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The Discursive Dilemma as a Lottery Paradox

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Outline of presentation

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❷ The lottery paradox

❸ Isomorphy of the paradoxes

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❶ Discursive dilemma

Imagine a parliament of three members, voting on the following policy statements.

statement \ voter	v_1	v_2	v_3	majority
A_1	1	0	1	1
A_2	0	1	1	1
$\neg (A_1 \wedge A_2)$	1	1	0	1

If the collective profile is assumed to be closed under conjunction, it is inconsistent.

❶

Impossibility theorem

List and Pettit [2002] prove roughly the following:

The following conditions on judgment aggregation are jointly inconsistent:

(1) the agenda has at least two independent propositions;

(2) voters have universal domain and anonimity;

(3) the voting rule R *satisfies independence and neutrality;*

(4) R *leads to consistent and complete collective opinions.*

There have been many refinements of this result, but we employ this early version as base case.

❷ Lottery paradox

Now imagine that we are considering propositions A_i stating that ticket i will lose in a lottery of three tickets:

statement	prob	accept
A_1	2/3	1
A_2	2/3	1
A_3	2/3	1
$\neg (A_1 \wedge A_2 \wedge A_3)$	1	1

Assuming that we also accept the deductive closure of accepted propositions, the rule $Accept(\varphi)$ *if* $Prob(\varphi) > ½$ gives inconsistent sets of accepted propositions.

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Acceptance rules

Take any value for the threshold t in the rule $Accept(\varphi)$ if $Prob(\varphi) > t$. There is always a sufficiently large lottery to generate inconsistency.

Accept φ *if* $Prob(\varphi) > t$*, unless some formally specified defeater* $\mathrm{D}(\varphi)$ *holds.*

Example: $\mathrm{D}(\varphi)$ *holds if* φ *is included in some minimal inconsistent set of* ψ_i *for which* $Prob(\psi_i) > t$*.*

If we want to maintain that sets of accepted propositions are the deductively closed, we must an acceptance rule of the above kind, which incorporates further conditions.

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Structural acceptance

Douven and Williamson [2006] proved a general result on the lottery paradox concerning acceptance rules with defeaters, of which we use the following corrollary.

> *The following conditions on rational acceptance of propositions* φ *are jointly inconsistent:*
>
> *(1) the possible worlds interpreting the propositions* φ *are equally probable;*
>
> *(2) the acceptance rule defines a structural property;*
>
> *(3) the accepted propositions are consistent, closed under conjunction, and include* φ *with* $Prob(\varphi) < 1$.

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No strictly formal solution

The result on structural acceptance is quite general. It covers all rules that can be defined in (higher order) logic, set theory, etc.

A function f *over propositions* φ *is an automorphism iff*

(1) $f(\varphi \wedge \psi) = f(\varphi) \wedge f(\psi)$*;*

(2) $f(\neg\varphi) = \neg f(\varphi)$*;*

(3) $Prob(\varphi) = Prob(\, f(\varphi)\,)$.

A property A *of propositions* φ *is structural iff it is invariant under all automorphisms* f.

This also means that excluding the acceptance of inconsistent conjunctions of accepted propositions does not help.

❸ Isomorphic paradoxes

Note that we can represent the probability assignment figuring in the lottery paradox by means of equally probable possible worlds.

world / statement	w_1	w_2	w_3	prob	accept
A_1	0	1	1	2/3	1
A_2	1	0	1	2/3	1
A_3	1	1	0	2/3	1
$\neg (A_1 \wedge A_2 \wedge A_3)$	1	1	1	1	1

A representation of a probability assignment over the propositions φ in terms of equiprobable worlds can always be given.

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Worlds are voters

Possible worlds can be considered as anonimous voters. The equal probability of the worlds translates into the equal say that voters have in the collective opinion.

voter / statement	v_1	v_2	v_3	vote	accept
A_1	0	1	1	2/3	1
A_2	1	0	1	2/3	1
A_3	1	1	0	2/3	1
$\neg (A_1 \wedge A_2 \wedge A_3)$	1	1	1	1	1

The acceptance rule $Accept(\varphi)$ *if* $Prob(\varphi) > ½$ then is a majority vote.

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Employing the isomorphy

We want to use the result on rational acceptance rules as an impossibility theorem concerning voting rules. For this we must establish the following translations.

- Acceptance rules $Accept(\varphi)$ translate into aggregation rules $\mathrm{R}(\varphi)$.

- Because possible worlds translate into voters, these voters are essentially characterised by their opinion profile. So voters cannot have identical profiles.

- Relatedly, the voting agenda consists of the powerset of all voters.

Agenda and domain assumptions

Both the interplay between agenda and voters and the fact that the voters are identifiable by their profiles require some further explanation.

- As opposed to other impossibility theorems, the present result employs a fixed profile to derive the inconsistency.

- The voting body may also be divided into equal parties with identifiable profiles. Such voting bodies are called party-wise opinionated.

- The agenda consisting of the powerset of parties may still be unusually rich. On the flip side, this enables us to widen the scope of voting rules significantly.

❹ A new impossibility result

The translation leads to the following generalised impossibility result, in which votes need not be independent.

The following conditions on voting rules are jointly inconsistent:

(1) the agenda allows for party-wise opinionated profiles;

(2) the domain of the voting rule consists of these profiles;

(3) the voting rule satisfies structuralness;

(4) the collective opinion profile is consistent, closed under conjunction, and it includes propositions that are not unanimously accepted.

❹

Relations to other results

The conditions in this theorem relate in rather intricate ways to the conditions of other theorems, and this requires explicit attention.

- The theorem concerns the possibility of consistent collective opinion at specific points in the domain of the voting rule. Thus unanimity need only apply at those points.

- The impossibility result nevertheless reflects back on voting rules in general because we cannot at the onset exclude these specific points.

- The condition of structuralness entails that the voters are anonimous, and further that the voting rule is neutral with respect to types of propositions.

❹

Discussion

We conclude with some considerations on the isomorphy and the impossibility result that can be derived from it.

- The main quality of the present result is that it allows votes on propositions to be interdependent. We can drop this assumption because we assume a rich agenda.

- We may expand voting rules to include non-formal properties of propositions, such as modal notions or semantics. This is perhaps bad for rational acceptance, but fine for voting.

- Given the liveliness of the judgment aggregation literature, there may very well be applications of the isomorphy in opposite direction.