Natural for Optimal Debates: Preliminaries for a Combinatorial Exploration

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Example

Two agents (0,1) holds conflicting views on a given issue. The issue is decided on the basis of 5 arguments : c_1, c_2, c_3, c_4, c_5. Each argument supports either 0 or 1.

$\Rightarrow \langle 0, 0, 1, 1, 0 \rangle \rightsquigarrow$ *winning position for agent 0 (majority rule)*

You want to decide the result but can ask agents to reveal only a limited number of arguments.

$\Rightarrow$ *"Please show me arguments c_1 and c_2".*

$\Rightarrow$ *Would give you the right outcome in this state, but would induce an error if the state was, e.g., $\langle 0, 1, 0, 0, 1 \rangle$.*

Mechanism Design Problem

How should you design the rule so as to minimize the number of mistakes induced by that rule ?

Glazer and Rubinstein's model

Procedural rules —how agents raise arguments :

- agents are sincere ;
- agents cannot reveal arguments in favour of their opponents ;
- different kinds of debate : single speaker, simultaneous, sequential

Persuasion Rule —how the designer makes his decision.
[GR01] study optimal persuasion rules in the 5-arguments case.

Our objective in this paper

Designing rules for the **single-speaker** when the number of arguments involved becomes large.

Basics (i)

States are n-bits vectors , only k bits allowed in the debate.
A persuasion rule is defined as

$$E = \{S_1, S_2, \ldots, S_i, \ldots, S_n\} \text{ s.t. } |S_i| = k$$

$\Rightarrow$ *"I will declare you winner if you can show me all the arguments in set S_1, or in set S_2, etc. "*

Now we will be more precisely interested in **natural rules**, i.e. rules that can easily be stated in natural language.

$\Rightarrow$ *"Give me k adjacent bits"*

$\Rightarrow$ *"Give me k bits of this set"*

$\Rightarrow$ *"Give me this set"*

$\Rightarrow$ *"Give me any set of size k"*

Basics (ii)

error —a state where you would wrongly declare agent x winner/loser on the basis of the rule.

- *minority error* : x is (objectively) loser but declared winner.
- *majority error* : x is (objectively) winner but declared loser.

covering —a rule is covered by a state (for agent x) when

$$\exists S_i \text{ s.t. } \forall j \in S_i, c_j = x$$

Find the rule that minimize the covering of objectively (c_m) while maximizing the covering of majority states (c_M).

$$n_{err} = c_m + (2^{n-1} - c_M)$$

Example

$\Rightarrow$ $E = \{\{1,2\},\{2,3\}\}$
Then (for agent 1), we have :
$\Rightarrow$ *Minority error in states*
$\langle 1,1,0,0,0\rangle$, $\langle 0,1,1,0,0\rangle$
$\Rightarrow$ *Majority error in states*
$\langle 0,1,0,1,1\rangle$, $\langle 1,0,1,0,1\rangle$, $\langle 1,0,1,1,0\rangle$, $\langle 1,0,1,1,1\rangle$
Hence 6 errors overall (out of 2^5) states : 18.75% error ratio.

[GR01] prove that the optimal rule for the single-debate is $E = \{\{1,2\},\{2,3\},\{1,3\},\{4,5\}\}$ (induces 4 errors).

Exploring Persuasion Rules

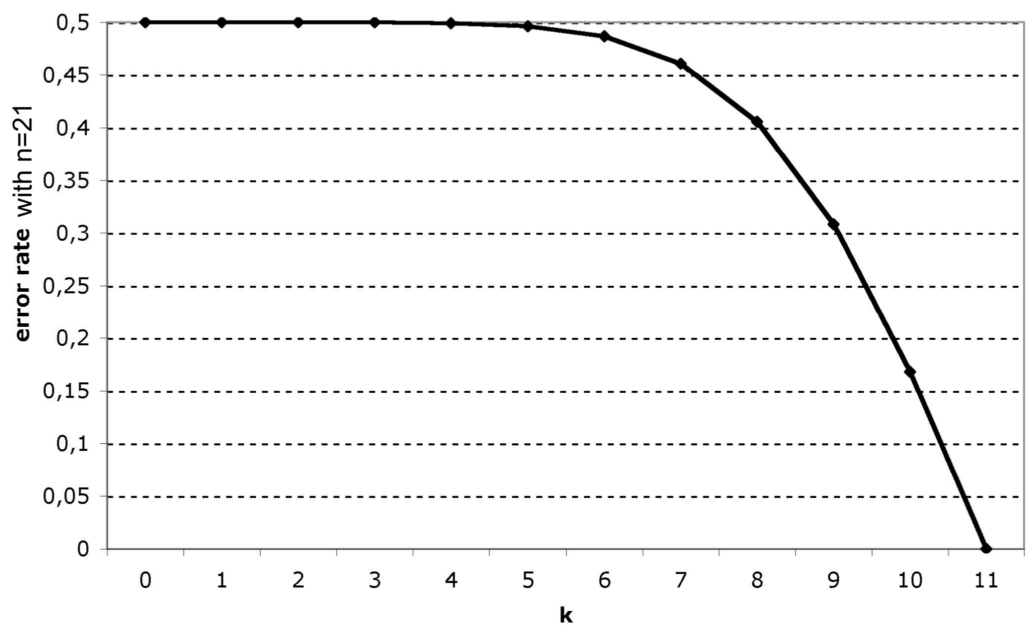
- Two “extremal” rules:
  - Show me *any set of* k argument
    E={ all subsets of [1..n] of size 3 }
  - Show me *this set of* k arguments
    E={ {1,2,3} }
- All rules “in between”
  - Show k arguments such that …

