The Complexity of Bribery in Elections

Piotr Faliszewski, UR
Edith Hemaspaandra, RIT
Lane A. Hemaspaandra, UR

Context

- Manipulation [BTT89, many others]: Can voter(s) make Hillary a winner by voting strategically?
- Control [BTT92, not many others]: Can the “chair” of an election make Hillary win by addition/suppression/partition of voters/candidates?
- Bribery: Can a briber make Hillary win by spending a certain amount of money to bribe voters?

Setting

1. E-bribery (E an election system):
   - **Given**: A set of candidates C, a set of voters V specified by their preference lists, a preferred candidate p in C, and an integer k.
   - **Question**: Is it possible to make p a winner by changing the preference lists of at most k voters?
2. E-$bribery:
   - Same except that each voter has a price and that k is now a spending limit.
3. E-weighted-bribery and E-weighted-$bribery:
   - Same as 1 and 2, except that voters have weights.

Plurality-$bribery Example

Voters:

1. John > Pat > Hillary > Ralph ($10)
2. John > Pat > Hillary > Ralph ($10)
3. John > Hillary > Ralph > Pat ($10)
4. John > Ralph > Pat > Hillary ($10)
5. Ralph > Hillary > John > Pat ($1)
6. Pat > John > Ralph > Hillary ($2)
7. Hillary > Ralph > John > Pat ($1)

Can we make Hillary a winner by spending at most $11?

Plurality-$bribery Example

Voters:

1. Hillary > Ralph > John > Pat ($10)
   (John > Pat > Hillary > Ralph)
2. John > Pat > Hillary > Ralph ($10)
3. John > Hillary > Ralph > Pat ($10)
4. John > Ralph > Pat > Hillary ($10)
5. Hillary > John > Pat > Ralph ($1)
   (Ralph > Hillary > John > Pat)
6. Pat > John > Ralph > Hillary ($2)
7. Hillary > Ralph > John > Pat ($1)

And Hillary is a winner (for only $11).

Results I: Plurality

	unweighted	weighted
No prices	P (a)	P (b)
$	P (c)	NP-complete (d)

(a) Easy greedy.

(b)/(c) Trickier.

(d) Even for just two candidates. But if weights or prices are unary, we drop down to P.

Election Systems: Scoring Protocols

Scoring Protocols (for m candidates) are described by an integer vector $\alpha = (\alpha_1,\ldots,\alpha_m)$ $(\alpha_1 \geq \ldots \geq \alpha_m \geq 0)$; i'th position gives α_i (times weight of voter) points.The candidate(s) with the most points win.

Particularly important scoring protocols:

- plurality: $\alpha = (1,0,0,\ldots,0)$
- veto: $\alpha = (1,1,\ldots,1,0)$
- k-approval: $\alpha = (1^k,0^{m-k})$
- Borda count: $\alpha = (m-1,m-2,\ldots,0)$.

Results II: Dichotomy Theorems

- Goal: Completely characterize when bribery is hard and easy.
- Theorem A: For each scoring protocol $\alpha = (\alpha_1,\ldots,\alpha_m)$, if $\alpha_1 = \alpha_m$, then α-weighted-$bribery is in P; otherwise, it is NP-complete.
- Theorem B: For each scoring protocol $\alpha = (\alpha_1,\ldots,\alpha_m)$, if $\alpha_2 = \alpha_m$, then α-weighted-bribery is in P; otherwise, it is NP-complete.
- Method: By linking the complexity of bribery to the complexity of (restricted versions of) manipulation using (a modified version of) the manipulation dichotomy result from [HH05] (see also PR 2006 and CS/CLS 2005 combined version).

Results II (continued): Dichotomy Theorems

Comments:

- Theorem B clearly implies that veto-weighted-bribery is NP-complete, even for 3 candidates. However, weights are crucial, since veto-bribery is in P.
- Theorem B was proven by connecting the complexity of manipulation and bribery for scoring protocols. However, this won't work in all settings:
  - approval-bribery is NP-complete, but approval-manipulation is in P.
  - We have designed an artificial voting system for which bribery is easy, but manipulation is NP-complete.

Summary and Conclusions

- We introduced and determined the complexity of bribery for various electoral systems (many more results and all the proofs are available in our 49-page UR TR).
- We related bribery to manipulation to obtain complexity transfer and meta-results.
- We've seen that hardness results are "fragile."

Future Directions

- Other election systems.
- Approximation algorithms.
- Average case behavior.
- Incomplete information.
- More complicated bribe structures.
- Competing bribers.