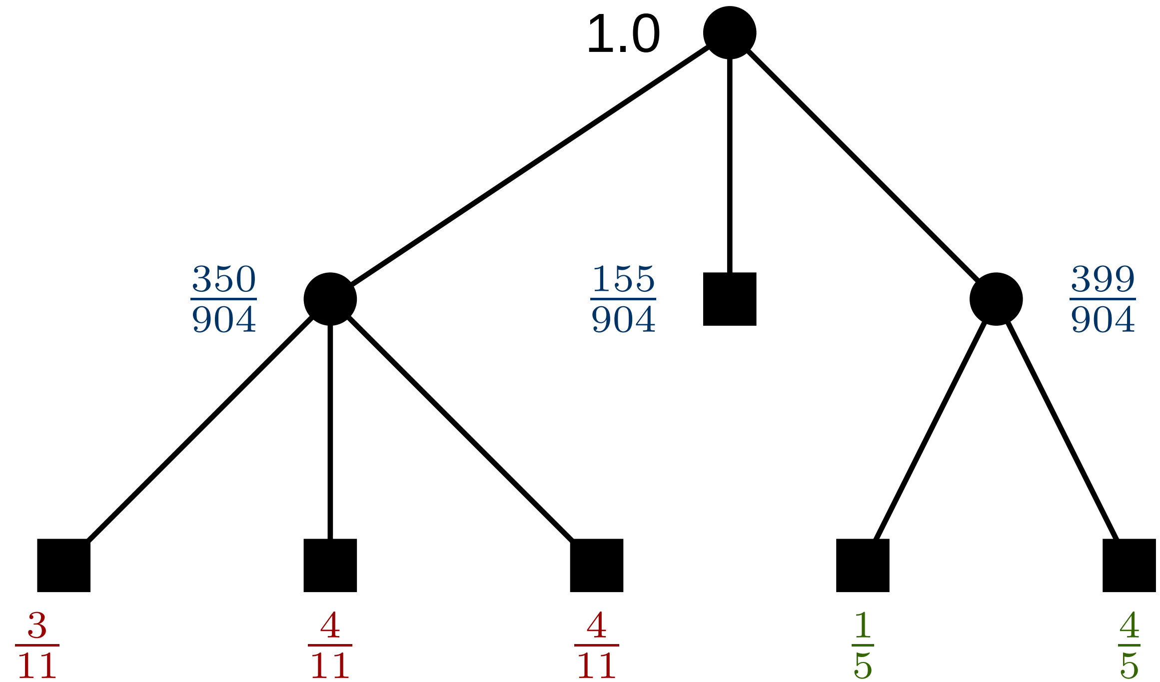


Apply Indegree to rank the subcommunity:

$$\begin{bmatrix} \frac{14}{11} & 0 & 0 \\ \frac{4}{11} & 1 & 0 \\ \frac{12}{11} & 1 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.1 \\ 0.4 \end{bmatrix} = \begin{bmatrix} \frac{7}{11} \\ \frac{31}{110} \\ \frac{399}{550} \end{bmatrix}$$

QuickRank: A Sample Calculation



Collapse rankings.