    E={ {1,4,6} , {2,3,4} , … }

Show me *any* set of k args

E={ all subsets of [1..n] of size k }

- $n_{err} = \sum_{t=k}^{\lfloor n/2 \rfloor} \binom{n}{t}$



- **Good error rate only when k≈n/2**

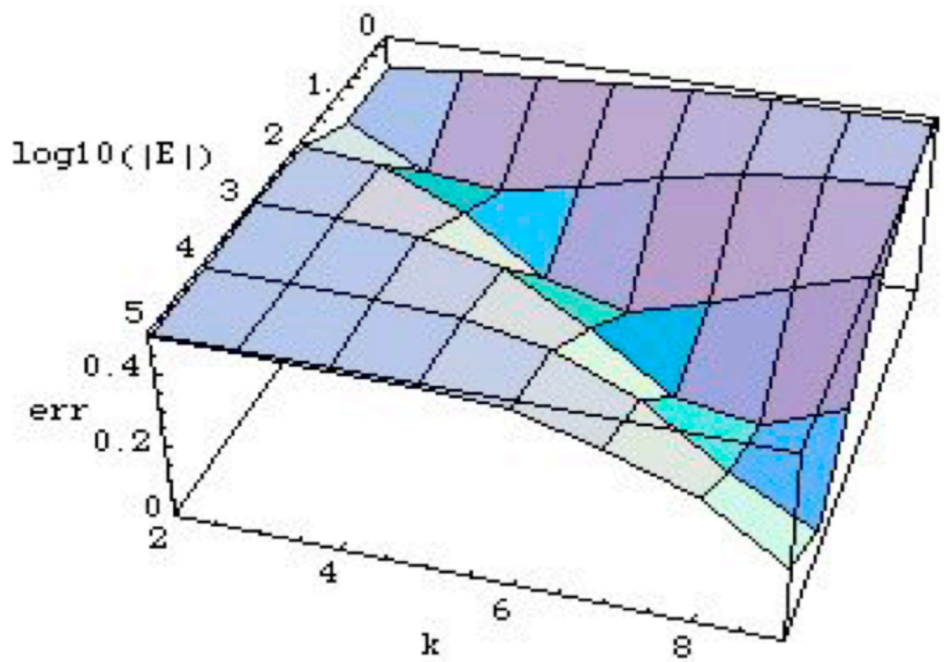
Show me *this set* of k args

E={ {1,2,3,…,k} }

- **Lemma**: error rate increases as k increases
- best rule is E={ {1} }
- Still, has a huge error rate :
  with n=20, n_{err}=40%

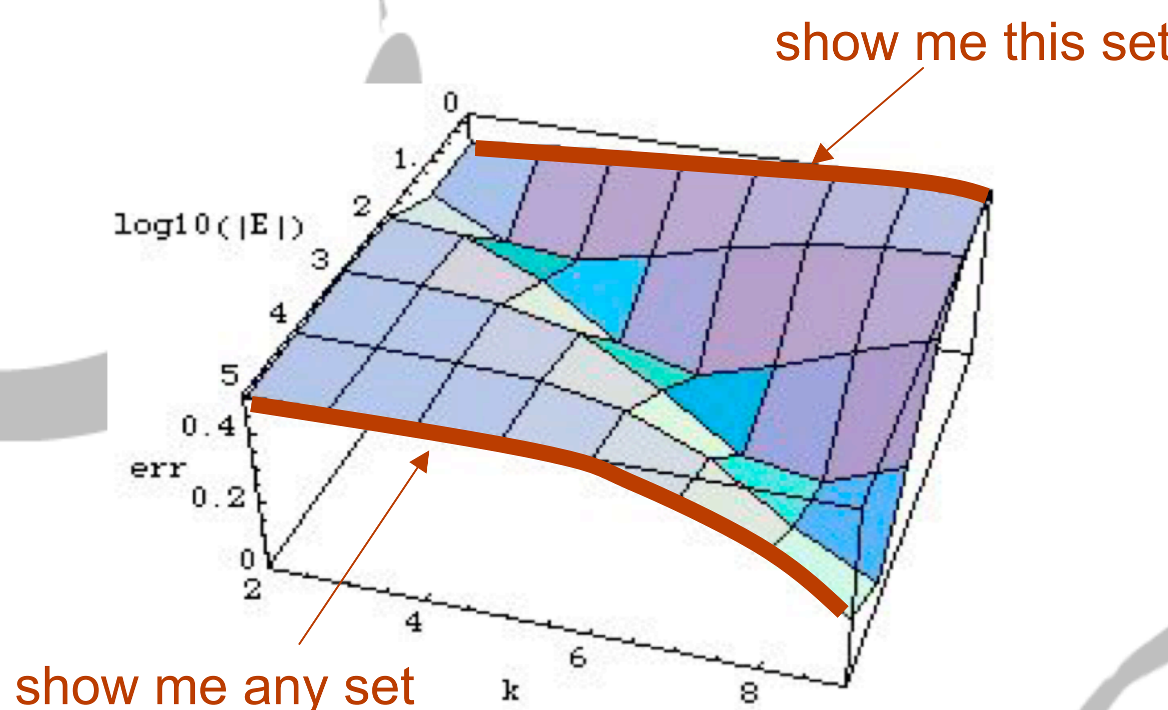
Rules "in between"