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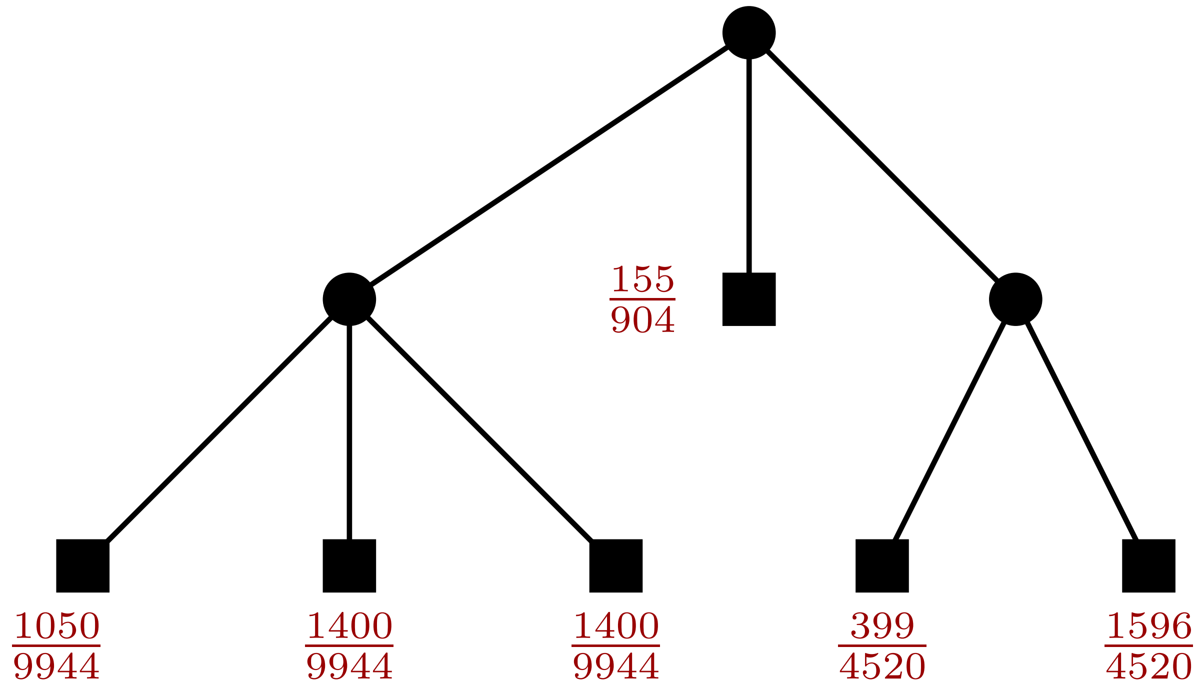

Collapse ranking.

Comparison with TREC 2003

P@10	AP	P@R	α	Alg	Depth
0.124	0.154	0.164	-	csiro03td03	-
0.090	0.099	0.114	0.97	Indegree	1
0.086	0.097	0.105	0.97	Indegree	0
0.082	0.089	0.086	1.00	Lucene	-
0.074	0.088	0.092	0.97	PageRank	0
0.062	0.087	0.078	0.97	PageRank	1
0.092	0.070	0.092	-	meijihilw1	-
0.032	0.023	0.028	-	C2B	-

Comparison with TREC 2004

S@1	S@5	S@10	P@10	R@M	AP	α	Alg	Depth
0.507	0.773	0.893	0.249	0.777	0.179	-	uogWebCAU150	-
0.213	0.680	0.773	0.151	0.590	0.123	0.95	Indegree	1
0.253	0.680	0.813	0.163	0.590	0.120	0.95	Indegree	0
0.333	0.64	0.76	0.199	0.647	0.115	-	MU04web1	-
0.227	0.587	0.707	0.135	0.586	0.093	0.95	PageRank	0
0.080	0.400	0.573	0.109	0.569	0.075	1.00	Lucene	-
0.187	0.533	0.600	0.097	0.582	0.074	0.95	PageRank	1
0.067	0.147	0.173	0.029	0.147	0.018	-	irttil	-

Desdirata for Ranking Algorithms

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  If all individual importance judgments express a common ranking, the posterior ranking should always equal this consensus, independent of prior ranking.

- **Spam Resistance**

  If *sybils* are introduced into the community, but are given 0 prior ranking, the posterior ranking of the original members should be unaffected.

Properties of QuickRank

Preserves:

- consensus
- spam-resistance
- identity

Future Work

- Applications

Examples of HSN's	Individuals	Network	Hierarchy
the Web	web pages	hyperlinks	domains, subdomains, etc.
citation index	publications	references	fields, subfields, etc.
the Enron email DB	employees	emails	organizational chart
Suggestions	?	?	?

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- Generalization to weighted DAG's.

Thanks