- Experiments:
  - Generate E by randomly picking k-subsets of [n]
  - Plot k,log(|E|),error rate



- **Result: best rule is for k$\propto$*ln* |*E*|**

Rules "in between"

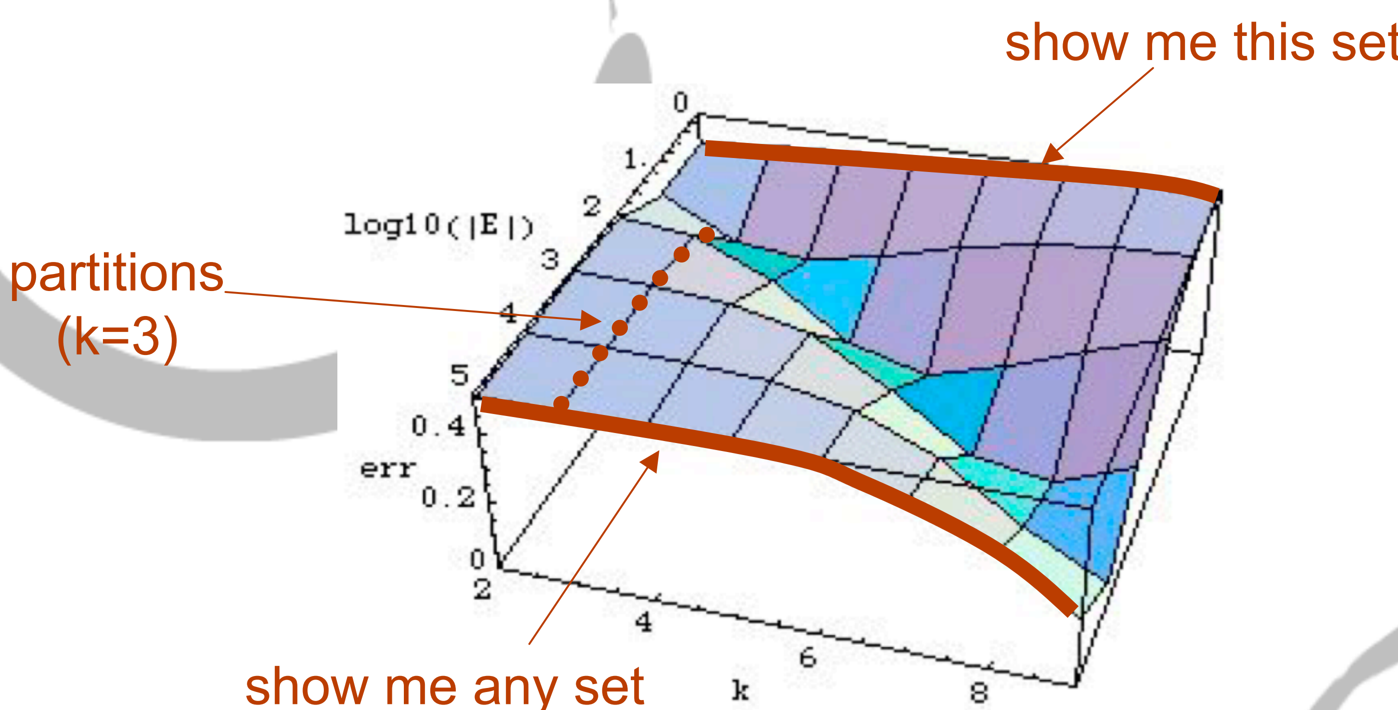


- **Result: best rule is for k∝$\boldsymbol{ln\ |E|}$**

Partitioning rules

- Rule shown to be optimal with n=5 [Rubinstein]
- e.g.
  Let P={{1,2,3},{4,5}}
  show me 2 args in P_1 or in P_2
  E={ {1,2} ,{2,3},{1,3},{4,5} }
- **Observation:** for low values of k,
  as |P| increases, |E| decreases and error rate decreases

Rules "in between"



Conclusion

- Simple rules, quite ineffective in general
- Experimentally, best result is for **k**$\propto \boldsymbol{ln}\ |E|$

Future work, perspectives

- Understand & exploit analytical formula
- "approximately correct" Communication complexity with very low bit-rates